### Abstract

The fuzzy logic constant controller (FLCC) is introduced in this paper. Unlike traditional method, a simplest controller is proposed via fuzzy logic design and Lyapunov direct method. Controllers in traditional method by Lyapunov direct method are always complicated or the functions of errors. We propose a new idea to design constant numbers as controllers, while the constant numbers are decided by the upper bound and the lower bound of the error derivatives. Via fuzzy logic rules, the strength of controllers in our new approach can be adjusted according to the error derivatives. Consequently, the slave system becomes exactly and efficiently synchronized to the trajectory of master system through FLCC. Two examples, Lorenz system and four order Chen–Lee system, are presented to illustrate the effectiveness of the new controllers in chaos generalized synchronization.

### Keywords

Generalized synchronization
Different orders
Fuzzy logic constant controller
FLCC

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### 1. Introduction

Since Pecora and Carroll (1990) proposed the concept of chaotic synchronization, chaos synchronization has become a hot subject in the field of nonlinear science due to its wide-scope potential application in various disciplines. The past two decades has witnessed significant progress on chaotic synchronization in secure communication, life science and information engineering. Typical application of synchronization techniques are in the remote control of nuclear systems and control of distributed power systems. Chaotic synchronization has been investigated extensively. Many kinds of synchronization phenomena and methods have been found in variety of chaotic systems, such as generalized synchronization (Chen, 2009; Chen, Chang, Yan, & Liao, 2008), phase synchronization (Egbert & Momani, 2008; Li, Chen, & Huang, 2008), lag synchronization (Chen, Chen, & Gu, 2007; Ge & Lin, 2007), inverse synchronization (Chang, Li, & Lin, 2009; Li, 2009), partially synchronization (Chen & Chen, 2009; Wu & Chen, 2009), projective synchronization (Chen, 2005; Hu, Yang, Xu, & Guo, 2008), Q–S synchronization (Hu & Xu, 2008; Wang & Chen, 2006), etc.

In recent years, some chaos synchronizations based on fuzzy systems have been proposed since the fuzzy set theory was initiated by Zadeh (1988), such as fuzzy sliding mode controlling technique (Bagheri & Moghaddam, 2009; Chen, Chen, & Chiang, 2009; Hung & Chung, 2007; Hung, Lin, & Chung, 2007), LMI-based synchronization (Wang, Guan, & Wang, 2003) and extended backstepping sliding mode controlling technique (Li & Khajepour, 2005). The fuzzy logic control (FLC) scheme have been widely developed for almost 40 years and have been successfully applied to many applications (Li, Kuo, & Guo, 2007). Recently, Yau and Shieh (2008) proposed an amazing new idea in designing fuzzy logic controllers – constructing fuzzy rules subject to a common Lyapunov function such that the master–slave chaos systems satisfy stability in the Lyapunov sense. In Yau and Shieh (2008), there are two main controllers in their slave system. One is used in elimination of nonlinear terms and the other is built by fuzzy rules subject to a common Lyapunov function. Therefore, the resulting controllers are nonlinear form. In Yau and Shieh (2008), the regular form is necessary. In order to carry out the new method, the original system must to be transformed into their regular form. In this paper, we propose a new strategy which is also constructing fuzzy rules subject to a Lyapunov direct method. Error derivatives are used to be upper bound and lower bound. Through this new approach, a simplest controller, i.e. constant controller, can be obtained and the difficulty in realization of complicated controllers in chaos synchronization by Lyapunov direct method can be also coped. Unlike conventional approaches, the resulting control law has less maximum magnitude of the instantaneous control command and it can reduce the actuator saturation phenomenon in real physic system.

The layout of the rest of the paper is as follows. In Section 2, generalized synchronization by fuzzy logic constant controller (FLCC) scheme is presented. In Section 3, simulation results are shown. In Section 4 conclusions are given.
2. Generalized synchronization by FLCC scheme

2.1. Generalized synchronization scheme

There are two nonlinear dynamical systems, while the master system controls the slave system. The master system is given by

\[ \dot{x} = Ax + f(x) \]  

where \( x = [x_1, x_2, \ldots, x_n]^T \in \mathbb{R}^n \) denotes a state vector, \( A \) is an \( n \times n \) constant coefficient matrix and \( f \) is a nonlinear vector function.

The slave system is given by

\[ \dot{y} = By + g(y) + u \]  

where \( y = [y_1, y_2, \ldots, y_n]^T \in \mathbb{R}^n \) denotes a state vector, \( B \) is an \( n \times n \) constant coefficient matrix, \( g \) is a nonlinear vector function, and \( u = [u_1, u_2, \ldots, u_n]^T \in \mathbb{R}^n \) is a constant control input vector.

Our goal is to design appropriate fuzzy rules and corresponding constant controllers \( u \) so that the state vector of the chaotic system (2-1) asymptotically approaches the state vector of the master system (2-2).

The generalized chaos synchronization can be accomplished in the sense that the limit of the error vector \( e(t) = [e_1, e_2, \ldots, e_n]^T \) approaches zero:

\[ \lim_{t \to \infty} e = 0 \]  

where \( e = H(x) - y \)  

where \( H(x) \) is a given vector function of \( x \). From Eq. (2-4) we have

\[ \dot{e} = \frac{\partial H(x)}{\partial x} \dot{x} - y \]  

\[ \dot{\varepsilon} = \frac{\partial H(x)}{\partial x} [Ax + f(x)] - Ay - f(y) - u \]  

A Lyapunov function \( V(e) \) is chosen as a positive definite function

\[ V(e) = \frac{1}{2} e^T e \]  

Its derivative along any solution of the differential equation system consisting of Eq. (2-6) is

\[ \dot{V}(e) = e^T \left( \frac{\partial H(x)}{\partial x} [Ax + f(x)] - Ay - f(y) - u \right) \]  

If fuzzy constant controllers \( u \) can be appropriately chosen so that \( \dot{V} = Ce^T e \), \( C \) is a diagonal negative definite matrix, and \( V \) is a negative definite function of \( e \). By Lyapunov theorem of asymptotical stability:

\[ \lim_{t \to \infty} e = 0 \]  

The generalized synchronization is obtained. The design process of FLCC is introduced in the following section.

2.2. Fuzzy logic constant controller design process

The basic configuration of the fuzzy logic system is shown in Fig. 1. It is composed of five function blocks (Shieh, 2003):

1. A rule base contains a number of fuzzy if-then rules.
2. A database defines the membership functions of the fuzzy sets used in fuzzy rules.
3. A decision-making unit performs the inference operations on the rules.

4. A defuzzification interface transforms the crisp inputs into degrees of match with linguistic value.
5. A defuzzification interface transforms the fuzzy results of the inference into a crisp output.

The fuzzy rules base consists of collection of fuzzy if-then rules expressed as the form if \( a \) is \( A \) then \( b \) is \( B \), where \( a \) and \( b \) denote linguistic variables, \( A \) and \( B \) represent linguistic values which are characterized by membership functions. All of the fuzzy rules can be used to construct the fuzzy associated memory.

We use two signals, \( e(t) = [e_1, e_2, \ldots, e_m]^T \) in Eq. (2-4) and \( e(t) = [e_1, e_2, \ldots, e_m, \ldots, e_n]^T \) Eq. (2-5), as the antecedent part of the proposed FLCC to design the control input \( u \) in Eq. (2-8) that will be used in the consequent part of the proposed FLCC as follows:

\[ u = [u_1, u_2, \ldots, u_m, \ldots, u_n]^T \]  

where \( u \) is a constant column vector and the FLCC accomplishes the objective to stabilize the error dynamics (2-6). In this paper, we are not going to use the original fuzzy rule base, but using it in each error dynamics separately. In order to obtain the simplest controllers, the \( i \)th if-then rule of the fuzzy rule base of the FLCC is of the following form:

**Rule i** : if \( e_m \) is \( X_i \) then \( e_m \) is \( Y_i \) and \( u_m = \) constant

where \( X_i \) is the input fuzzy sets of \( e_m \), \( m = 1 \sim n \), \( Y_i \) is the output fuzzy sets of \( e_m \) and \( u_m \) is the \( i \)-th output of \( e_m \) is a constant controller. For given input sign of the process variables \( e_m \), then the output sign of \( e_m \) would be decided and its degree of membership \( H_{m} \), \( i = 1 \sim 3 \) called rule-antecedent weights are calculated. The centroid defuzzifier evaluates the output of all rules as follows:

\[ u_m = \frac{\sum_{i=1}^{3} H_{m} \times u_{m}}{\sum_{i=1}^{3} H_{m}} \]  

The fuzzy rule base is listed in Table 1, in which the input variables in the antecedent part of the rules are \( e_m \) and the output variable, in the consequent part are \( e_m \) and \( u_m \).

<table>
<thead>
<tr>
<th>Rule</th>
<th>Antecedent</th>
<th>Consequent part 1</th>
<th>Consequent part 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>e_m</td>
<td>Positive (P)</td>
<td>Positive (P)</td>
<td>u_m</td>
</tr>
<tr>
<td>e_m</td>
<td>Negative (N)</td>
<td>Negative (N)</td>
<td>u_m</td>
</tr>
<tr>
<td>e_m</td>
<td>Zero (Z)</td>
<td>Zero (Z)</td>
<td>u_m</td>
</tr>
</tbody>
</table>

The membership function is obtained via the method shown in Fig. 2. After designing appropriate fuzzy logic constant controllers,
a negative definite of $\dot{V}$ in Eq. (2-9) can be obtained and the asymptotically stability of Lyapunov theorem can be achieved.

3. Simulation results

3.1. Example 1: synchronization of master and slave Lorenz system

The master Lorenz system (Lorenz, 1963) is:

$$\begin{align*}
\frac{dx_1(t)}{dt} &= a(x_2(t) - x_1(t)) \\
\frac{dx_2(t)}{dt} &= cx_1(t) - x_1(t)x_2(t) - x_2(t) \\
\frac{dx_3(t)}{dt} &= x_1(t)x_2(t) - bx_3(t)
\end{align*}$$

(3-1-1)

When initial condition $(x_{10}, x_{20}, x_{30}) = (-0.1, 0.2, 0.3)$ and parameters $a = 10$, $b = 8/3$ and $c = 28$, chaos of the Lorenz system appears. The chaotic behavior of Eq. (3-1-1) is shown in Fig. 3.

The slave Lorenz system is:

$$\begin{align*}
\frac{dy_1(t)}{dt} &= a(y_2(t) - y_1(t)) + u_1 \\
\frac{dy_2(t)}{dt} &= cy_1(t) - y_1(t)y_2(t) + y_2(t) + u_2 \\
\frac{dy_3(t)}{dt} &= y_1(t)y_2(t) - by_3(t) + u_3
\end{align*}$$

(3-1-2)

When initial condition $(y_{10}, y_{20}, y_{30}) = (0.5, 0.7, 1.5)$ and parameters are the same as that of Eq. (3-1-1), chaos of the slave Lorenz system appears as well. $u_1$, $u_2$ and $u_3$ are FLCC to synchronize the slave Lorenz system to master one, i.e.,

$$\lim_{t \to \infty} e = 0$$

(3-1-3)

where the error vector

$$[e] = \begin{bmatrix} e_1(t) \\ e_2(t) \\ e_3(t) \end{bmatrix} = \begin{bmatrix} x_1(t) - y_1(t) \\ x_2(t) - y_2(t) \\ x_3(t) - y_3(t) \end{bmatrix}$$

(3-1-4)

From Eq. (3-1-4), we have the following error dynamics:

$$\begin{align*}
\dot{e}_1 &= a(x_2 - x_1) - (a(y_2 - y_1) + u_1) \\
\dot{e}_2 &= cx_1 - x_1x_3 - x_2 - ((cy_1 - y_1y_3 - y_2) + u_2) \\
\dot{e}_3 &= x_1x_2 - bx_3 - ((y_1y_2 - by_3) + u_3)
\end{align*}$$

(3-1-5)

Choosing Lyapunov function as:

$$V = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2)$$

(3-1-6)

Its time derivative is:

$$\dot{V} = e_1\dot{e}_1 + e_2\dot{e}_2 + e_3\dot{e}_3 = e_1(a(x_2 - x_1) - (a(y_2 - y_1) + u_1)) + e_2(cx_1 - x_1x_3 - x_2 - ((cy_1 - y_1y_3 - y_2) + u_2)) + e_3(x_1x_2 - bx_3 - ((y_1y_2 - by_3) + u_3))$$

(3-1-7)

In order to design FLCC, we divide Eq. (3-1-7) into three parts as follows: Assume $V = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2) = V_1 + V_2 + V_3$, then $V = e_1\dot{e}_1 + e_2\dot{e}_2 + e_3\dot{e}_3 = V_1 + V_2 + V_3$, where $V_1 = \frac{1}{2}e_1^2$, $V_2 = \frac{1}{2}e_2^2$ and $V_3 = \frac{1}{2}e_3^2$.

Part 1: $V_1 = e_1\dot{e}_1 = e_1(a(x_2 - x_1) - (a(y_2 - y_1) + u_1))$

Part 2: $V_2 = e_2\dot{e}_2 = e_2(cx_1 - x_1x_3 - x_2 - ((cy_1 - y_1y_3 - y_2) + u_2)$

Part 3: $V_3 = e_3\dot{e}_3 = e_3(x_1x_2 - bx_3 - ((y_1y_2 - by_3) + u_3)$

Part 1: FLCC in Part I can be obtained via the fuzzy rules in Table 1 as follows and the maxima value and minima value of $\dot{e}_i$ (without any controller) can be observed in time history of error

Fig. 2. Membership function.

Fig. 3. Projections of phase portrait of chaotic Lorenz system with $a = 10$, $b = 8/3$ and $c = 28$. 
derivatives drawn in Fig. 4. We choose $f_1$ to be the upper bound value and $g_1$ to be the lower bound value of $\dot{e}_1$(without any controller), they are satisfied with $f_1 < e_1$ (without any controller) < $g_1$ and $f_1, g_1$ are all constants.

Rule 1: if $e_1$ is $P$, then $e_1$ is $N$ and we take $u_{11} = f_1$

Rule 2: if $e_1$ is $N$, then $e_1$ is $P$ and we take $u_{12} = g_1$

Rule 3: if $e_1$ is $Z$, then $e_1$ is $Z$ and we take $u_{13} = 0 = e_1$

where $f_1 = - g_1 = constant = 400$ and we choose $u_{13} = 0 = e_1$ when $e_1$ approaches to zero. We take Rule 1~3 in Part 1, $V_1 = e_1 e_1$, for explaining:

Rule 1: if $e_1$ is $P$, then $e_1$ is $N$ and we take $u_{11} = f_1$

$V_1 = e_1 e_1 = e_1 (a(x_2 - x_1) - a(y_2 - y_1) - f_1)$

where $e_1 > 0$ and $(a(x_2 - x_1) - a(y_2 - y_1) - f_1) = (e_1$(without controller) $- f_1) < 0$. Therefore, $V_1 = e_1 e_1 = e_1 (a(x_2 - x_1) - a(y_2 - y_1) - f_1) < 0$ and is going to approach asymptotically stable.

Rule 2: if $e_1$ is $N$, then $e_1$ is $P$ and we take $u_{12} = g_1$

$V_1 = e_1 e_1 = e_1 (a(x_2 - x_1) - a(y_2 - y_1) - g_1)$

where $e_1 < 0$ and $(a(x_2 - x_1) - a(y_2 - y_1) - g_1) = (e_1$(without controller) $- g_1) > 0$. Therefore, $V_1 = e_1 e_1 = e_1 (a(x_2 - x_1) - a(y_2 - y_1) - g_1) < 0$ and is going to approach asymptotically stable.

Rule 3: if $e_1$ is $Z$, then $e_1$ is $Z$ and we take $u_{13} = 0 = e_1$

$V_1 = e_1 e_1 = e_1 (a(x_2 - x_1) - a(y_2 - y_1) - e_1)$

where $e_1 = 0$ and we donot need any controller now. Therefore, $V_1 = e_1 e_1 = 0$ and achieve asymptotically stable. As a results, FLCC in Part 1 can be obtained from Rule 1, 2 and 3:

$$u_1 = \frac{\mu_p \times u_{11} + \mu_h \times u_{12} + \mu_g \times u_{13}}{\mu_p + \mu_h + \mu_g} \quad (3-1-8)$$

Part 2: FLCC in Part 2 can be obtained via the fuzzy rules in Table 1 as follows and the maxima value and minima value of $\dot{e}_2$ (without any controller) can be observed in time history of error derivatives drawn in Fig. 4. We choose $f_2$ to be the upper bound value and $g_2$ to be the lower bound value of $\dot{e}_2$ (without any controller), they are satisfied with $f_2 < \dot{e}_2$ (without any controller) < $g_2$ and $f_2, g_2$ are all constants.

Rule 1: if $e_2$ is $P$, then $e_2$ is $N$ and we take $u_{21} = f_2$

Rule 2: if $e_2$ is $N$, then $e_2$ is $P$ and we take $u_{22} = g_2$

Rule 3: if $e_2$ is $Z$, then $e_2$ is $Z$ and we take $u_{23} = 0 = e_2$

where $f_2 = - g_2 = constant = 500$ and we choose $u_{23} = 0 = e_2$ when $e_2$ approaches to zero. The process of FLCC designing is the same as Part 1, as a results, FLCC in Part 2 can be obtained from Rule 1, 2 and 3 and are going to take $V_2 = e_2 e_2 < 0$:

$$u_2 = \frac{\mu_p \times u_{21} + \mu_h \times u_{22} + \mu_g \times u_{23}}{\mu_p + \mu_h + \mu_g} \quad (3-1-9)$$

Part 3: FLCC in Part 3 can be obtained via the fuzzy rules in Table 1 as follows and the maxima value and minima value of $\dot{e}_3$ (without any controller) can be observed in time history of error derivatives drawn in Fig. 4. We choose $f_3$ to be the upper bound value and $g_3$ to be the lower bound value of $\dot{e}_3$ (without any controller), they are satisfied with $f_3 < \dot{e}_3$ (without any controller) < $g_3$ and $f_3, g_3$ are all constants.

Rule 1: if $e_3$ is $P$, then $e_3$ is $N$ and $u_{31} = f_3$

Rule 2: if $e_3$ is $N$, then $e_3$ is $P$ and $u_{32} = g_3$

Rule 3: if $e_3$ is $Z$, then $e_3$ is $Z$ and $u_{33} = 0 = e_3$

where $f_3 = - g_3 = constant = 500$ and we choose $u_{33} = 0 = e_3$ when $e_3$ approaches to zero. The process of FLCC designing is the same as Part 1, as a results, FLCC in Part 3 can be obtained from Rule 1, 2 and 3 and are going to take $V_3 = e_3 e_3 < 0$:

$$u_3 = \frac{\mu_p \times u_{31} + \mu_h \times u_{32} + \mu_g \times u_{33}}{\mu_p + \mu_h + \mu_g} \quad (3-1-10)$$

FLCC are proposed in Part 1, 2 and 3 and are going to take $V_1 = e_1 e_1 < 0$, $V_2 = e_2 e_2 < 0$ and $V_3 = e_3 e_3 < 0$. Hence, we have $V = V_1 + V_2 + V_3 < 0$. It is clear that all of the rules in our FLCC can lead the Lyapunov function to approach asymptotically stable and the simulation results are shown in Figs. 5 and 6.
3.2. Example 2: generalized synchronization of different order chaotic system- Lorenz and New Chen–Lee system

Chen & Lee (2004) gave a new chaotic system, which is now called the Chen–Lee system (Tam & Tou, 2008). The system is described by the following nonlinear differential equations and is denoted as system (1):

\[
\begin{align*}
\frac{dz_1}{dt} &= -z_2(t)z_3(t) + a_1z_1(t) \\
\frac{dz_2}{dt} &= z_1(t)z_3(t) + b_1z_2(t) \\
\frac{dz_3}{dt} &= -\frac{1}{3}z_1(t)z_2(t) + c_z(t)
\end{align*}
\]

where \(z_1, z_2, \) and \(z_3\) are state variables, and \(a_1, b_1, \) and \(c_1\) are three system parameters. When \((a_1, b_1, c_1) = (5, -10, -3.8)\), system (3-2-1) is a chaotic attractor. The positive Lyapunov exponent of this attractor is \(\lambda_1 = 0.88\), while the other ones are \(\lambda_2 = 0\) and \(\lambda_3 = -13.57\), respectively. It is clear that the Chen–Lee system is a regular chaotic system. For more-detailed dynamics of the Chen–Lee system, see Chen & Lee (2004).

It is known that in order to obtain hyper-chaos, there are two important requisites: (1) the minimal dimension of the phase space that embeds a hyper-chaotic attractor should be at least four, which requires a minimum of four couple first-order autonomous ordinary differential equations; and (2) the number of terms in

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**Fig. 5.** Time histories of errors for Example 1- the FLCC is coming into after 30s.

**Fig. 6.** Time histories of states for Example 1- the FLCC is coming into after 30s.
the couple equations giving rise to instability should be at least two, of which at least one should be a nonlinear function. In (Chen et al., 2009), Chen and Lee introduce a nonlinear feedback controller to the third equation of system (3-2-1), the following dynamic system can be obtained:

\[
\begin{align*}
\frac{dz_1(t)}{dt} & = -z_2(t)z_3(t) + a_1z_1(t) \\
\frac{dz_2(t)}{dt} & = z_1(t)z_2(t) + b_1z_2(t) \\
\frac{dz_3(t)}{dt} & = \frac{1}{2}z_1(t)z_2(t) + c_1z_3(t) + \frac{1}{2}z_4(t) \\
\frac{dz_4(t)}{dt} & = d_1z_1(t) + \frac{1}{2}z_2(t)z_3(t) + \frac{1}{2}b_2z_4(t)
\end{align*}
\]

(3-2-2)

where \(d\) is a constant, determining the dynamic behaviors of the system (3-2-2) and \(a_1\), \(b_1\), and \(c_1\) are three system parameters. Thus, controller \(z_4\) causes chaotic system (3-2-1) to become a four-dimensional system, which has four Lyapunov exponents. This may lead to a hyper-chaotic system. When \((a_1,b_1,c_1) = (5,-10,-3.8)\) and we choose \(d = 1.3\), system (3-2-2) is a hyper-chaotic attractor. The projection of phase portraits of system (3-2-2) with hyper-chaotic behaviors is shown in Fig. 7.

Eq. (3-1-2) is chosen as slave system to be synchronized with the master system (3-2-2). Our goal is \([e] = [e_1(t),e_2(t),e_3(t)] = [z_1(t)−y_1(t),z_2(t)−y_2(t),z_3(t)−y_3(t)]\). As a result, we get the following error dynamics:

\[
\begin{align*}
\dot{e}_1 & = -z_2z_3 + a_1z_1 - (a(y_2 - y_1) + u_1) \\
\dot{e}_2 & = \frac{1}{2}z_1z_2 + c_1z_3 - ((Cy_1 - y_1y_3 - y_2) + u_2) \\
\dot{e}_3 & = d_1z_1 + \frac{1}{2}z_2z_3 + \frac{1}{2}b_2z_4 - ((y_1y_2 - by_3) + u_3)
\end{align*}
\]

(3-2-3)

Choosing Lyapunov function as:

\[
V = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2)
\]

(3-2-4)

Its time derivative is:

\[
\dot{V} = e_1\dot{e}_1 + e_2\dot{e}_2 + e_3\dot{e}_3
\]

\[
= e_1(-z_2z_3 + a_1z_1 - (a(y_2 - y_1) + u_1)) + e_2(\frac{1}{2}z_1z_2 + c_1z_3 - ((Cy_1 - y_1y_3 - y_2) + u_2)) + e_3(d_1z_1 + \frac{1}{2}z_2z_3 + \frac{1}{2}b_2z_4 - ((y_1y_2 - by_3) + u_3))
\]

(3-2-5)

We divide Eq. (3-2-5) into three parts as follows:

Assume \(V = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2) = V_1 + V_2 + V_3\), then \(\dot{V} = e_1\dot{e}_1 + e_2\dot{e}_2 + e_3\dot{e}_3 = V_1 + V_2 + V_3\), where \(V_1 = \frac{1}{2}e_1^2\), \(V_2 = \frac{1}{2}e_2^2\) and \(V_3 = \frac{1}{2}e_3^2\).

Part 1: \(\dot{V}_1 = e_1\dot{e}_1 = e_1(-z_2z_3 + a_1z_1 - (a(y_2 - y_1) + u_1))\)

Part 2: \(\dot{V}_2 = e_2\dot{e}_2 = e_2(\frac{1}{2}z_1z_2 + c_1z_3 - ((Cy_1 - y_1y_3 - y_2) + u_2))\)

Part 3: \(\dot{V}_3 = e_3\dot{e}_3 = e_3(d_1z_1 + \frac{1}{2}z_2z_3 + \frac{1}{2}b_2z_4 - ((y_1y_2 - by_3) + u_3))\)

Part 1: FLCC in Part 1 can be obtained via the fuzzy rules in Table 1 as follows and the maxima value and minima value of \(e_1\) (without any controller) can be observed in time history of error derivatives drawn in Fig. 8. We choose \(f_4\) to be the upper bound value and \(g_4\) to be the lower bound value of \(e_1\) (without any controller), they are satisfied with \(f_4 < e_1\) (without any controller) < \(g_4\). \(f_4\) and \(g_4\) are all constants.

Rule 1: If \(e_1\) is P, then \(e_1\) is N and we take \(u_{11} = f_4\).

Rule 2: If \(e_1\) is N, then \(e_1\) is P and we take \(u_{12} = g_4\).

Rule 3: If \(e_1\) is Z, then \(e_1\) is Z and we take \(u_{13} = 0 = e_1\).

where \(f_4 = -g_4 = \text{constant} = 2000\) and we choose \(u_{13} = 0 = e_1\) when \(e_1\) approaches to zero. We take Rule 1~3 in Part 1, \(V_1 = e_1\dot{e}_1\), for explaining:

Rule 1: If \(e_1\) is P, then \(e_1\) is N and we take \(u_{11} = f_4\).

\[V_1 = e_1\dot{e}_1 = e_1(-z_2z_3 + a_1z_1 - (a(y_2 - y_1) - f_4))\]

where \(e_1 > 0\) and \((-z_2z_3 + a_1z_1 - (a(y_2 - y_1) - f_4)) = (e_1\text{(without controller)}) - f_4 < 0\). Therefore, \(V_1 = e_1\dot{e}_1 = e_1(-z_2z_3 + a_1z_1 - a(y_2 - y_1) - f_4) < 0\) and is going to approach asymptotically stable.

Fig. 7. Projections of phase portrait of chaotic Chen–Lee system.
Rule 2: if $e_1$ is $N$, then $\dot{e}_1$ is $P$ and we take $u_{12} = g_4$

$$V_1 = e_1 \dot{e}_1 = e_1(-x_2 x_3 + a_1 x_1 - a(y_2 - y_1) - g_4)$$

where $e_1 < 0$ and $(-x_2 x_3 + a_1 x_1 - a(y_2 - y_1) - g_4) = (\dot{e}_1$ (without controller) $- g_4) > 0$. Therefore, $V_1 = e_1 \dot{e}_1 = e_1(-x_2 x_3 + a_1 x_1 - a(y_2 - y_1) - g_4) < 0$ and is going to approach asymptotically stable.

Rule 3: if $e_1$ is $Z$, then $\dot{e}_1$ is $Z$ and we take $u_{13} = 0 = e_1$

$$V_1 = e_1 \dot{e}_1 = e_1(-x_2 x_3 + a_1 x_1 - a(y_2 - y_1) - e_1)$$

where $e_1 = 0$ and we donot need any controller now. Therefore, $V_1 = e_1 \dot{e}_1 = 0$ and achieve asymptotically stable. As a results, FLCC in Part 1 can be obtained from Rule 1, 2 and 3:

\[
u_1 = \frac{\mu_P \times u_{11} + \mu_N \times u_{12} + \mu_Z \times u_{13}}{\mu_P + \mu_N + \mu_Z}
\] (3.2.6)

Part 2: FLCC in Part 2 can be obtained via the fuzzy rules in Table 1 as follows and the maxima value and minima value of $\dot{e}_2$ (without any controller) can be observed in time history of error derivatives drawn in Fig. 8. We choose $f_5$ to be the upper bound value and $g_5$ to be the lower bound value of $\dot{e}_2$ (without any controller), they are satisfied with $f_5 < \dot{e}_2$ (without any controller) $< g_5$ and $f_5, g_5$ are all constants.

Rule 1: if $e_2$ is $P$, then $\dot{e}_2$ is $N$ and $u_{21} = f_5$

Rule 2: if $e_2$ is $N$, then $\dot{e}_2$ is $P$ and $u_{22} = g_5$

Rule 3: if $e_2$ is $Z$, then $\dot{e}_2$ is $Z$ and $u_{23} = 0 = e_2$

Fig. 8. Time histories of error derivatives for master and slave chaotic systems without controllers.

Fig. 9. Time histories of errors for Example 2- the FLCC is coming into after 30 s.
where \( f_3 = -g_3 = \text{constant} = 1000 \) and we choose \( u_{32} = 0 = e_2 \) when \( e_2 \) approaches to zero. The process of FLCC designing is the same as Part 1, as a results, FLCC in Part 2 can be obtained from Rule 1, 2 and 3 and are going to take \( V_2 = e_2\dot{e}_2 < 0 \): \[
u_2 = \frac{\mu_p \times u_{21} + \mu_y \times u_{32} + \mu_y \times u_{22}}{\mu_p + \mu_y + \mu_2} \quad (3-2-7)\]

Part 3: FLCC in Part 3 can be obtained via the fuzzy rules in Table 1 as follows and the maxima value and minima value of \( e_3 \) (without any controller) can be observed in time history of error derivatives drawn in Fig. 8. We choose \( f_6 \) to be the upper bound value and \( g_6 \) to be the lower bound value of \( e_3 \) (without any controller), they are satisfied with \( f_6 < e_3 \) (without any controller) < \( g_6 \), and \( f_6, g_6 \) are all constants.

- Rule 1: if \( e_3 = P \), then \( e_3 = N \) and \( u_{31} = f_6 \)
- Rule 2: if \( e_3 = N \), then \( e_3 = P \) and \( u_{32} = g_6 \)
- Rule 3: if \( e_3 = Z \), then \( e_3 = Z \) and \( u_{33} = 0 = e_3 \)

where \( f_3 = -g_3 = \text{constant} = 2000 \) and we choose \( u_{33} = 0 = e_3 \) when \( e_3 \) approaches to zero. The process of FLCC designing is the same as Part 1, as a results, FLCC in Part 3 can be obtained from Rule 1, 2 and 3 and are going to take \( V_3 = e_3\dot{e}_3 < 0 \): \[
u_3 = \frac{\mu_p \times u_{31} + \mu_y \times u_{32} + \mu_y \times u_{33}}{\mu_p + \mu_y + \mu_2} \quad (3-2-8)\]

FLCC are proposed in Eq. (3-2-6), (3-2-7) and (3-2-8) and are going to take \( V_1 = e_1\dot{e}_1 < 0 \), \( V_2 = e_2\dot{e}_2 < 0 \) and \( V_3 = e_3\dot{e}_3 < 0 \) separately. Hence, we have \( V = V_1 + V_2 + V_3 < 0 \). It is clear that all of the rules in our FLC can lead the Lyapunov function to approach asymptotically stable and the simulation results are shown in Figs. 9 and 10.

**4. Conclusions**

In this paper, a simplest controller - fuzzy logic constant controller (FLCC) is introduced. Based on Lyapunov direct method and the upper bound and lower bound of the error derivatives, we construct the fuzzy rules and the simplest corresponding constant controllers. Complicated and nonlinear controllers would no longer appear and are replaced with simple and constant controllers through our new strategy. Simulation results in synchroni-

**References**


