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子計畫一：光纖都會核心網路技術研究

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Optical Metro Core Network Technology

一、中文摘要

全光分波多工網路通常包含多重粗細粒度交換能力之全光交錯連接器，譬如以交換於單一波長、一個波段或整條光纖為基準。此外，在任意一個分波多工網狀網路進行路由與波長分配(RWA)是屬於NP-complete問題。在本篇報告中,我們提出一種高效率的近似法,稱作Lagrangean Relaxation with Heuristics (LRH),打算解決多重粗粒度之分波多工網路內之RWA問題,特別是在具有波長與光纖交換器之網路。首先將問題公式化為將瓶頸鏈結使用率降低至最小化之組合性最佳化問題。LRH方法執行限制解限並且根據由subgradient為基礎之疊代法產生之Lagrangean multipliers集合導出下限指標解。同時利用已產生之Lagrangean multipliers，LRH使用一種新的heuristic演算法去達到一個近乎最佳上限解。在具備上下限之情形下,我們對LRH的正確性與匯合速度在不同參數設定之下進行績效研究,並進一步以模擬隨機產生與幾個廣為人知的大型網路的方式,對LRH和現存實際方法進行比較。結果顯示出LRH勝過現有方法,不論是於準確性或計算時間複雜度上，特別是應用於較大型網路。

關鍵詞：波長分波多工(WDM),多重粗細粒度交換能力，路由與波長分配(RWA)，組合性最佳化問題，Lagrangean relaxation。

二、英文摘要

Optical WDM networks often include optical cross-connects with multi-granularity switching capability, such as switching on a single lambda, a waveband, or an entire fiber basis. In addition, it has been shown that Routing and Wavelength Assignment (RWA) in an arbitrary mesh WDM network is an NP-complete problem. In this report, we propose an efficient approximation approach, called Lagrangean Relaxation with Heuristics (LRH), aimed to resolve RWA in multi-granularity WDM networks particularly with lambda and fiber switches. The task is first formulated as a combinatorial optimization problem in which the bottleneck link utilization is to be minimized. The LRH approach performs constraint relaxation and derives a lower-bound solution index according to a set of Lagrangean multipliers generated through subgradient-based iterations. In parallel, using the generated Lagrangean multipliers, the LRH approach employs a new heuristic algorithm to arrive at a near-optimal upper-bound solution. With lower and upper bounds, we conduct a performance study on LRH with respect to accuracy and convergence speed under different parameter settings. We further draw comparisons between LRH and an existing practical approach via experiments over randomly generated and several well-known large sized networks. Numerical results demonstrate that LRH outperforms the existing approach in both accuracy and computational time complexity, particularly for larger sized networks.

Keywords: Wavelength Division Multiplexing (WDM), Multi-granularity switching capabilities, Routing and Wavelength Assignment (RWA), Combinatorial optimization problem, Lagrangean relaxation.
One major traffic engineering challenge in such WDM networks has been the Routing and Wavelength Assignment (RWA) problem [3,6]. The problem deals with routing and wavelength assignment between source and destination nodes subject to the wavelength-continuity constraint [7] in the absence of wavelength converters. It has been shown that RWA is an NP-complete problem [7]. Numerous approximation algorithms [3,6] have been proposed with the aim of balancing the trade-off between accuracy and computational time complexity. In general, some algorithms [8,9] focused on the problem in the presence of sparse, limited, or full-range wavelength converters. Some others made an effort to either reduce computational complexity by solving the routing and wavelength assignment sub-problems separately [7], or increase accuracy by considering the two sub-problems [10] jointly. However, with the multi-granularity switching capability taken into consideration, most existing algorithms become functionally or economically unviable.

In this report, our aim is to resolve the RWA problem in multi-granularity WDM networks particularly with Fiber Switch Capable (FSC-OXC) and Lambda Switch Capable (LSC-OXC) devices. It is worth mentioning that, as shown in Figure 1, an MG-OXC node is logically identical to an individual FSC-OXC node in conjunction with an external separated LSC-OXC node. For ease of illustration, we adopt the separated node form throughout the rest of the report.

The problem is in short referred to as RWA+. To tackle the problem, we propose an efficient approximation approach, called Lagrangean Relaxation with Heuristics (LRH). RWA+ is first formulated as a combinatorial optimization problem in which the bottleneck link utilization is to be minimized. The LRH approach performs constraint relaxation and derives a lower-bound solution index according to a set of Lagrangean multipliers generated through subgradient-based iterations. In parallel, using the generated Lagrangean multipliers, the LRH approach employs a new primal heuristic algorithm to arrive at a near-optimal upper-bound solution. With lower and upper bounds, we conduct a performance study on LRH with respect to accuracy and convergence speed under different parameter settings and termination criteria. We further draw comparisons between LRH and an existing practical approach [7] via experiments over randomly generated and several well-known large sized networks. Numerical results demonstrate that LRH outperforms the existing approach in both accuracy and computational time complexity, particularly for larger sized networks.

The remainder of this report is organized as follows. In Section 4.1, we first give the RWA+ problem formulation. In Section 4.2, we present the LRH approach and its primal heuristic algorithm. In Section 4.3, we demonstrate numerical results of the performance study and comparisons under randomly generated and large sized networks. Finally, concluding remarks are made in Section 4.4.
additional $K \times K$ phantom links connecting the $2K$ phantom nodes. These phantom links describe possible configuration combinations inside an FSC node. For ease of description, we summarize the notation used in the formulation as follows.

Input values:
- $N^F$: the set of FSC nodes in the network;
- $N^L$: the set of LSC nodes in the network;
- $L^p$: the set of physical optical links;
- $L^F$: the set of phantom links within FSC nodes;
- $V^m_n$: the set of phantom input nodes for node $n$;
- $V^o_n$: the set of phantom output nodes for node $n$;
- $W$: the set of available wavelengths on each link (assumed to be the same for simplicity);
- $S$: the set of source-destination (SD) pairs requesting lightpath set-up;
- $S_n$: the set of SD pairs where node $n$ is the source node;
- $P_{sd}$: candidate path set for SD pair $sd$;
- $y_{sd}$: lightpath demand for SD pair $sd$;
- $\delta_p$: =1, if path $p$ includes link $l$; =0, otherwise;
- $\sigma_{lv}$: =1, if link $l$ is incident to node $v$; =0, otherwise;

Decision variables:
- $\alpha$: most congested link utilization (lightpath no./$|W|$);
- $x_{pw}$: =1, if lightpath $p$ uses wavelength $w$; =0, otherwise;
- $z_l$: =1, if phantom link $l$ is selected; =0, otherwise;

Problem (P):
\[
\begin{align*}
\text{min } & \alpha \\
\text{subject to } & \sum_{s \in S} \sum_{p \in P_{sd}} \sum_{w \in W} x_{pw} \delta_{pl} \leq \alpha |W| & \forall l \in L \\
& \sum_{p \in P_{sd}} x_{pw} = y_{sd} & \forall sd \in S \\
& \sum_{s \in S} \sum_{p \in P_{sd}} x_{pw} \delta_{pl} \leq 1 & \forall l \in L^p, w \in W \\
& \sum_{s \in S} \sum_{p \in P_{sd}} x_{pw} \delta_{pl} \leq z_l & \forall l \in L^F, w \in W \\
& \sum_{l \in L^p} z_l \sigma_{lv} = 1 & \forall v \in V^m_n, n \in N^F \\
& \sum_{l \in L^F} z_l \sigma_{lv} = 1 & \forall v \in V^o_n, n \in N^F \\
& x_{pw} = 0 \text{ or } 1 & \forall p \in P_{sd}, sd \in S, w \in W \\
& 0 \leq \alpha \leq 1 & \\
& z_l = 0 \text{ or } 1 & \forall l \in L^F 
\end{align*}
\]
resolving the Lagrangean relaxation problem. Next, due to constraint relaxation, the lower bound solutions generated during the computation might be infeasible for the original primal problem. However, these solutions and the generated Lagrangean multipliers can serve as a base to develop efficient primal heuristic algorithms for achieving a near-optimal upper-bound solution to the original problem. Based on LR, the work reported in [15] and [16] resolved the RWA problems for multi-fiber WDM networks and WDM networks with limited-range wavelength converters, respectively. To the best of our knowledge, the LR approach is first time used in this report to resolve an RWA problem for multi-granularity WDM networks.

In the sequel, we first give the transformed dual problem and the derivation of the lower bound. We then present the primal heuristic algorithm for obtaining the upper-bound solution.

4.2.1. The Dual Problem and Lower Bound

In the relaxation process, Constraints (1), (3), and (4) are first relaxed from the constraint set. As shown in the first line of Equation (11), the three expressions corresponding to the three constraints, are respectively multiplied by Lagrangean multipliers $s$, $q$, and $r$, and then summed with the original objective function. Problem (P) is thus transformed into a dual problem, called Dual_P, given as follows:

**Problem (Dual_P):**

\[
Z_{\text{dual}}(\rho) = \min \left[ \sum_{s, q, r} \left( s \sum_{i \in s} \sum_{j \in q} \sum_{k \in r} z_{ij} \right) - \alpha \sum_{i \in s} \sum_{j \in q} \sum_{k \in r} w_{ij} \delta_{ik} \right]
\]

subject to Constraints (2), (5), (6), (7), (8), (9) and (10) where $\rho = (q, r, s)$ is the non-negative Lagrangean multiplier vector. To compute the Lagrangean multipliers, we adopt the subgradient method as delineated in the Lagrangean Relaxation with Heuristics (LRH) algorithm outlined in Figure 2. By separating decision variable $\alpha$, and decision variable vectors, $x$, $z$. Problem (Dual_P) in Equation (11) can be decomposed into three independent sub-problems $S_1$, $S_2$ and $S_3$. Specifically, we have

\[
Z_{\text{dual}} = Z_{S_1} + Z_{S_2} + Z_{S_3} - \sum_{i \in s} \sum_{j \in q} \sum_{k \in r} w_{ij} \delta_{ik},
\]

where sub-problem $S_1$ is given by

\[
Z_{S_1}(s) = \min \left( \sum_{i \in s} \sum_{j \in q} \sum_{k \in r} w_{ij} \delta_{ik} \right), \text{ subject to Constraint (8)}
\]

sub-problem $S_2$ is given by

\[
Z_{S_2}(q, r) = \min \left( \sum_{i \in s} \sum_{j \in q} \sum_{k \in r} w_{ij} \delta_{ik} + \sum_{i \in s} \sum_{j \in q} \sum_{k \in r} w_{ij} \delta_{ik} \right),
\]

subject to Constraints (2), (7) and (10); and sub-problem $S_3$ is given by

\[
Z_{S_3}(r) = \min \left( \sum_{i \in s} \sum_{j \in q} \sum_{k \in r} w_{ij} \delta_{ik} \right), \text{ subject to Constraints (5), (6) and (9)}
\]

First, sub-problem $S_1$ is to determine the decision variable, $\alpha$. Clearly, $\alpha$ is set to 1 if the corresponding cost $1 - \sum_{i \in s} \sum_{j \in q} \sum_{k \in r} w_{ij} \delta_{ik}$ is negative; otherwise $\alpha$ is set to 0. Thus, $S_1$ requires $O(L)$ computation time. Second, sub-problem $S_2$ is to compute the decision variable vector, $x$. There exist $|S_n|$ (one for each source node) independent problems, each of which is an edge-disjoint-path problem, starting from the given source node and destined to all destination nodes of

```
Algorithm LRH;
begin
initialize the Lagrangean multiplier vector
$s:=0$, $q:=0$, and $r:=0$;
UB:=1 and LB:=0; /*upper and lower bounds on $\alpha$ */
quiescence_age:=0;
step size coefficient $\lambda:=2$;
for each $k:=1$ to Iteration_Number do
begin
Solve sub-problem $S_1$;
Solve sub-problem $S_2$; /* by MSSP Algorithm */
Solve sub-problem $S_3$;
end;
if $Z_{\text{dual}} > LB$
then $LB:=Z_{\text{dual}}$ and quiescence_age:=0;
else quiescence_age:=quiescence_age+1;
if quiescence_age $\geq$ Quiescence_Threshold
then $\lambda:=\lambda/2$ and quiescence_age:=0;
run Primal Heuristic Algorithm;
/* $ub$ is the newly computed upper bound */
if $ub<UB$ then $UB:=ub$;
/* by subgradient method */
update the step size and multiplier vector;
end;
end.
```

Figure 2. Lagrangean Relaxation with Heuristics (LRH).
the SD pairs with non-zero lightpath demands. Due to multiple wavelengths on each link, the network can be viewed as a layered graph with a total of \(|W|\) layers, where each layer corresponds to each wavelength. Each layer then contains \((L+L^f)\) links and \((N^d+N^f)\) nodes. Notice that each link can be designated with unit flow capacity and a non-negative cost, for example, \(s_l+q_{lw}\), for each non-phantom link.

Accordingly, the edge-disjoint-path problem for each source corresponds to a minimum-cost flow problem. Ultimately, with \(|W|\) layers considered as a whole, the minimum-cost flow problem can be solved by the Successive Shortest Path (SSP) algorithm [14]. However, the integrated problem requires high computational time complexity provided with large values of \(|W|\). To reduce the complexity, we employ a modified successive shortest path (MSSP) algorithm as shown in Figure 3. In the algorithm, we treat each layer graph individually and perform incremental selection of minimum-cost edge-disjoint path (from one layer). The computational complexity of MSSP for each SD pair is \(O(k(m+n\log n))\), where \(m=L+L^f\), \(n=N^d+N^f\), and \(k = \max\{y_{sd}, |W|\}\). All decision variables \(x's\) for S2 can be obtained by repeatedly applying the MSSP algorithm for all sources. Finally, sub-problem S3 is to resolve decision variable, \(z\). The problem can be further decomposed into \(|N^f|\) (one for each FSC node) independent problems, each of which can be optimally solved by a bipartite weighted matching algorithm. Thus for an \(n \times n\) bipartite graph, the problem requires \(O(n^3)\) computation time.

According to the weak Lagrangean duality theorem [14], \(Z_{\text{dual}}\) in Equation (12) is a lower bound of the original Problem (P) for any non-negative Lagrangean multiplier vector \(\rho= (q, r, s) \geq 0\). Clearly, we are to determine the greatest lower bound. Equation (12) can be solved by the subgradient method, as shown as a part of the LRH approach in Figure 2. As shown in Figure 2, the algorithm is run for a fixed number of iterations (i.e., Iteration Number). (Notice that the algorithm can also be driven by given a termination requirement, as will be shown in the next subsection). In every iteration, the three sub-problems (S1-S3) are solved (described above), resulting in the generation of a new Lagrangean multiplier vector value. Then, according to Equation (12), a new lower bound is generated. If the new lower bound is tighter (greater) than the current best achievable lower bound (LB), the new lower bound is designated as the LB. Otherwise, the LB value remains unchanged.

Significantly, if the LB value remains unimproved for a number of iterations that exceeds a threshold, called Quiescence Threshold (QT), the step size coefficient (\(\lambda\)) of the subgradient method is halved, in an attempt to reduce oscillation possibility. Specifically, to update the step size and multiplier vector as specified in Figure 2, the Lagrangean

**Algorithm MSSP;**

```plaintext
begin
  for each LSC node src \(\in N^d\) do
    for each wavelength \(w \in W\) do /* initialization */
      begin
        \(x:=0;\) /*decision variable vector*/
        \(\pi_w:=0;\) /*node potential vector*/
        for each link \(l \in L\) do costlw:=\(r_{lw}\); /* link cost*/
        for each link \(l \in L\) do costlw:=\(s_l+q_{lw}\);
      end;
    for each SD pair \(sd \in |S_{src}|\) do
      begin
        dest:=destination(sd);
        for each \(w \in W\) do
          begin
            ready_layerw:="Unknown"; num-path-setupsd:=0;
          end;
        repeat
          for each \(w \in W\) do
            begin
              if ready_layerw="Unknown" then
                begin
                  run Dijkstra’s shortest-path(cost, src, dest) on layer \(w\);
                  if the shortest path exists then
                    denote the path cost as \(k_w\);
                    ready_layerw:="Yes";
                  else /* no more path on the layer for sd */
                    ready_layerw:="No";
                end;
            end;
          if there exists a layer \(w^*\) with smallest cost \(k_{w^*}\) and ready_layerw="Yes" then
            begin
              update \(x_{pw^*}\), \(\pi_{w^*}\), costlw; /* SSP algorithm */
              num-path-setupsd:=num-path-setupsd+1;
              ready_layerw:="Unknown";
            end;
          else /* all ready_layer’s are “No” */
            return "infeasible";
        until num-path-setupsd=\(y_{sd}\);
      end;
end.
```

Figure 3. MSSP Algorithm.
multiplier vector $\rho$ is updated as $\rho_{k+1} = \rho_k + \theta_k b_k$, where $\theta_k$ is the step size, determined by $\theta_k = \lambda_k (UB - Z_{dual}(\rho_k))/\|b_k\|^2$, in which $\lambda_k$ is the step size coefficient, $UB$ is the current achievable least upper bound obtained from the Primal Heuristic Algorithm described next, and $b_k$ is a subgradient of $Z_{dual}(\rho)$ with vector size $|L+LW + L^2W|$.

4.2.2. The Primal Heuristic Algorithm and Upper Bound

The primal heuristic algorithm in the LRH approach is used to find an updated upper bound $ub$. Similar to the lower bound case, as given in Figure 2, if the new upper bound ($ub$) is tighter (smaller) than the current best achievable upper bound (UB), the new upper bound is designated as the UB.

As shown in Figure 4, the algorithm first settles the phantom links suggested by the solution to sub-problem S3 for all FSC nodes, reducing the problem complexity. The cost of each link is designated as the Lagrangean multipliers previously obtained. Clearly, the cost of unaccepted phantom links are set to $\infty$, excluding them from subsequent path consideration. The algorithm then repeatedly applies the Dijkstra’s shortest path algorithm in an effort to satisfy the lightpath demands of all SD pairs.

At the end of the computation, the costs of those links associated with the selected wavelengths/paths are set to $\infty$ to prevent the links from being considered by other upcoming iterations. If the number of wavelengths (lightpaths) used on a link is greater than the current tightest lower bound multiplied by $|W|$, indicating potential congestion, the cost of the link is then scaled by multiplying by a constant, referred to as the penalty term. This is to avoid further lightpath set-up through this link. The process repeats until either the lightpath demands of all SD pairs are satisfied (i.e., feasible), or there is no remaining resource (i.e., infeasible) in the network.

4.3. Experimental Results

We have carried out a performance study on the LRH approach, and drawn comparisons between LRH and the Banerjee&Mukherjee approach [7] via experiments over randomly generated networks. Given the total number of nodes, say $n$, the greatest possible number of bi-directional links is $C(n,2)$, where $C$ is the combination operation. Then, for a network with $n$ nodes and connectivity $v$, it is generated by randomly selecting $C(n,2) \times v$ out of the $C(n,2)$ bi-directional links of the network. In the experiments, we used 32 wavelengths on each fiber link (i.e., $|W| = 32$) for all networks.

4.3.1. Performance Study

We carried out two sets of experiments over 15-node random networks with two connectivities $v = 0.4$ and 0.8, which correspond to sparse and dense networks, respectively. In the first set of experiments, the LRH algorithm was terminated when the gap between the UB and the LB on $\alpha$ was less than or equal to one out of the maximum number of wavelengths, or the number of iterations exceeds 2000. While the former condition corresponds to reaching a near-optimal upper bound solution, the latter condition represents abnormal termination due

```
Algorithm Primal Heuristic;
begin
for each wavelength $w \in W$ do
begin
for each link $l \in L^F$ do
if $z_l^w = 1$ then $cost_{lw} := r_{lw}$ else $cost_{lw} := \infty$;
for each link $l \in L$ do $cost_{lw} := cost_{lw} + q_{lw}$;
end;
for each SD pair $sd = (s, d)$ do $num-path-setup_{sd} := 0$;
repeat
for each SD pair $sd = (s, d)$ do $num-path-setup_{sd} := 1$ to $|S|$ do
begin
if $num-path-setup_{sd} \leq y_{sd}$ then begin
src := source($sd$);
dest := destination($sd$);
run Dijkstra’s shortest-path($cost$, src, dest) on each wavelength layer; /* $cost$ is vector of costs of all wavelengths and links*/
if the shortest path exists then begin
 designate the wavelength associated with the shortest path as $w^*$;
for all links $l$ on the shortest path do
begin
$cost_{lw} := \infty$;
if # of allocated paths on link $l > LB \times |W|$ then
for each wavelength $w \in W$ do
$cost_{lw} := cost_{lw} \times Penalty$;
end;
end;
else return “infeasible”;
end;
end;
until all SD demand satisfied;
update upper bound $ub$;
end.
```

Figure 4. Primal Heuristic Algorithm.
to the failure of achieving such accuracy or solution infeasibility. We examine the total number of iterations required as a function of the mean lightpath demand under different QT values. Numerical results are plotted in Figure 5. Notice that the absence of data under certain demands corresponds to abnormal termination.

First, we observe that the dense network in general requires less number of iterations before reaching a near-optimal solution. Significantly, we discover from the figure that parameter QT plays a key role in the performance trade-off between convergence speed and accuracy. Smaller values of QT, which imply frequent updates of the subgradient step-size coefficient, yield faster convergence to near-optimal solutions but at the cost of failing to reach accurate solutions under heavier lightpath demands. Greater QT values on the other hand result in completely opposite performance.

In the second set of experiments, the LRH algorithm was terminated when the number of iteration exceeded a pre-determined Iteration_Number, ranging from 0 to 1500. Numerical results are displayed in Figure 6. We study both the lower and upper bounds on $\alpha$ under different QT values. We observe that while the upper bound performance is irrelevant to QT, the lower bound performance is highly dependent on the QT setting in the same manner as above. Specifically, smaller QT values yield faster convergence but only to looser lower bounds, while larger QT values result in tighter lower bounds through gradual convergence over a larger number of iterations. This fact reveals that, by adjusting the QT value, the LRH approach is capable of balancing the trade-off between accuracy and efficiency for resolving various types of RWA problems.

4.3.2. Performance Comparisons

We further draw comparisons of accuracy and computation time between our LRH approach and a Linear Programming Relaxation (LPR)-based method, i.e., Banerjee&Mukherjee [7]. For generating networks, it is impractical to experiment on networks with smaller numbers of nodes and links. However,
for networks with greater than 11 nodes, we experienced that the computation time using the LPR method became unmanageable. Accordingly in the experiment, we considered three random networks, NET1, NET2, and NET3, as shown in Figure 7. NET1 consists of 7 nodes including 2 FSC nodes, and 14 bi-directional links, corresponding to a connectivity \((v)\) of 0.66. NET2 consists of 10 nodes including 2 FSC (nodes 1-2) or 4 FSC (nodes 1-4) nodes, and 20 bi-directional links, corresponding to a connectivity \((v)\) of 0.44. Finally, NET3 consists of 11 nodes including 2 FSC (nodes 1-2) or 4 FSC (nodes 1-4) nodes, and 22 bi-directional links, corresponding to a connectivity \((v)\) of 0.4. Results are plotted in Figures 8-10.

In the computation using our LRH approach, we adopted QT=50 and three different termination criteria. The three criteria are: Iteration Number =1000, 2000, and requirement \((UB-LB)\leq1/32\). The algorithm was written in the C language and operated on a PC running Windows XP with a 2.53GHz CPU power. In the LPR-based method, by removing Constraints (7) and (9), the original Integer Linear Programming (ILP) problem is relaxed to a Linear Programming (LP) problem. Thus, the solution to the relaxed problem is a legitimate lower bound of the original ILP problem. The upper bound is then obtained according to the randomization procedure proposed in [7]. In the experiment, the LP problem was solved using the CPLEX software, operating in the same PC environment. For both approaches, the accuracy is measured in terms of the Gap(%) which is defined as the ratio of the difference of the UB and LB values to the LB value in percentage.

First of all, we draw comparisons of accuracy and computation time between the LRH approach and the LPR method for random network NET1, as plotted in Figure 8. Notice that the LRH approach using fixed iteration numbers outperforms the LPR method in accuracy under all lightpath demands. However, it appears that the LRH method using the termination requirement yields a high gap under low demands. This is only due to the magnification of the gap resulting from being divided by a small LB value under low demands. In particular, under demand=1, the algorithm was terminated with UB=2/32 and LB=1/32, resulting a 100% gap. Surprisingly, we discover from part (b) of Figure 8 that the LPR

![Network topology](image)

**Figure 7. Network topology.**

![Accuracy for NET1](image)

**Figure 8. Accuracy for NET1**

![Computation time for NET1](image)

**Figure 8. Computation time for NET1.**
method requires less computation time than that of the LRH approach using fixed iterations. This indicates that LPR is an efficient approach particularly for smaller size networks.

For random networks with size over 10 nodes (NET2 and NET3) as shown in Figures 9 and 10, the LPR method yields larger gaps, namely poorer accuracy, and demands exponentially increasing computation time. In contrast, the LRH approach achieves identical lower and upper bounds, namely the optimal solutions under several lightpath demand cases. In fact, we discover that, both LRH and LPR approaches achieve tight lower bounds. Significantly, the LRH heuristic algorithm arrives at much improved upper bounds due to the use of the Lagrangean multipliers derived upon seeking the Lagrangean relaxation solution. It is worth noticing that the results of the LRH approach using the termination requirement are not shown in Figures 9 and 10. This is due to its high accuracy and low computation time, yielding impossible plotting within the figures. Specifically, we discover from Figure 9 that the LRH approach using the 1000 iterations achieves as high accuracy as that using the 2000 iterations under most demand cases. Significantly, the approach using the (UB-LB) ≤ 1/32 requirement for NET2 reaches the small gap within only a total of (8,40,164,480,339,287,137,424) iterations for lightpath demands ranging from 1 to 8, respectively.

Furthermore, as shown in Figure 10, the LRH approach outperforms the LPR method in computation time by at least one order of magnitude under all cases. Notice that, the LRH approach using the termination requirement incurs exceptionally low computation times that are equal to (0,1,7,24,18,17,9,31) for eight lightpath demands, respectively. In this case, compared to the LPR method, the LRH approach offers an improvement of computation time by more than two orders of magnitude.

To observe the performance of our LRH approach for large sized networks, we carried out experiments on two well-known networks, i.e., USA and ARPA, as shown in Figures 11(a) and 12(a). The USA network consists of 28 nodes including 3 FSC nodes and 90 bi-directional links, corresponding to a connectivity (\( v \)) of 0.12. The ARPA network has 61 nodes including 4 FSC nodes and 148 bi-directional

![Figure 9](image-url)
links, which corresponds to a connectivity ($v$) of 0.04. There are 64 wavelengths on each fiber for both networks. Numerical results are displayed in Figures 11 and 12.

In the experiment, we adopted QT=50 and two different termination criteria, namely Iteration_Number=500 and 1000. For the USA network, LRH achieves a guarantee of no more than 8% Gap between the upper and lower bounds under both termination criteria. For the ARPA network, the LRH achieves a guarantee of no more than 9.3% Gap in less than 9400 sec computation time. We particularly observe from Figure 12(c) that the accuracy of the LRH approach based on the 500-iteration termination criterion is as high as that based on the 1000-iteration termination criterion under most lightpath demand cases. This again demonstrates the superiority of the LRH approach to the RWA$^+$ problem with respect to both computation accuracy and time complexity for large sized networks.

4.4. Conclusions

In this report, we have resolved a RWA$^+$ problem using the LRH method, which is a Lagrangean Relaxation based approach augmented with an efficient primal heuristic algorithm. With the aid of generated Lagrangean multipliers and lower bound indexes, the primal heuristic algorithm of LRH achieves a near-optimal upper-bound solution. A performance study delineated that the performance trade-off between accuracy and convergence speed can be manipulated via adjusting the Quiescence Threshold parameter in the algorithm. We have drawn comparisons of accuracy and computation time between LRH and the Linear Programming Relaxation (LPR)-based method, under three random networks. Experimental results demonstrated that, particularly for small to medium sized networks, the LRH approach using a termination requirement profoundly outperforms the LPR method and fixed-iteration-based LRH, in both accuracy and computational time complexity. Furthermore, for large sized networks, i.e., the USA and ARPA networks, numerical results showed that LRH achieves a near optimal solution within acceptable computation time. The above numerical results justify
that the LRH approach can be used as a dynamic RWA$^+$ algorithm for small to medium sized networks, and as a static RWA$^+$ algorithm for large sized networks.

五、計畫成果自評

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Figure 11. The USA network and LRH results.

Figure 12. The ARPA network and LRH results.
六、參考文獻


