Pricing display ads and contextual ads: Competition, acquisition, and investment

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1. Introduction

The contribution made by advertising is to deliver the message of individual products to consumers [63,47,48,17]; thus, higher exposure for a product means that it impresses more of the potential market [60]. According to “Internet Advertising Revenue Report” announced by IAB in April 2006, Internet ad revenue in the United States amounted to over $12.5 billion for the full year 2005, up from $9.6 billion reported in 2004 [30]. Roughly, display ads and text-based search ads occupy, respectively, 40% of online ad sales. Large-scale Web portals or content sites, such as AOL, CNET, and ESPN.com, may place display ads on their sites and optimize ad revenues by making the best use of their ad inventory. Leading channel providers, such as DoubleClick, help advertisers deliver their display ads onto these sites and allow them to monitor ad effectiveness. In this way, advertisers can reach their target audience, enhance customer's taste by ad content, and even drive potential customers to their websites via hyperlinks.

Display advertising (also known as banner ads) is a rectangular graphic image which ranges from 120 to 500 pixels wide and from 45 to 120 pixels high [50], most of them can be found in large-scale Web portals or content sites. Compared to display ads, text-based search ads not only utilize the advantage of search engines to improve ad relevance by matching Web content with the advertiser's ad, but also make online advertising accessible to small advertisers for the first time [65]. In contrast, association with display advertising and Web content will be difficult if Web content varies with time. Contextual advertising, such as Google AdSense[1], is a type of text-based search ad which often presents three to five different ads in a frame, each including title, description, and display URL. Publishers can apply Google AdSense by completing an application form, logging into their private accounts, and putting a block of HTML into Web pages. Then, when someone views the Web page containing the code (i.e., requests and responses between publisher’s Web server and visitor’s browser), the visitor’s browser downloads the Web page and displays it. Since the code indicates that some components (i.e., ad content) must be downloaded from a particular Web server, more requests and responses will not exhaust until all materials are ready to show.

The order of contextual ads is determined by specific rules based on a combination of factors, such as relevancy, bid amount, click-thru rate, and so on. By sharing a common resource pool, advertisers are willing to adopt contextual advertising as their marketing tools if time pressure caused by sharing is low or moderate. Here, contextual advertising is different from keyword advertising, where the latter signifies that advertisers bid for keywords on search engines appearing next to the search results [42]. Even so, their operation principles and pricing mechanisms are similar; as a result, much research on keyword advertising has contributed to our study [21,43,67].

On the other hand, even if contextual advertising has given rise to competition pressure, the role of display advertising is still important. First, due to the development of multimedia techniques, display ads can capture visitors visually, by incorporating 3D graphics, video, sound, and more user interactivity. For example, rich media advertising, such as Flash ads, provides an interactive interface for advertisers to directly collect their customers'...
responses. Second, because these channel providers offering display advertising are the first movers in the market, they already have thousands of relationships with excellent publishers that have come out on top around the world [11]. Finally, building brand equity remains an important task for advertising. Therefore, the current trend shows that these two different advertising types tend to merge into a giant in the Web advertising market, such asDoubleClick and Google. To date, Google is the winner of the global search market, controlling 70% market share, whereas DoubleClick has garnered 80% of the online display advertising market [10]. According to the article published on official Google Blog [65], Google considers that buying DoubleClick can complement Google’s search and content-based advertising capabilities, thereby benefiting all parties in the online advertising business. However, many individuals and organizations worry that Google’s legal framework for consumer privacy is insufficient and that the acquisition will make Google become the dominant player in the Web advertising market [10].

1.1. Problems and motivation

An online advertising market may involve the following parties: advertisers, channel providers, publishers, and visitors (see Fig. 1). The role of publishers is to manage their websites and offer content which visitors are interested in viewing, whereas the function of channel providers is to serve as a bridge between advertisers and publishers, helping advertisers publish their ads in suitable websites managed by publishers and transferring advertising fees paid by advertisers to publishers with a committed ratio. The traditional ad pricing strategy used in newspapers, radio, and television is based on cost per thousand, known as CPM, which means the cost for showing an ad to one thousand viewers. Indeed, channel providers may guarantee an advertiser a certain number of impressions; however, people have the ability to read preferred content and avert their eyes from an ad trap set by the advertiser [55]. As a result, click-thru rate, derived from dividing the number of visitors who clicked on an ad embedded in a Web page by the number of times that the Web page was delivered to visitors, has become a popular benchmark for measuring advertisement performance. In order to win market share and gain better revenue, the channel providers have to carefully consider many factors when choosing their ad prices, such as click-thru rate, the strength of a given impression, user bases, market power, and even the order of the entrance to the market. Therefore, we utilize the Hotelling-type linear city model to better understand how two heterogeneous channel providers make their pricing decisions in the Web advertisement market, such as DoubleClick and Google. Several key parameters are studied in this research, including impression effect, click benefit, expected delay time, and click-thru rate. We also consider the scenario where these two heterogeneous channel providers are merged into a dominant player with monopolistic power in the market.

We aim to address the following questions. First, could the higher quality of a search engine boost the profit of the channel provider offering contextual ads? Moreover, under competition, what are the conditions that would lead to the increase (decrease) of the channel provider’s profit? Second, what is the sufficient condition for ensuring that a market becomes efficient? Is a search engine company joining the advertising market socially beneficial from the viewpoint of market efficiency (social welfare)? Finally, it is considered that contextual text links lack the impression effect; therefore, do the quite high quality display ads threaten the profit of contextual ads?

1.2. Contributions and findings

In this paper, we developed an economic model to examine the profitability among different market structures. We originally assumed that click benefit and impression benefit are two of the most important capabilities derived from Web advertising. Click benefit means the ability to deliver visitors who click a particular hyperlink in a Web page to an appointed website, whereas impression benefit represents the power to influence visitors’ impression of an advertised product or service. Consequently, we derived some interesting findings and counterintuitive results as follows.

First, could the higher quality of a search engine boost the profit of the channel provider offering contextual ads? In a duopolistic market, the answer is positive only if the impression benefit generated by display advertising is sufficiently low. Moreover, we find that the profitability of each channel in a duopolistic market may increase with the channel’s marketing power and waiting time. Second, what is the sufficient condition to ensure that a market becomes efficient? We suggest that the click-thru rate of display advertising can be used as the condition. When the click-thru rate of display advertising is high, a monopoly is more efficient than competition. However, when the click-thru rate of display advertising is high, the opposite holds true. Therefore, if statistical data shows that the click-thru rate of display advertising is at a higher level, a search engine company joining the advertising market will be beneficial to society. Third, it is considered that contextual text links lack the impression effect; therefore, do the quite high quality display ads threaten the profit gained by contextual ads? The answer is negative. It is considered that the profitability of a channel offering contextual advertising would decrease when its competitor’s ability to make an impression on visitors becomes more advanced. However, our result shows a contradictory result in a duopolistic market.

Furthermore, we investigate how channel providers control spending on multimedia technology applied to display ads and the quality of contextual search, to maximize their profits. Our results show that the actual strength of search engines cannot be utilized in a duopolistic market; therefore, it is likely to be one of the most important reasons why Google acquired DoubleClick. The rest of the paper is organized as follows. The literature review is given in Section 2. The single-stage model is developed in Section 3. In Section 4 we discuss pricing strategy and market efficiency (social welfare). In Section 5 the model is extended to a two-stage game where the channel providers can determine their investment in multimedia technology and search engines, in advance. Finally, we conclude this paper and emphasize our findings in Section 6.

2. Literature review

There are two types of advertising frequently analyzed in prior economics literature: informative advertising and persuasive advertising [59]. The former delivers basic product information, such as product prices and points of sale, to target customers,
whereas the primary goal of the latter is to strengthen consumer’s experience regarding an advertised product or service. The effect of advertising is mainly determined by ad expenditure level, time in the market, number of competitors, and order of entry [66]. According to Jedidi et al., advertising has a positive effect on “brand equity” in the long term [33]. Because of “two-way communication” and “conveying detailed information”, the Web is perceived to be different from most media [6,14]. Two-way communication, also known as interactivity, is the reason why Internet is more efficient than traditional media for high involvement products [70]. Statistical data also exhibits that Internet ads are more likely to be employed by companies in the automotive-and-durables and financial-services sectors [40]. Although advertisements provide a majority of revenue for websites, increasing advertising may repel visitors [18]. Therefore, determining how to allocate ads efficiently in a Web page, in order to enhance revenue from a long-term viewpoint, becomes a valuable research issue [52]. For instance, a technique report shows that experts in Microsoft Research had developed an approach for efficiently delivering banner advertisements on the msn.com Web site, running roughly 500 advertisements at a time [15]. In addition, Hofacker and Murphy [27] studied how many clickable banners ads should appear on a site. In their setting, the ad price is paid for clicking on a link and paid in opportunity cost, wasted time, connect time charges, and frustration. Their finding suggests that a single ad on a page can safely add a second banner to that page without impacting the click-thru rate for the first ad.

2.1. Banner ads

Display advertising (also known as banner ads) are one of the earliest formats of Web advertising. Briggs and Hollis [7] measured the impact of banner ads by a proprietary system and their findings confirm the dual role of banner ads in both direct response and brand building. Consumers often rely upon brands to guide their purchase decisions [8,46]. Ko et al. [37] found that attitude toward the brand was strongly affected by attitude toward the site, whereas attitude toward the brand will have a directly positive effect on purchase intention. Eng and Keh [20] investigated the impact of advertising and brand value on future operating and market performance. They showed that advertising effects for the top brands can last up to 4 years; more important, both advertising and brand value have positive effects on brand sales and profitability. Li and Bukovac [41] found that animated banner ads result in shorter reaction time and better recall than do non-animated banner ads, and that large banner ads lead to more clicks than do small banner ads. According to Xie et al. [68], providing incentives in banner ads is an effective technique for improving the click-thru rate.

2.2. Contextual ads

Web content may have a significant impact on how advertising messages are processed or evaluated by individuals [58]. Prior research has found that contextual materials are more likely to influence how consumers interpret advertising messages as well as the brand [25,69]. Since page content can powerfully self-select visitors to that page, webpage’s context should drastically influence click-thru behavior [27]. Therefore, even if search engine firms, such as Google, are new players in the Web advertisement market, their ad products, contextual advertising, can utilize their search technique advantage to gain revenue from the market.

2.3. Marketing strategy

Two of the most fundamental questions that marketers face in marketing are how to implement target advertising to specific consumers and how to allocate their media budgets [5,31,39]. Prior research suggests that developing one common Web advertisement for all users may not be the most effective way to drive intended visitors to a certain website [38]. In recent studies, Kim et al. used tree induction techniques and data-mining tools to generate marketing rules that match customer demographics, thereby providing personalized advertisement selection when a customer visits an Internet store [35]. Furthermore, because of the advancement of wireless technology, advertising may be driven by location information, so that greater interaction between a viewer and advertiser is possible [26,44]. Regarding advertising strategies, Nguyen and Shi address the issue of optimal advertising strategies that incorporate both market-share dynamics and market-size dynamics [49]. Saeed et al. highlighted that advertising spending alone has only a negligible impact on firm performance; thus, companies should provide a better product-ownership experience to execute effective advertising programs [53].

2.4. Advertising pricing

Advertiser’s willingness-to-pay is driven by the number of potential viewers that the ad message might reach [3]. Channel providers usually receive revenue in the following two ways. First, an advertiser pays by the impression, referred to as cost per thousand (CPM) pricing, or the advertiser pays for each time a visitor clicks on the ad placed in the content site and subsequently visits the advertiser’s site, referred to as click through pricing [27]. Dreze and Hushherr [19] suggested that surfers actually avoid looking at banner ads during their online activities, and revealed that banner ads will most likely operate at the pre-attentive processing level. Their results show that banner advertising has a long-term impact beyond the immediate click-through. Shen [57] pointed out that more than 90% of the respondents frequently used conventional CPM to price banner ads. On the contrary, contextual ads get paid just for clicks on the ads in the content sites. Although a lot of attention has been focused on click rates as a benchmark for success, advertisers’ faith may be shaken by click fraud, which is generated by automated hacker programs [9].

Banerjee and Bandyopadhyay constructed a multistage game theoretical model of advertising and price competition in a duopoly of differentiated products. Their finding exhibits that advertising spending by a large firm provides a positive externality on a small firm’s profit [4]. Anderson and Coate [2] considered the commercial broadcast market as a two-sided market and utilized the Hotelling model [28] to study the relation between advertisers and viewers. They showed that the equilibrium level of advertising levels may be too low or too high, depending on the nuisance cost to viewers, the substitutability of programs, and the expected benefits to advertisers from contacting viewers. In this paper, we also adopt the Hotelling model to formulate an advertising pricing game. The players in this setting are composed of advertisers and channel providers. A large number of surveys on this model can be found in the work of Greenhut, Norman, and Hung [24]. According to the blueprint of the model, the goods sold by a firm is characterized as \((p, \theta)\), where \(p\) and \(\theta\) are its price and “address”, respectively [54]. In addition to describing the distance between customers and firms (i.e., transportation costs), the address can also be adopted as the nature of the goods in some other “spectrum”, such as the preferences of customers [22]. Each firm first chooses its address (or chooses its price according to customer’s distribution) and then each customer would make his/her purchase decision [16,29,56].

Here, display ads and contextual ads are two types of product that a channel provider can offer. We compare display ads with contextual ads under a monopolistic setting [1,3,4,45] and duopolistic one [23,36,61]. In a monopolistic setting, a channel provider
can offer differentiated products with discriminated price levels. In a duopolistic market, two channel providers offer different types of ad channel (one offers display ad and the other offers contextual ad) and choose various price levels simultaneously or sequentially. The core of our research is to investigate how these channel providers, which offer ad services, manipulate market price to achieve maximal profit. In fact, there is little information available on comparisons of display advertising and contextual advertising. Therefore, we focus on how these channel providers' profits are altered with environment change, and then investigate the influence of several parameters, including the click-thru rate, impression effect, click benefit, and expected delay time.

3. The model

We consider an ad pricing problem where advertisers want to publish their advertising on websites through two different channels: display advertising or contextual advertising channel. A purchase action is executed by an advertiser and each advertiser decides whether to buy an ad or not to buy at all. The advertisers can choose one of the channels to publish their ads. Thus, we denote the total purchase amount of advertising during a certain time interval $T$ in this market as $\eta_0$. Also, we divide $\eta_0$ into $\eta_d$ and $\eta_c$, which represent the purchase amount with respect to display advertising and contextual advertising, respectively (see Tables 1 and 2 for a complete list of notations).

The fee for display advertising is charged on an impression basis; that is, no matter whether a visitor specifically clicks on the advertiser’s ad, the channel provider charges $p_d$ for impression as long as the Web page containing advertiser’s ad is delivered to the visitor’s browser. On the contrary, the fee for contextual advertising, denoted as $p_c$, is charged on a click basis; that is, no matter whether a visitor views an advertiser’s ad, the channel provider cannot charge any fee until someone clicks the advertiser’s ad. There are two main types of effects (benefits) caused by Web ads: click benefit and impression benefit. Impression benefit catches the value of impressing visitors regarding advertised products or business that will eventually result in a sale, whereas click benefit mainly considers the value generated at the moment when a visitor is delivered to an appointed website by clicking an ad. In other words, click benefit is measured by the expected revenue generated from direct response, whereas impression benefit is associated with increasing brand awareness and purchase intent.

In this study, we assume that every advertisement has the same layout. To emphasize the characteristics of these two advertising types, we assume that contextual advertising is composed of text only, whereas display advertising is presented in a multimedia format. Thus, display advertising offers better performance than contextual advertising in delivering impression benefit. We denote $\gamma_d/\gamma_c$ as click-thru rate, where the subscripts indicate to which channel the parameters belong. For example, ‘$d$’ represents a contextual advertising channel, whereas ‘$c$’ represents a display advertising channel. Also, according to the channel provider's marketing power, we denote click benefit and impression benefit as $\kappa_d(z_d)$ and $\kappa_c(p_c)$, respectively. In this paper, advertisers are heterogeneous with respect to click benefit and impression benefit, which are denoted as $\theta_d$ and $\theta_c$. Therefore, for advertiser $i$, she can receive $\mu_{d(i)}$ from an impression by choosing display advertising; however, the expected benefit of click effect she receives is $\kappa_{d(i)}$ because the probability of a click event occurring in a display advertising channel is $\gamma_d$. Similarly, she can receive $\mu_{c(i)}$ from an ad click by choosing contextual advertising; however, the expected benefit of impression effect she receives is $\kappa_{c(i)}$, because her ad might have been shown several times before the ad is clicked by someone.

Furthermore, because of the limited capacity of an advertising platform, we should consider the expected delay time. As the advertising platform is shared by all subscribed ads, advertisers must wait until the ad activity is committed (e.g., an impression for display advertising and a click for contextual advertising). We assume that each advertiser's ad has the same possibility to be drawn from the pool of ads when an ad request arrives. Thus, according to geometric distribution, the expected delay time to be selected (matched) is the number of advertisers in a channel. In this paper, we adopt linear waiting costs proposed by prior research [32,51,62] and then denote the costs of expected delay time as $w_d\eta_d$ and $w_c\eta_c$, where the coefficients, $w_d$ and $w_c$, are dependent on the capacity of respective advertising platforms. The assumption of linear waiting time stems from the fact of random choice. Thus, the expected waiting time of a service could be linear for any service types that the service order is determined by random

<p>| Table 1 |</p>
<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
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<tbody>
<tr>
<td>$\eta_0$</td>
<td>The total purchase amount of advertising in this market</td>
</tr>
<tr>
<td>$\eta_d(\eta_c)$</td>
<td>The market share of display advertising (contextual advertising)</td>
</tr>
<tr>
<td>$\gamma_d(\gamma_c)$</td>
<td>The click-thru rate of display ads (contextual ads)</td>
</tr>
<tr>
<td>$\theta_d(\theta_c)$</td>
<td>The sensitivity of impression effect on the advertiser</td>
</tr>
<tr>
<td>$p_d(p_c)$</td>
<td>The price of display ads (contextual ads)</td>
</tr>
<tr>
<td>$\delta$</td>
<td>The threshold where the payoff of the advertiser $i$ with type $\theta_i - \delta$ receives is indifferent between display ads and contextual ads</td>
</tr>
<tr>
<td>$\gamma_p$</td>
<td>The price heterogeneity of display ads and contextual ads, where $\gamma_p = p_d - p_c$</td>
</tr>
<tr>
<td>$\gamma_p^*$</td>
<td>The price heterogeneity of display ads and contextual ads in an efficient market</td>
</tr>
<tr>
<td>$c_d(c_c)$</td>
<td>The unit production cost of display advertising (contextual advertising)</td>
</tr>
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<p>| Table 2 |</p>
<table>
<thead>
<tr>
<th>Notation</th>
<th>Price</th>
<th>Market share</th>
<th>Profit</th>
<th>Social welfare</th>
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<tbody>
<tr>
<td>Market type</td>
<td>Channel</td>
<td>Price</td>
<td>Market share</td>
<td>Profit</td>
</tr>
<tr>
<td>Monopoly</td>
<td>Display</td>
<td>$p_d^{\text{mon}}$</td>
<td>$\eta_d^{\text{mon}}$</td>
<td>$\kappa_d^{\text{mon}}$</td>
</tr>
<tr>
<td></td>
<td>Contextual</td>
<td>$p_c^{\text{mon}}$</td>
<td>$\eta_c^{\text{mon}}$</td>
<td>$\kappa_c^{\text{mon}}$</td>
</tr>
<tr>
<td></td>
<td>Mixed</td>
<td>$p_d^{\text{mon}}$</td>
<td>$\eta_d^{\text{mon}}$</td>
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<tr>
<td></td>
<td>Display</td>
<td>$p_d^{\text{mon}}$</td>
<td>$\eta_d^{\text{mon}}$</td>
<td>$\kappa_d^{\text{mon}}$</td>
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<tr>
<td></td>
<td>Contextual</td>
<td>$p_c^{\text{mon}}$</td>
<td>$\eta_c^{\text{mon}}$</td>
<td>$\kappa_c^{\text{mon}}$</td>
</tr>
<tr>
<td>Duopoly</td>
<td>Display</td>
<td>$p_d^{\text{dio}}$</td>
<td>$\eta_d^{\text{dio}}$</td>
<td>$\kappa_d^{\text{dio}}$</td>
</tr>
<tr>
<td></td>
<td>Contextual</td>
<td>$p_c^{\text{dio}}$</td>
<td>$\eta_c^{\text{dio}}$</td>
<td>$\kappa_c^{\text{dio}}$</td>
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<tr>
<td>Efficient</td>
<td>Mixed</td>
<td>$p_d^{*}$</td>
<td>$\eta_d^{*}$</td>
<td>$\kappa_d^{*}$</td>
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<tr>
<td></td>
<td>Display</td>
<td>$p_d^{*}$</td>
<td>$\eta_d^{*}$</td>
<td>$\kappa_d^{*}$</td>
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<tr>
<td></td>
<td>Contextual</td>
<td>$p_c^{*}$</td>
<td>$\eta_c^{*}$</td>
<td>$\kappa_c^{*}$</td>
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* The symbol is omitted because we do not discuss or analyze it in this paper.
selection by the server in which any job has the same probability to be chosen for processing. Since the waiting costs increase with the number of advertisers in the channels, we call $w_{d(t)}$ (or $w_{c(t)}$) the maximum waiting cost of a display advertising channel (contextual advertising channel). Consequently, the payoff of a typical advertiser $i$ is given by

$$
\Pi_i = \begin{cases} 
2a_i\phi_i + b_i\phi_i - w_{d(i)} - p_d & \text{display ads (an impression),} \\
\alpha_i\phi_i + (b_i/\gamma_i)\phi_i - w_{c(i)} - p_c & \text{contextual ads (a click).}
\end{cases}
$$

(3.1)

Since the weakness of text makes it hard to enhance visitor’s taste, it is natural that the value of $\phi_i$ is slight in this paper. For the purpose of exposition, we set $\phi_i$ as zero; thus, $\phi_i$ can be interpreted as the comparative impression benefit from ads with multimedia display. Since the click-thru rate of contextual advertising disappears by our assumption (i.e., the term $(b_i/\gamma_i)$ is removed from $\Pi_i$ due to $\phi_i = 0$), for the purpose of concise writing, we sometimes use the abbreviated term “CTR” to represent the click-thru rate of display advertising in the following sections. Thus, the payoff of a typical advertiser $i$ can be rewritten as

$$
\Pi_i = \begin{cases} 
2a_i\phi_i + b_i\phi_i - w_{d(i)} - p_d & \text{display ads (an impression),} \\
\alpha_i\phi_i - w_{c(i)} - p_c & \text{contextual ads (a click).}
\end{cases}
$$

(3.2)

In practice, if an advertiser has more click benefit due to her brand power, she should also have more impression benefit. Therefore, we assume $\phi_i = \theta_i + e$, where $\theta_i$ is uniformly distributed within a unit interval. Furthermore, we make the usual assumption that the market is fully covered in equilibrium when both channels are offered; that is, the following constraints are to ensure that each advertiser in the market will purchase an ad. First, in this market there exists at least one advertiser who considers that improving brand awareness is more important than direct response (i.e., $\beta_i > (\epsilon + \lambda)x_c$). Second, for each advertiser in contextual advertising channel, the benefit of delivering a visitor to her website outweighs twice[2] the maximal waiting cost (i.e., $e_\pi > 2w_{c(i)}\pi_c$). Third, $e$ is assumed to be considerably larger than $\lambda$. For the sake of simplifying mathematical expressions and equations, we define the following symbols: $\Delta p = p_d - p_c$, $\gamma^\prime = \lambda x_d + b_c$, $\Omega = \lambda x_c$, $\Gamma = e x_d + a$, and $\Lambda = e x_c$

Solving the market-segmentation equation $\psi(\theta + \Gamma) = w_{d(i)} - p_d$ yields $\theta = (\Lambda - \Gamma + w_{d(i)} - w_{c(i)} + p_c)/\psi$, where the payoff an advertiser with type $\theta$ receives is indifferent between these two ad services. All advertisers indexed by $\theta_i \in [0, \theta]$ choose contextual advertising and the others indexed by $\theta_i \in \{0, 1\}$ prefer display advertising. Finally, according to the conditions $\eta_d = \eta_d(\theta)$ and $\eta_c = \eta_c(\theta)$, the demand functions of display advertising and contextual advertising are given by

$$
\eta_d = (\psi(\theta) - (\Lambda - \Gamma + w_{d(i)} - \Delta p))/((\psi(\theta)/\eta_d) + w_{d(i)} + w_{c(i)}),
$$

(3.3)

$$
\eta_c = ((\psi(\theta) - (\Lambda - \Gamma + w_{d(i)} - \Delta p))/((\psi(\theta)/\eta_c) + w_{d(i)} + w_{c(i)}).)
$$

(3.4)

4. Pricing in the web advertisement markets

In the first part of this section, we consider a duopoly market where one channel offers display ads and the other provides contextual ads. For instance, Google and DoubleClick are two representative channels in the Web advertisement market. In order to maximize individual profit, these channel providers set their advertising prices based on their own consideration. Subsequently, we consider the scenario where these two channels are merged. For instance, the fact that Google has acquired DoubleClick for $3.1$ billion [11–13] incurs its rivals’ complaints that it would give Google an unfair monopolistic advantage in the Web advertisement market [64]. In this paper, we use the term “mixed advertising” to represent the business strategy where the monopolistic channel provider simultaneously offers display ads and contextual ads. Finally, we investigate the features of an efficient market from the viewpoint of social welfare.

4.1. Heterogeneous channel in a duopoly market

Consider a duopoly market where two channels offer different ad services, forming a price competition between display advertising and contextual advertising. The unit production costs of contextual advertising and display advertising are denoted as $c_s$ and $c_d$, respectively. $c_s$ is slight because text-only format can save designing cost and reduce traffic; here, without the loss of generality, we assume $c_s$ as zero to simplify the profit-maximization problem. Moreover, $c_d$ is composed of multimedia design cost and transmission cost, where $c_d > c_s$ ensures that the channel offering display ads can make a profit.

4.1.1. Simultaneous pricing competition

Let us first examine a simultaneous price competition between display advertising and contextual advertising. Since each channel has individual best-response function (see Appendix A) to the price announced by the other, the optimal prices can be derived by equating theses two best-response functions simultaneously, which are given by

$$
p_d^o = \frac{2(\psi - \Omega) - (\Lambda - \Gamma) + (w_d + 2w_c)\eta_d + 2c_d}{3},
$$

(4.1)

$$
p_c^o = \frac{(\psi - \Omega) - (\Lambda - \Gamma) + (w_d + w_c)\eta_d + c_d}{3},
$$

(4.2)

Plugging (4.1) and (4.2) into (3.3) and (3.4) yields the demand functions of the duopoly market which are given by

$$
\eta_d^o = \frac{2(\psi - \Omega) - (\Lambda - \Gamma) + (w_d + 2w_c)\eta_d - c_d}{3((\psi - \Omega)/\eta_d + w_{d(i)} + w_{c(i)}),}
$$

(4.3)

$$
\eta_c^o = \frac{(\psi - \Omega) - (\Lambda - \Gamma) + (w_d + w_c)\eta_d + c_d}{3((\psi - \Omega)/\eta_d + w_{d(i)} + w_{c(i)}).
$$

(4.4)

Thus, we can derive each channel’s profit from $\pi_d^o = (p_d^o - c_d)\eta_d^o$ and $\pi_c^o = (p_c^o - c_c)\eta_c^o$ as follows:

$$
\pi_d^o = \frac{(2(\psi - \Omega) - (\Lambda - \Gamma) + (w_d + 2w_c)\eta_d - c_d)^2}{9((\psi - \Omega)/\eta_d + w_{d(i)} + w_{c(i)}),}
$$

(4.5)

$$
\pi_c^o = \frac{(\psi - \Omega) - (\Lambda - \Gamma) + (w_d + w_c)\eta_d + c_d)^2}{9((\psi - \Omega)/\eta_d + w_{d(i)} + w_{c(i)}).
$$

(4.6)

(All related analytical results can be found in Table 3.)

For the sake of simplifying the discussion, we term the channel offering display advertising as channel 1 and the other as channel 2. Intuitively, each channel’s price (profit) should increase with its own marketing power and decrease with the other’s. This can be shown by differentiating $p_d^o(\pi_d^o)$ and $p_c^o(\pi_c^o)$ with respect to $\pi_d$ and $\pi_c$.

**Proposition 1** (simultaneous competition in a duopoly web ad market).

(a) As long as the impression benefit of display advertising is sufficient, each channel’s profit becomes greater when the expected delay time in any channels increases. Formally, $(\partial p_d^o/\partial \omega_d) > 0$, $(\partial p_c^o/\partial \omega_d) > 0$. Moreover, $(\partial p_d^o/\partial \omega_d) > 0$ if $\beta_d > (\epsilon + \lambda)(x_d - x_c)/\lambda$ and $(\partial p_c^o/\partial \omega_c) > 0$ (all the proofs can be found in the appendix).
Table 3
Price, market share, and profit in a monopoly market and duopoly market

<table>
<thead>
<tr>
<th>Market and channel type</th>
<th>Price</th>
<th>Market share</th>
<th>Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monopoly</td>
<td>D</td>
<td>( \frac{P + f + c_d}{2} )</td>
<td>( \frac{(P + f - c_d)}{2} ) ( \eta_0 )</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>( P - w_n \eta_0 ) ( \frac{(P - Q - (A - f) + 2w_n \eta_0 + c_d)}{2(\eta - D)/\eta_0 + w_n + w_d} )</td>
<td>( (P - Q - (A - f) - 2w_n \eta_0 + c_d) ) ( \frac{(P - Q - (A - f) + 2w_n \eta_0 + c_d)}{2(\eta - D)/\eta_0 + w_n + w_d} )</td>
</tr>
<tr>
<td></td>
<td>M</td>
<td>( P - w_n \eta_0 ) ( \frac{(P - Q - (A - f) + 2w_n \eta_0 + c_d)}{2(\eta - D)/\eta_0 + w_n + w_d} )</td>
<td>( (P - Q - (A - f) - 2w_n \eta_0 + c_d) ) ( \frac{(P - Q - (A - f) + 2w_n \eta_0 + c_d)}{2(\eta - D)/\eta_0 + w_n + w_d} )</td>
</tr>
<tr>
<td>Duopoly (simultaneous)</td>
<td>D</td>
<td>( 2(P - Q - (A - f) + w_n \eta_0 + c_d) )</td>
<td>( \frac{(P - Q - (A - f) + w_n \eta_0 + c_d)}{2} )</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>( \frac{(P - Q - (A - f) + w_n \eta_0 + c_d)}{2} )</td>
<td>( \frac{(P - Q - (A - f) + w_n \eta_0 + c_d)}{2} )</td>
</tr>
<tr>
<td>Duopoly (display first)</td>
<td>D</td>
<td>( 2(P - Q - (A - f) + w_n \eta_0 + c_d) )</td>
<td>( \frac{(P - Q - (A - f) + w_n \eta_0 + c_d)}{2} )</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>( \frac{(P - Q - (A - f) + w_n \eta_0 + c_d)}{2} )</td>
<td>( \frac{(P - Q - (A - f) + w_n \eta_0 + c_d)}{2} )</td>
</tr>
<tr>
<td>Duopoly (contextual first)</td>
<td>D</td>
<td>( \frac{(P - Q - (A - f) + w_n \eta_0 + c_d)}{2} )</td>
<td>( \frac{(P - Q - (A - f) + w_n \eta_0 + c_d)}{2} )</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>( \frac{(P - Q - (A - f) + w_n \eta_0 + c_d)}{2} )</td>
<td>( \frac{(P - Q - (A - f) + w_n \eta_0 + c_d)}{2} )</td>
</tr>
</tbody>
</table>

D, display advertising; C, contextual advertising; M, mixed advertising.

4.1.2. Sequential pricing competition

By the same approach as simultaneous pricing competition, the demand functions and profits of the duopoly market are given by

\[
P_{1,2} = \frac{2(P - Q - (A - f) + w_n \eta_0 + c_d)}{2}.
\]

Utilizing the backward induction approach, the optimal prices are determined by

\[
P_{1,2} = \frac{2(P - Q - (A - f) + w_n \eta_0 + c_d)}{2}.
\]

In Proposition 1, we find that the negative factors with respect to service quality, including \( w_n \) and \( c_d \), are positive to both channels.
If the channel provider shuts down the display adver-
sion channel, the demand function of contextual
ads (a click) is equal to \( \pi^{c}_d \). Mixed advertising
would be the best strategy except when the
importance of brand building and CTR are sufficiently small.

Thus, we can derive the monopolistic channel’s profit \( \pi_m \) as follows:

\[
\pi_m = (\Psi - \Omega) - (\Lambda - \Gamma - 2\omega_c h_0 - c) - \frac{\omega_c}{4} (\Psi - \Omega - (\Lambda - \Gamma) - 2\omega_c h_0 - c) - c_d
\]

If both channels can be merged into one giant provider in the Web
advertisement market, the profit of mixed advertising will be supe-
rior because the monopolistic channel provider can segment adver-
sers according to their interests.

**Proposition 2** (monopoly pricing in a web advertisement market).

(a) Mixed advertising would be the best strategy except when the
importance of brand building and CTR are sufficiently small.

(b) If providing mixed advertising is the best strategy, the price of
display ads (an impression) is greater than that of contextual
ads (a click).

(c) The demand of contextual advertising (display advertising) in
the monopolistic setting is higher (lower) than that in the duop-
opoly market. Formally, \( \eta_c^c < \eta_c^m \) and \( \eta_a^c > \eta_a^m \).

Here, we use comparative static analysis by varying the expected
delay time to investigate the pricing strategy of the channel pro-
vider offering mixed advertising. We find that the monopolistic
channel seeks to maximize the total profit in order to utilize its
resource optimally; therefore, when the resource is shrinking, the
only response of the channel for declining quality is to lower the
price of each advertisement, which can be shown by \( (\Psi^{c}_d / \omega_c) < 0, (\Psi^{m}_d / \omega_c) < 0, (\Psi^{c}_d / \omega_a) < 0, \) and \( (\Psi^{m}_d / \omega_a) < 0 \). Consequently, when the expected delay time increases, the number of advertise-
ments in a congested channel decreases and that in the other
channel increases, which can be verified by \( (\Psi^{c}_d / \omega_a) < 0, (\Psi^{m}_d / \omega_a) > 0, (\Psi^{c}_d / \omega_a) > 0, \) and \( (\Psi^{m}_d / \omega_a) < 0 \). Fig. 3 [17] shows the compari-
sion of profit between the monopolistic channel provider and two
heterogeneous channel providers in the Web advertisement market,
where the x-axis of the figure is the click-thru rate of display advertising. We find that the monopolistic channel provider can utilize contextual advertising to acquire enormous benefit; contrarily, because of competition, the strength of contextual advertising is limited in the duopoly market.

4.3. Analysis of market efficiency

At the end of this section, we consider an efficient market in which display advertising channel and contextual advertising channel are offered, where the scope of our considered market is composed of advertisers and channel providers. The efficiency of a market refers to the welfare aspect of the market interaction. In other words, market efficiency measure investigates whether the interaction in the market leads to a desired outcome. There is a tradeoff between the advertiser’s benefit derived from acquiring the visitor’s attention and the channel provider’s operating cost imposed on showing the advertisements, which defines a welfare function. Our goal is to utilize the welfare function to determine the optimal balance between the advertiser’s benefit and the cost of channel allocation. For instance, a third party, such as the Better Business Bureau, often plays the role of resolving individual disputes and offers effective dispute resolution programs. Here, the efficiency of an advertising market is given by \( g (\varepsilon > 0) \). Proposition 3

Thus, the demands in the efficient market are given by

\[
\begin{align*}
\sum_{H} d_{i} & = (\Omega_{i} + A - w_{i} \eta_{i} - p_{d} \eta_{d} + \pi_{c} + (\pi_{E} \eta_{i} + \Gamma - w_{d} \eta_{d} - p_{d} \eta_{d}) \eta_{d} + \pi_{d}) \\
\text{s.t.} & \quad \eta_{i} \in \left[0, \frac{\eta_{d}}{\eta_{d}} \right] \quad \text{and} \quad \eta_{d} \in \left[\frac{\eta_{d}}{\eta_{d}}, 1\right].
\end{align*}
\]

(4.26)

Thus, the demands in the efficient market are given by

\[
\begin{align*}
\eta_{d}^{-} & = \frac{(\pi_{E} - \Omega) - (A - \Gamma) + 2w_{d} \eta_{d} - c_{d} \left(\pi_{E} - \Omega\right)}{(\pi_{E} - \Omega) \eta_{d} + 2w_{d} + 2c_{d}} \\
\eta_{d}^{+} & = \frac{(A - \Gamma) + 2w_{d} \eta_{d} + c_{d}}{(\pi_{E} - \Omega) / \eta_{d} + 2w_{d} + 2c_{d}}.
\end{align*}
\]

(4.27)

Moreover, if a market is efficient, its price heterogeneity (i.e., \( p_{d} - p_{s} = \Delta \hat{p} \)) must satisfy

\[
\Delta \hat{p} = \frac{(\pi_{E} - \Omega) w_{d} + ((\pi_{E} - \Omega) / \eta_{d} + w_{d} + c_{d} - \left(2w_{d} + 2c_{d}\right) (A - \Gamma))}{(\pi_{E} - \Omega) / \eta_{d} + 2w_{d} + 2c_{d}}.
\]

Proposition 3 (efficient market). Form the viewpoint of an efficient market, the channel provider which has monopolistic power should increase (decrease) the number of advertisers in display advertising channel (contextual advertising channel). Formally, \( \eta_{d}^{-} > \eta_{d}^{+} \) and \( \eta_{d}^{+} < \eta_{d}^{*} \).

The market share of display advertising in the efficient market demonstrates that the monopolistic channel provider utilizes display advertising to serve top advertisers in the market; however, from the viewpoint of market efficiency, the monopolistic channel provider should boost the market share of display advertising by reducing the price of display advertising. Moreover, using the same parameters as in Figs. 2 and 3 shows that market efficiency in the monopolistic setting is better than that in the duopoly market, when CTR is low; however, the opposite holds true when CTR is high enough. The result reveals that the goal of the monopolistic channel provider is to maximize her profit; therefore, some advertisers cannot benefit from the high CTR because the price of display advertising is higher than their willingness-to-pay.

5. Investment of contextual search and multimedia technologies

In this section, we discuss the case where the impression effect of display advertising and the click-thru rate of contextual advertising can be adjusted by channel providers. Since the impression benefit of display advertising is related to the multimedia effect of advertising, and meanwhile, the click-thru rate of contextual advertising is associated with the quality of contextual search, the cost of producing multimedia advertising and the cost of improving the quality of search engines can be assumed to be \( c_{d} = c_{d} \hat{p}_{d}^{2} \) and \( K = \mu \hat{p}_{d}^{2} \), respectively. Consequently, the channel providers may charge a higher price due to their better ability to enhance a visitor’s impression or match Web content to ads; however, the increasing marginal cost becomes an obstacle to their profitability. Therefore, we first consider that there is a monopolistic channel provider offering mixed advertising in a market. The channel provider can decide the impression benefit and click-thru rate before determining the prices of her ad services. In other words, the optimal spending on multimedia technology and search engines are the solution to the following constrained maximization problem.

\[
\begin{align*}
\text{Max} \quad & \pi_{d}^{\max} = \Gamma \eta_{d} - w_{d} \eta_{d}^{2} + \left(\frac{(\pi_{E} - \Omega + 2w_{d} \eta_{d} - (A - \Gamma)^{2} - c_{d}^{2}}{4(\pi_{E} - \Omega) / \eta_{d} + 2w_{d} + 2c_{d}}\right) \\
\text{s.t.} & \quad \hat{\beta}_{d} > c_{d}, \quad \hat{\beta}_{d} > (\varepsilon + \lambda) s_{d}, \quad \text{and} \quad \varepsilon_{d} > 2w_{d} \eta_{d}.
\end{align*}
\]

(5.1)

In this mathematical program, all constraints had been mentioned and discussed in Section 4. For example, \( \varepsilon_{d} > 2w_{d} \eta_{d} \) is to guarantee...
that the search engine can serve all advertisers in the market, whereas \( \beta_a > (e + \lambda)x_s \), is to highlight the importance of brand building. Moreover, in order to gain positive profit from display advertising, the unit production cost of display advertising should be less than the impression benefit derived from carrying a display ad. Therefore, the unit production cost of display advertising should be less than the impression benefit derived from carrying a display ad. Here, because the closed form of analytic solution is difficult to handle, we use a numerical example to observe the channel provider’s behavior in the monopolistic setting, as shown in Fig. 4. The optimal solution is given by \((\beta_a^*, \gamma_s^*) = (864.7, 0.167)\), as shown in Fig. 5.

Subsequently, we introduce a dynamic model of duopoly in which the channel offering display advertising is still termed as channel 1 and the other as channel 2. The timing of the game is as follows: (1) Channel provider 1 and channel provider 2 can decide the impression benefit of display advertising and the click-thru rate of contextual advertising, respectively. (2) The channel providers observe \( \beta_a \) and \( \gamma_s \). (3) Then, both channel providers choose respective ad prices. Since the second period-profit functions are known in Section 4, channel 1’s best response function is given by solving the optimization problem as follows:

\[
\max_{\beta_a} p_d^c(\beta_a; \gamma_s^*) = \frac{(2(\Psi - \Omega) - (A - \Gamma) + (w_d + 2w_s)e_0 - c_d)^2}{9((\Psi - \Omega)/\eta_0 + w_d + w_s)}
\]

s.t. \( \beta_a > c_d \), \( \beta_a > (e + \lambda)x_s \).

Similarly, channel 2’s best response function is given by solving the optimization problem as follows:

\[
\max_{\gamma_s} \pi_2^c(\beta_a^*, \gamma_s) = \left( \frac{(\Psi - \Omega) + (A - \Gamma) + (2w_d + w_s)e_0 - c_d}{\Psi - \Omega}/\eta_0 + w_d + w_s \right)^2 - K
\]

s.t. \( e_0x > 2w_0\eta_0 \).

For the same reason as the above monopolistic case, we use a numerical example to illustrate the first period-Nash equilibrium of the dynamic model. Given the same parameters as in Fig. 4, the numerical result of each channel’s best response is shown in Fig. 6, where \((\beta_a^*, \gamma_s^*) = (1393.1, 0.0125)\). From Fig. 6, we find that both channel providers tend to gain more profit by decreasing their effort on multimedia technology and the quality of contextual search.

Comparing these two scenarios, we find that the amount of investment in multimedia technology in the duopoly market is higher than that in the monopolistic setting. Furthermore, we find that the channel provider in the monopolistic setting would reduce expenses on multimedia technology and concentrate on the quality of search engines. In addition, in a duopolistic market, the channel offering contextual advertising would provide the lowest quality search engine to serve its advertisers. That is, in a duopolistic market, the value of contextual advertising cannot truly be revealed due to price competition. Therefore, these numerical results may shed light on why acquisition is intensive among these channels offering contextual advertising in the real world.

6. Summary and Discussion

In a duopolistic market, contextual advertising becomes an innovative means to create another income stream for channel providers taking advantage of search engines to penetrate the Web advertising market which is handled by a few large-scale channels offering display advertising. On the other hand, in a monopolistic setting, contextual advertising can be treated as a vertical product differentiation strategy with which to acquire enormous profit. Our major premise was that there exist some advertisers in the market who consider that brand building is more important than delivering visitors to appointed websites. In addi-
tion, the channel provider offering contextual advertising has the ability to serve all advertisers in the market. Consequently, we give the causes and effects for each question proposed in Introduction as follows.

6.1. Summary of Findings & Managerial Implications

Question 1: Could the higher quality of a search engine boost the profit of the channel provider offering contextual ads? Moreover, under competition, what are the conditions that would lead to the increase (decrease) of the channel provider’s profit?

In a duopolistic market, the answer is positive only if the impression effect generated by display advertising is not useful. Moreover, we find that the profitability of each channel in a duopolistic market may increase with the channel’s marketing power and waiting time. As long as impression benefit generated by display advertising is sufficient, the profitability of each channel provider in a duopolistic market may increase when any channel provider’s waiting cost becomes higher (i.e., increasing delay time for an exposure chance). Actually, increasing service quality can improve profitability in a monopolistic setting; however, this result in a competitive setting shows a contradictory outcome. The reason for this counterintuitive result is as follows. In the competitive setting, each channel provider’s pricing decision is conditional on the other channel provider’s. If one channel can reduce its service rate, then the other would boost ad price in response. Thus, although increasing waiting time may mean losing some customers, the channel can compensate for its loss by boosting the ad price due to its competitor’s response. As a consequence, equilibrium profits of both channels may increase.

Question 2: What is the sufficient condition for ensuring that a market becomes efficient? Is a search engine company joining the advertising market socially beneficial from the viewpoint of market efficiency (social welfare)?

We suggest that the click-thru rate of display advertising can be used as the condition. When the click-thru rate of display advertising is low, a monopoly is more efficient than competition. However, when the click-thru rate of display advertising is high, the opposite holds true. Therefore, if statistical data shows that the click-thru rate of display advertising is at a higher level, a search engine company joining the advertising market will be beneficial to society. However, if both channels are merged, the monopolistic channel provider would sacrifice some advertisers’ benefit and enforce them to purchase contextual advertising by raising the price of display ads. Therefore, the government should reject the deal because the acquisition would lead to an inefficient Web advertisement market and advertisers would become the losers in the market.

Question 3: It is considered that contextual text links lack the impression effect; therefore, do the quite high quality display ads threaten the profit gained by contextual ads?

Our result shows a contradictory result in a duopolistic market. The reason for this result is as follows: (1) one channel provider would raise its ad price when the other channel provider does the same; (2) the target consumers of contextual advertising involve top-level advertisers who have higher valuation on brand building in the market and those buyers are willing to pay more for a better display ad. Therefore, the price of contextual advertising may follow the price of display advertising, which increases considerably such that both channel providers could make more profits.

In addition, we investigate how channel providers control spending on multimedia technology applied to display ads and the click-thru rate of contextual ads to maximize their profits. The monopolistic channel provider would boost the click-thru rate and reduce her investment on multimedia technology. On the contrary, in a duopolistic market, the channel provider offering contextual ads has no incentive to improve the click-thru rate, whereas the channel provider offering display ads would spend much more capital than would the monopolistic channel provider on multimedia technology. This reveals that the actual strength of search engines cannot be utilized in the duopolistic market; therefore, it is likely to be one of the most important motivations why Google acquired DoubleClick. By constructing the framework that relates causes and effects in the online advertising market, the impact of this study to each stakeholder in Fig. 1 is given as follows:

6.1.1. Channels

In a monopolistic market, both stronger brand power and higher service rate have a positive effect on revenue. In a duopolistic market, because higher brand power is helpful to the revenue of channels, they should enhance their brand power as much as possible. Meanwhile, because reducing waiting time may negatively affect their profitability, they should keep their service rate at an “appropriate” level. Comparing both online advertising market structures, we find that the amount of investment in multimedia technology in the duopolistic market is higher than that in the monopolistic setting. Furthermore, we find that the channel provider in the monopolistic setting invests more on the quality of contextual search. On the contrary, in a duopolistic market, the contextual advertising channel has no incentive to raise the quality of contextual search. Therefore, for the contextual advertising channel in the duopolistic market, because its profit cannot be enhanced by improving the click-thru rate, we suggest that the channel provider should consider buying her major competitor in order to utilize advantage of search engines. For the display advertising channel in the duopolistic market, it can safely develop better multimedia ads because this action leads to a win-win result.

6.1.2. Advertisers

For small advertisers, the existence of contextual advertising offers them a channel to carry Web advertising for the first time. Our results show that small advertisers with lower branding power would prefer contextual ads than display ads, whereas top advertisers with stronger branding power would prefer display ads than contextual ads. In addition, according to the analysis of market efficiency, a monopolistic ad channel would sacrifice some advertisers’ benefit by raising the price of display ads. Therefore, if the click-thru rate of display advertising is at a higher level, it is better for them to block two competing ad channels merged. Meanwhile, the government also should attach great importance to the issue.

6.1.3. Publishers

In general, contextual advertising and display advertising are the major source of revenue for small publishers and top publishers, respectively. Because channel providers can transfer advertising fees paid by advertisers to publishers with a committed ratio, the impact of this study on publishers is as follows. First, if the revenue of publishers is positively associated with the profit of channels, such as some kinds of revenue sharing, the revenue gained from contextual ads in the monopolistic setting is higher than that in the duopolistic market. On the other hand, if the click-thru rate of display ads is sufficiently low, the revenue gained from display ads in the monopolistic setting is lower than that in the duopolistic market. If the click-thru rate of display ads is sufficiently high, the opposite holds true. Second, if the revenue of publishers is positively associated with market share of channels, the revenue gained from display ads is greater than in a monopolistic market, whereas the revenue gained from display ads in the monopolistic setting is lower than that in the duopolistic market.
6.1.4. Visitors

Because visitors would receive more useful information from contextual ads when the quality of contextual search increases and see more entertaining content from display ads when the multimedia effect becomes stronger, IT investment on contextual search and multimedia technology would attract visitors deeply in the study. Therefore, the quality of contextual search in the monopolistic setting is better than that in the duopolistic market; visitors would see more useful banner ads when both channels coexist in the online advertising market.

6.2. Limitation and directions for future study

Admittedly, the strategy, reducing service capacity in the duopolistic market, may boost both channel providers’ profits in a short run. However, from the long-term viewpoint, the expected delay time to be selected should be reduced, such that current advertisers can be served expeditiously; thus, oncoming advertisers may consider placing their orders for the ad service due to efficient service rate. Since increasing service capacity leads to price wars and also appeals to prospective advertisers, the question of optimal delay time remains to be solved. Furthermore, for facilitating mathematical analysis, an advertiser’s utility and the cost of delay time are linear in this paper. Indeed, a nonlinear utility function (the cost of delay time) works well conceptually; however, it would lead to no closed form of market share function.

Therefore, we sacrifice a little reality of the model; however, it would lead to no closed form of market share function. Thus, all related analytic results can be derived by suitable differentiation and substitution. By letting $\eta_0^m = 0$, we can find that the service (i.e., display ads) will be terminated as long as $\beta_d \leq (1 + \lambda) (x_d - x_d g) - 2w_d \eta_0 + c_d$ holds.

\textbf{Appendix A. Proof of Proposition 1}

By first-order conditions, we can derive the best-response functions of these two ad services, which are given by

$$p_d^*(\bar{p}_d^*) = \frac{p_d^0 + (\Psi - \Omega) + w_d \eta_0 - (\Gamma - \Gamma_d) + c_d}{2}$$

and

$$p_c^*(\bar{p}_c^*) = \frac{w_c \eta_0 - (\Gamma - \Gamma_d) + c_d}{2}$$

Solving both equations simultaneously, we have Nash equilibrium $p_d^0$ and $p_c^0$; thus, the demand functions and the profit functions can be solved by plugging $p_d^0$ and $p_c^0$ into $\eta_d$, $\eta_c$, $\pi_d^0$, and $\pi_c^0$.

\textbf{Appendix B. Proof of Proposition 2}

By first-order condition, the optimality conditions of the maximization problem are given by $(\partial \pi_m^0 / \partial p_m^0) \leq 0$ and $(\partial \pi_m^0 / \partial p_c^0) \leq 0$. Obviously, these two equations cannot be solved simultaneously because no such $\Delta p$ can satisfy this system. Notice that $p_m^0 = A - w_m \eta_0$ is the maximal price for contextual advertising. However, for any interior solution (i.e., $p_m^0 < A - w_m \eta_0$), there exists some feasible direction $d$ such that $d^\top \nabla(p_m^0, p_c^0) > 0$, which implies that the relative maximal point does not locate in the inside region; in other words, we only consider the boundary case. Plugging $p_m^0 = A - w_m \eta_0$ into $\eta_m^0$, the relationship between $p_m^0$ and $\eta_m^0$ is given by $\eta_m^0((\Psi - \Omega)/\eta_0 + w_d) + \Gamma - w_d \eta_0 = p_d^0$, and then the problem can be rewritten as follows:

$$\begin{align*}
\text{Max } & \pi_m^0 = (\eta_m^0((\Psi - \Omega)/\eta_0 + w_d) + \Gamma - w_d \eta_0 - \eta_d) (\eta_0 - \eta^m_d) \\
\text{s. t. } & \eta_m^0 \leq \eta_0.
\end{align*}$$

Thus, all related analytic results can be derived by suitable differentiation and substitution. By letting $\eta_0^m = 0$, we could find that the service (i.e., display ads) will be terminated as long as $\beta_d \leq (1 + \lambda) (x_d - x_d g) - 2w_d \eta_0 + c_d$ holds.

\textbf{Appendix C. Proof of Proposition 3}

The mathematical program of market efficiency (social welfare) given by (4.26) can be transformed as follows:

$$\begin{align*}
\text{Max } & W = \frac{Q(\eta_0 - \eta_d)^2}{2\eta_0} + A(\eta_0 - \eta_d) - w_d (\eta_0 - \eta_d)^2 \\
& + \Psi \frac{(2\eta_0 - \eta_d \eta_d)}{2\eta_0} + \Gamma \eta_d - w_d \eta_0 - c_d \eta_d.
\end{align*}$$

Solving the program and utilizing $\eta_d = \eta_d^0 + \eta_d^0$, yield the result of efficient allocation for each channel given by $\eta_d^0$ and $\eta_d^0$. Letting $\eta_d$ equal to $\eta_d^0$, we can solve the equation by viewing $\Delta p$ as an unknown variable; thus, we could derive $\Delta p$.

\textbf{References}


