Abstract—In the real world, most object shapes are perspectively transformed when imaged. How to recognize or locate such shapes in images is interesting and important. The conventional generalized Hough transform (GHT) is useful for detecting or locating translated two-dimensional (2D) planar shapes. However, it cannot be used for detecting perspectively transformed planar shapes. A new version of the GHT, called perspective-transformation-invariant generalized Hough transform (PTIGHT), is proposed to remove this weakness. The PTIGHT is based on the use of a new perspective reference table that is built up by applying both forward and inverse perspective transformations on a given template shape image from all viewing directions and positions. Due to the use of the point spread function to express the perspective reference table, the required dimensionality of the Hough counting space (HCS) for the PTIGHT is reduced to 2D. After performing the PTIGHT on an input image, the peaks in the HCS whose values are larger than a threshold is picked out as the candidate locations of the perspective shape to be detected in the input images. By performing an inverse PTIGHT on the candidates, one of the candidate locations whose corresponding shape matches best with the input shape is selected and the desired parameters of the perspective transformation can be obtained. Some experimental results are included to demonstrate the applicability of the proposed PTIGHT. © 1997 Pattern Recognition Society. Published by Elsevier Science Ltd.

1. INTRODUCTION

In practical applications, most object shapes are perspectively transformed when imaged. How to recognize or locate such object shapes in images is interesting and important. The Hough transform is useful for detecting or locating straight lines or analytic curves. A generalized version of the Hough transform, called the generalized Hough transform (GHT), was proposed for detecting arbitrary two-dimensional (2D) planar shapes. It has been used in many applications of computer vision such as shape detection and recognition, image registration, etc. However, a weakness of the GHT is that it can only be used to detect translated shapes and is unsuitable for detecting perspectively transformed planar shapes from unknown viewpoints. To improve the GHT, Silberberg et al. proposed an iterative Hough procedure for 3D object recognition, and Jeng and Tsai proposed a scale- and orientation-invariant GHT to detect rotated and scaled shapes.

To improve the GHT further, a new version of the GHT, called perspective-transformation-invariant GHT (PTIGHT), is proposed in this study. The PTIGHT can be used for detecting a perspectively transformed planar shape in a perspective image that is taken from an unknown viewpoint. The new approach is used on the GHT using a new perspective reference table (PR-table). In order to build a PR-table from a given template shape image, we first apply an inverse perspective transformation on the given template shape image to derive the points of a certain 2D shape called the original planar shape. Next, all possible perspective shape images are derived by applying forward perspective transformations on the derived original planar shape from all viewing directions and positions. The position of each shape point in each perspective shape image is represented by a displacement vector relative to a reference point (usually selected to be the shape centroid). The displacement vectors are then rotated 180° with respect to the reference point. The PR-table, one kind of point spread function, is constituted by superimposing the displacement vectors of all the perspective shapes with respect to an identical reference point. The PR-table contains the information of all possible perspective transformations of the original planar shape. The process of constructing the PR-table is time-consuming but it can be completed in advance before the PTIGHT is performed.

To perform the PTIGHT, a 2D Hough counting space (HCS) consisting of accumulator cells is constructed.
The cell value incrementation strategy of the PTIGHT is similar to that of the conventional GHT except that the PR-table instead of the conventional R-table is used here. Although more than two perspective transformation parameters are to be found, only two parameters, namely, the x- and y-translations, are required so that the HCS is reduced to 2D as that required by the conventional GHT. Furthermore, because the PR-table is built in the form of a point spread function which contains all the perspective-transformation informations, the processing time for cell value incrementation can be shortened. After performing the PTIGHT, the peaks whose values are larger than a threshold in the HCS are detected as the candidate locations of the perspective planar shape in the input perspective image. By performing an inverse process of the PTIGHT proposed in this study, called inverse PTIGHT, to the candidate locations and selecting one of them whose corresponding shape matches best with the input shape, the desired parameters of the perspective transformation of the input shape image can be obtained finally.

The remainder of this paper is organized as follows. In Section 2, we present the formulas for general forward and inverse perspective transformations between two planes. In Section 3, we review the conventional GHT and describe the proposed inverse GHT. In Section 4, the proposed PTIGHT is described in detail. Several experimental results are given in Section 5. Some concluding remarks and discussions are given in Section 6.

2. FORWARD AND INVERSE PERSPECTIVE TRANSFORMATIONS

The formulas for general forward and inverse perspective transformations between two planes are described in many papers. Consider the perspective transformations relationship between the plane π of a perspective planar shape and its image plane π as shown in Fig. 1. The transformation can be described with two coordinate systems. One is the camera coordinate system (CCS) denoted as X, Y, Z. The other is the natural coordinate system (NCS) denoted as X₀, Y₀ on perspective shape plane π₀. The observer’s viewpoint is located at the origin O=(0, 0, 0) of the CCS and the visual axis coincides with the Z-axis. Point F=(0, 0, C₂) in plane π₀ is called the fixation point, where the parameter C₂ represents the distance between the observer’s viewpoint and the fixation point. Let the origin of the NCS coincide with the fixation point and its Y₀-axis have the direction of the cross product of the vector normal to plane π₀ and that normal to plane π. It is convenient to use the NCS to

![Fig. 1. Perspective transformation relationship between a perspective shape plane and its image plane. The reference point of the perspective planar shape is (X₀, Y₀, Z₀) and the reference point of the perspective shape image is (X', Y', f).](image_url)
describe a planar shape in any perspective shape plane. Two other parameters of plane $\pi_0$ are the pan angle $\tau$ and the tilt angle $\sigma$, where $\tau$ is the angle between the $X$-axis and the projection of the normal to perspective shape plane $\pi_0$ on image plane $\pi$, and $\sigma$ is the angle between the $Z$-axis and the normal. Assume that the direction of angle $\tau$ in Fig. 1 is positive and that of $\sigma$ is negative. From the geometric relations shown in Fig. 1, the transformations between perspective shape plane $\pi_0$ and image plane $\pi$ can be derived according to the following steps:

1. The equation of image plane $\pi$ in the CCS is
   \[ Z = f, \]
   where $f$ is the distance between the observer’s viewpoint (or the lens center of the camera) and the image plane.

2. The normal to perspective shape plane $\pi_0$ in the CCS is
   \[ n = (-\sin \sigma \cos \tau, -\sin \sigma \sin \tau, \cos \sigma). \]

3. The equation of perspective shape plane $\pi_0$ in the CCS is $n \cdot (X', Y', Z') = C_2 \cos \sigma$ which can be transformed into
   \[ Z' = AX' + BY' + C_2, \]
   where $A = \tan \sigma \cos \tau$, $B = \tan \sigma \sin \tau$.

4. Let the viewpoint $O$, point $P$ in $\pi$ with CCS coordinates $(X, Y, f)$, and point $Q$ in $\pi_0$ with CCS coordinates $(X', Y', Z')$ be collinear. Then $P$ is the image of $Q$. This implies the following equations:
   \[ \frac{X'}{X} = \frac{Y'}{Y} = \frac{Z'}{Z}. \]
   Then, we can solve $X'$, $Y'$, and $Z'$ from equations (3) and (4) to be
   \[ \begin{align*}
   X' &= \frac{C_2 X}{f - AX - BY}, \\
   Y' &= \frac{C_2 Y}{f - AX - BY}, \\
   Z' &= \frac{f C_2}{f - AX - BY}.
   \end{align*} \]

5. Now, we can derive the inverse perspective transformation from image plane $\pi$ to perspective shape plane $\pi_0$. First, we transform the CCS coordinates $(X', Y', Z')$ of $Q$ in $\pi_0$ into its NCS coordinates $(320, 110)$ as follows:
   \[ \begin{align*}
   X_0 &= C_2 \frac{X \cos \tau + Y \sin \tau}{\cos \sigma} - \frac{Y \sin \sigma \cos \tau - Y \sin \sigma \sin \tau}{\cos \sigma}, \\
   Y_0 &= C_2 \frac{-X \sin \tau + Y \cos \tau}{\cos \sigma}, \\
   \end{align*} \]
   where the peak detection method is found unstable. A peak so found sometimes does not necessarily correspond to a real template shape, but just a collection of shape pieces which come from different object shapes and happen to vote in the identical cell where the peak is located.

To overcome the above difficulty, one way is to detect those cells in the HCS that have values exceeding a certain threshold, regard the corresponding locations of
these cells in the image space as candidate shape locations, and verify the existence of the template shape at each of these locations.

For the purpose of such template shape verification, we propose in this study a new technique called inverse GHT. To apply the inverse GHT, first an inverse R-table is constructed in all the same way as that for the R-table except that the template shape is not rotated 180° before the displacement vectors are collected. Then, to verify if the template shape exists at a candidate location in the input image, the inverse R-table is superimposed on the location and a counter is created for this location. The counter is incremented by one for each shape point in the input image whose location is pointed to by a displacement vector of the inverse R-table. If the final counter value is close to the number of the total pixels in the template shape, then it is decided that the template shape exists at the candidate location. This process is illustrated in Figs 2(d) and (e). Sometimes, several candidate locations may be verified, and the one with the largest counter value is selected, which will be called the optimal candidate.

4. PROPOSED PERSPECTIVE-TRANSFORMATION-INVARIANT GENERALIZED HOUGH TRANSFORM

In actual applications, given a template image of a planar shape and an input perspective image containing a perspectively transformed version of the planar shape, how do we verify that the shape in the input image is a perspectively transformed version of the planar shape in the template image? It is mentioned previously that the conventional GHT is unsuitable for this problem. So the PTIGHT is proposed. The basic steps are similar to those of the conventional GHT, including construction of a PR-table, creation of an HCS, incrementation of cell values, and HCS peak detection. However, an additional process, called inverse PTIGHT, is proposed for verification of candidate shape locations. The details are described in the following.

4.1. PR-table construction

The purpose of constructing the PR-table is to include the information of all possible perspective shapes. Before describing the steps for constructing the PR-table, we have to derive appropriate formulas for creating all possible perspective shapes from the given template shape image. For this, basically we do the following two major works: (1) perform an inverse perspective transformation to derive the original planar shape from the given template image; (2) perform forward perspective transformations to derive all possible perspectively transformed versions of the original planar shape.

4.1.1. Formula derivation. The detailed steps we employ to derive formulas for creating possible perspective shapes are as follows.

(1) Assume that the shape $T$ on the given template image is the image of a certain shape $S$, called the original planar shape, taken without any pan and tilt at a distance $C_1$ from the origin of the CCS under the condition that the template image plane and the plane on which $S$ appears (called original shape plane henceforth) are parallel (see Fig. 3 for an illustration).

(2) Let $P_t$ be a point with CCS coordinate $(x, y, f)$ of shape $T$ on the given template image, as shown in Fig. 3.

(3) Derive the CCS coordinates $(x', y', C_1)$ of point $Q_t$ on original planar shape $S$ corresponding to $P_t$ on $T$ using equations (5). Note that the original shape plane is just a special case of the perspective shape plane mentioned previously (see Fig. 1), so equations (5) are applicable here with tilt $\sigma=0$, pan $\tau=0$, and $C_2=C_1$. The derivation results are as follows:

$$x' = C_1 \frac{x}{f}, \quad y' = C_1 \frac{y}{f}.$$
Perspective-transformation-invariant generalized Hough transform

Z or \( X \) of 

\[
\begin{align*}
(0, 0, C_3) \\
\end{align*}
\]

Original Shape Plane

\( f(Xf \cos \sigma - Yf \sin \sigma)/\cos \sigma \\
C_2 \\

\( f - Xf \tan \sigma \cos \tau - Yf \tan \sigma \sin \tau \),

Y of \( Xf \sin \sigma + Yf \cos \sigma \\
C_2 \\

\( f - Xf \tan \sigma \cos \tau - Yf \tan \sigma \sin \tau \).

(10)

(5) Since the shape appearing in the original shape plane (see Fig. 3) is actually identical to that appearing in perspective shape plane \( \pi_0 \) (see Fig. 1), each pair of corresponding points in the two shapes have the same relative displacements with respect to their respective reference points. Moreover, the NCS coordinates \((X_0, Y_0)\) of each point \(Q\) of the shape on perspective shape plane \( \pi_0 \) (see Fig. 1) can be expressed as

\[
X_0 = X_0' + \Delta X_0, \quad Y_0 = Y_0' + \Delta Y_0,
\]

where \((\Delta X_0, \Delta Y_0)\) are the relative displacements of \(Q\) with respect to the reference point \((X_0', Y_0')\). Also, the relative displacement of point \(Q_i\) on original planar shape \(S\) (see Fig. 3), which corresponds to point \(Q\) on perspective shape plane \(\pi_0\), with respect to the reference point \((0, 0, C_1)\) is just \((x', y', C_1)\). Therefore, we have \(\Delta X_0 = x', \Delta Y_0 = y'\). Further from equations (9), we get

\[
\begin{align*}
X_0 &= X_0' + C_1 x' f, \\
Y_0 &= Y_0' + C_1 y' f. \\
\end{align*}
\]

(11)

(6) Thus, from equations (10) and (11), equation (8) can be rewritten as follows:

\[
X = f \frac{X_0 \cos \sigma \cos \tau - Y_0 \sin \tau}{X_0 \sin \sigma + C_2} = f \left[ \frac{(X_0/C_2) + (C_1/C_2)(x'/f)}{\cos \sigma \cos \tau - [(Y_0/C_2) + (C_1/C_2)(y'/f)] \sin \tau} \right],
\]

\[
Y = f \frac{X_0 \cos \sigma \sin \tau + Y_0 \cos \tau}{X_0 \sin \sigma + C_2} = f \left[ \frac{(X_0/C_2) + (C_1/C_2)(x'/f)}{\cos \sigma \sin \tau + [(Y_0/C_2) + (C_1/C_2)(y'/f)] \cos \tau} \right].
\]

(12)

Note that in the derivation we have \(X = x, X' = x', Y = y, Y' = y'\) in equations (5).

(4) When using the PHTHT to check if a perspective-transformed shape exists in the input image, it will result in the form of locating a reference point of the perspective-transformed shape in the input image. Let the CCS coordinates of the reference point of the perspective-transformed shape be \((X_f, Y_f, f)\). Then the NCS coordinates \((X_0, Y_0)\) of the corresponding reference point of the shape in perspective shape plane \(\pi_0\) (see Fig. 1) can be derived from equations (7) as follows:

\[
X_0 = C_2 \frac{(X_f \cos \tau + Y_f \sin \tau)/\cos \sigma}{f - X_f \tan \sigma \cos \tau - Y_f \tan \sigma \sin \tau},
\]

\[
Y_0 = C_2 \frac{-X_f \sin \tau + Y_f \cos \tau}{f - X_f \tan \sigma \cos \tau - Y_f \tan \sigma \sin \tau}.
\]

where \(X_0/C_2\) and \(Y_0/C_2\) are computed by equations (10) and \(C = C_1/C_2\). Note that in equations (12), there are totally five controllable parameters \(C, \sigma, \tau, (X_f, Y_f)\) where \((X_f, Y_f)\) appears in the computation of \((X_0/C_2, Y_0/C_2)\) by equations (10).

4.1.2. Algorithm for construction of PR-table. Now, given a template image \(T\) of a planar shape, we can construct the PR-table from it using the relevant formulas derived previously. As an illustrative example, let the given template shape image \(T\) be shown in Fig. 4(a) which includes a planar arrow shape. The PR-table are constructed from image \(T\) by the following major steps:

(1) Regard each image pixel in the image plane with CCS coordinates \((X_f, Y_f, f)\) as the reference point.

Fig. 3. Transformation relationship between the original planar shape \(S\) and its template shape image \(T\). The reference point of the original planar shape is \((0, 0, C_3)\) and the reference point of its template shape image is \((0, 0, f)\).
of the certain perspective shapes, and compute the corresponding reference point with NCS coordinates \((X_0, Y_0)\) on the perspective shape plane by equations (10).

(2) Compute the locations of the image points of all possible perspective shapes in the image plane by equations (12) with different perspective parameters. For example, three possible perspective shape images denoted as a, b and c are shown in Fig. 4(b).

(3) Rotate each perspective shape in the image plane through 180° with respect to its reference point and then translate it in such a way that its reference point coincides with a preselected common reference point selected to be the centroid of \(T\). Finally, the PR-table is constituted by superimposing all the perspective shape images, as shown in Fig. 4(c).

The detailed steps to build the PR-table are described in the following as an algorithm.

**Algorithm 1. Building a PR-table from a given template shape image.**

*Input:*
1. A given template image \(T\) of a planar shape.
2. The focus length \(f\) of the camera.
Perspective-transformation-invariant generalized Hough transform

Fig. 5. Illustration of cell value incrementation: (a) an input image containing one perspective transformed shape; (b) superimposition of part of the PR-table on two shape points (denoted as 1 and 2) of the input image; (c) 3D shape of the HCS after completing cell value incrementation; (d) the HCS of (c) shown as an image.

Output:
A PR-table.

Steps:
1. Initialization: Form a 2D array $B$ as the PR-table, and set the values of all the array elements to zero.
2. Building PR-table: Select the centroid $Q$ of the template image $T$ as a reference point and let its CCS coordinates be $(0, 0, f)$. Also, regard $Q$ as a common reference point for all perspective shapes on the image plane to be derived. Let the corresponding reference point of each possible perspective planar shape be denoted as $E$ and the corresponding reference point of the perspective shape on the image plane be denoted as $R$.
   2.1. For each possible location with CCS coordinates $(X, Y, f)$ in the image plane (regarded as a reference point $R$ of possible perspective shapes),
   2.2. for $C = 0.5$ to 2.0 with increment of 0.1,
   2.3. for $\tau = -45^\circ$ to $45^\circ$ with increment of $5^\circ$,
   2.4. for $\sigma = -45^\circ$ to $45^\circ$ with increment of $5^\circ$,
   2.5. compute the NCS coordinates $(x_{gr}, y_{gr})$ of the reference point $E$ in the perspective shape plane corresponding to $R$ by equations (10) [note that $(x_{gr}, y_{gr})$ are used in computation of equations (12) of Step 2.7];
   2.6. for each point $P_i$ of the template shape in the given image $T$,
   2.7. compute the location of the corresponding point $P_i'$ of the perspective shape in the image plane by equations (12), rotate $P_i'$ through $180^\circ$ with respect to $R$, and translate the displacement vector $V_i$ between $P_i'$ and $R$ in such a way that $R$ coincides with the common reference point $Q$;
   2.8. increment by 1 the value of the cell in $B$ which is pointed to by the displacement vector $V_i$;
   2.9. end;
3. End.

Note that in the above algorithm, the possible ranges for the five parameters are limited to those found ade-
quate for our experiments and most applications. Continuing the illustrative example, after performing Algorithm 1 in Fig. 4(a), we get the complete PR-table as shown in Figs 4(d) and (e). The PR-table not only records the displacement vector between each pixel and the reference point of each perspective shape image but preserves the perspective information. Furthermore, the PR-table is built up intrinsically as a point spread function.

4.2. Cell value incrementation

The cell value incrementation strategy is similar to that of the conventional GHT except that the PR-table instead of the R-table is used and that the cell increment value is not always 1. To perform the PTIGHT, a 2D HCS consisting of accumulator cells is constructed first. Then, superimpose the common reference point of the PR-table on each shape point in the input image and add the value of each array element $B_i$ of the PR-table to the corresponding cell of the HCS which is pointed to by the displacement vector of $B_i$. Note that the value of $B_i$ in the PR-table is used as the increment value here, which is not necessarily the value of 1 as used in the cell value incrementation stage of the conventional GHT. As an illustrative example, let an input image be shown in Fig. 5(a). An illustration of the result of superimposing part of the PR-table (including only three perspective shapes) on two shape points is shown in Fig. 5(b). The result of complete cell value incrementation is shown in Figs 5(c) and (d). Figure 5(c) shows the 3D shape of the resulting HCS. Figure 5(d) shows the HCS as an image.

4.3. Candidate shape locations and inverse PTIGHT

After the cell value incrementation process is completed, we locate candidate locations of the perspective shape in the input perspective image by finding the peaks whose values in the 2D HCS are larger than a threshold value. The reason why we do not detect the perspective shape simply by finding the peak in the HCS with the maximum value has been explained in Section 3 where we proposed the inverse GHT. In order to find the optimal candidate and retrieve the corresponding perspective parameters, we perform the inverse PTIGHT on all the candidate locations. The inverse PTIGHT is similar to the inverse GHT and the detailed steps are described in the following algorithm.

![Fig. 6. Illustration of perspective airplane detection: (a) the template airplane image; (b) input image simulated by using $\sigma=-45$, $\tau=30$, $C=0.8$ and reference point (r.p.) of the perspective airplane image at (32, -32); (c) another input image like (b) simulated by using $\sigma=30$, $\tau=15$, $C=1.4$ and the r.p. at (32, -32); (d) the PR-table shown as 3D shape; (e) the same as (d) but shown as an image; (f) and (g) the resulting HCSs after the PTIGHT are performed on (b) and (c) respectively; (h) and (i) all detected candidates in (f) and (g), respectively; (j) and (k) recomputed perspective shapes using retrieved sets of parameters.](image-url)
Algorithm 2. Inverse PTIGHT.

Input: A set of candidate shape locations in the HCS and the input template shape image $T$.

Output: A set of desired perspective transformation parameters of the input perspective shape in the input image.

Steps:

1. For each candidate shape location $(X_f, Y_f)$,
   1.1. for each possible set of perspective parameters $C, \sigma$ and $\tau$,
   1.2. Construct perspective shape: Construct the image of a perspective shape $U$ from original template shape image $T$ by equations (12) using $(X_f, Y_f), C, \sigma$ and $\tau$, and count the total number $C_1$ of pixels of $U$;
   1.3. Build inverse PR-table: Considering the perspective shape $U$ as a template shape, select the centroid $R$ of $U$ as a reference point, form a 2D cell array $S_i$ as the inverse PR-table, set all values of $S_i$ to zero, and for each point $P_i$ of $U$, increment by 1 the value of the cell in $S_i$ which is pointed to by the displacement vector $V_i$ between $P_i$ and $R$;
   1.4. Inverse counting: Create a counter $C_2$, set its content to zero, superimpose all the displacement vectors of the inverse PR-table on the candidate shape location, and increment $C_2$ by 1 each time a point of $U$ is pointed to by a displacement vector of the inverse PR-table.

1.5. end.

2. Optimal candidate detection: Find the optimal candidate location with the maximum ratio of $C_2$ to $C_1$ among all possible sets of perspective parameters and all candidate locations. The perspective transformation parameters of the corresponding perspective transformed template shape are output as the desired parameters.

3. End.

4.4. PTIGHT algorithm

Now we are ready to describe the overall PTIGHT algorithm as follows:

Algorithm 3. PTIGHT for detecting a template shape from an input perspective image.

Input:

1. A given template image $T$ of a known planar shape.
2. An input image containing a partial or full perspective transformed version of the known planar shape whose viewpoint is unknown.
3. The focus length $f$ of the camera and a threshold value $r_v$.

**Output:**
1. A location in the input image where the perspective transformed shape appears (or more specifically, where the reference point of the perspectively transformed shape is located).
2. A set of parameters for describing the perspective transformation from the perspective shape plane to the image plane.

**Steps:**
1. **Initialization:** Form a 2D HCS and set all values of the cells in the HCS to zero.
2. **Building PR-table:** Build the PR-table from the given template image $T$ by Algorithm 1.
3. **Cell value incrementation:** Increment the values of the cells in the HCS using the PR-table by the process described in Section 4.2.
4. **Detecting candidate shape locations:** Find the candidate shape locations in the HCS with all values exceeding $r_v$.
5. **Performing inverse PTIGHT:** Detect the optimal candidate and retrieve the corresponding perspective parameters by Algorithm 2.
6. **End.**

In the above algorithm, Step 2 is time-consuming but can be completed beforehand. Due to using the point
5. EXPERIMENTAL RESULTS

The proposed PTIGHT algorithm has been implemented on a SUN SPARC 10 workstation and several sets of images have been tested. Some experimental results are shown in Figs 6–8 using the focus length \( f = 1628 \) pixels.

In Fig. 6, an airplane template image with \( 32 \times 32 \) pixels is shown in Fig. 6(a) and two input perspective images with \( 128 \times 128 \) pixels are shown in Figs 6(b) and (c). The input perspective image of Fig. 6(b) is obtained by simulation with parameters \( \sigma = -45, \tau = 30, C = 0.8 \) and \( (X_f, Y_f) = (32, 32) \). The input perspective image of Fig. 6(c) is obtained by simulation with parameters \( \sigma = 30, \tau = -15, C = 1.4 \) and \( (X_f, Y_f) = (32, -32) \). By using these two sets of parameters to recompute the locations of the perspective template images, the results are shown in Figs 6(j) and (k). By comparing Fig. 6(b) with Fig. 6(j), and Fig. 6(c) with Fig. 6(k), we see that they have identical locations and shape points.

In Fig. 7, a key template image with \( 71 \times 157 \) pixels is shown in Fig. 7(a) and two input perspective images with \( 512 \times 400 \) pixels are shown in Figs 7(b) and (c). After the PTIGHT is performed in Figs 7(e) and (f), the resulting HCSs are shown in Figs 7(i) and (j). We detect all peaks that have cell values larger than the threshold value \( t_v \) and find the optimal candidate from them. Then, the perspective transformation parameters of the optimal candidate are retrieved. Figures 6(b) and (i) show all detected candidates. For Fig. 6(b), the retrieved parameters are \( \sigma = -45, \tau = 30, C = 0.8 \) and \( (X_f, Y_f) = (32, 32) \); and for Fig. 6(c), the parameters are \( \sigma = 30, \tau = -15, C = 1.4 \) and \( (X_f, Y_f) = (32, -32) \). By using these two sets of parameters to recompute the locations of the perspective template images, the results are shown in Figs 6(j) and (k). Comparing Fig. 6(b) with Fig. 6(j), and Fig. 6(c) with Fig. 6(k), we see that they have identical locations and shape points.
dates. For Fig. 7(b), the retrieved perspective parameters are $\sigma = -10, \tau = 35, C = 0.6$ and $(X_f, Y_f) = (39, -73)$ and for Fig. 7(c), the parameters are $\sigma = -5, \tau = 35, C = 1.0$ and $(X_f, Y_f) = (80, -125)$. By using these two sets of parameters to recompute the template shapes, the results are shown in Figs 7(m) and (n). Comparing Figs 7(b) and (e) with (m), and Figs 7(c) and (f) with (n), we see that the detected locations and the recomputed shapes of the keys are quite close to their original locations and the shapes.

In Fig. 8, a real image containing a film box template is shown in Fig. 8(a) and an input real perspective-transformed image containing three kinds of boxes as well as one chewing gum is shown in Fig. 8(d). Both of Figs 8(a)

Fig. 8. Illustration of practical perspective film box detection: (a) a real image containing film box template; (b) the segmented template image from (a); (c) the resulting feature detection from (b); (d) the input image; (e) the resulting feature detected from (d); (f) and (g) the constructed PR-table from (c) shown as a 3D shape and an image, respectively; (h) and (i) the resulting HCSs shown as a 3D shape and an image, respectively, after performing the PTIGHT on (e); (j) all detected candidates; (k) recomputed perspective template using retrieved parameters.
and (d), whose image sizes are 512×400 pixels, are obtained through a real TV camera. Figure 8(b) is the film box template, whose size is 110×84 pixels, segmented from Fig. 8(a). After performing the preprocessing steps of edge detection, inverse and thresholding in Figs 8(b) and (d), Figs 8(c) and (e) are obtained, respectively. Figure 8(f) is the PR-table with size 165×126 pixels constructed from Fig. 8(c), and Fig. 8(g) is the 2D image of Fig. 8(f). After the PTIGHT is performed in Fig. 8(e), the resulting HCS is shown as a 3D shape and an image in Figs 8(h) and (i), respectively. We detect all peaks that have cell values larger than the threshold value $t_v$ and find the optimal candidate from them. Then the perspective transformation parameters of the optimal candidate are retrieved. Figure 8(j) shows all the detected candidates. For Fig. 8(d), the retrieved perspective parameters are $\sigma=5$, $\tau=15$, $C=1.2$ and $(X_0, Y_0) = (-122, -51)$.

The result of recomputing the template shape using the parameters is shown in Fig. 8(k). We see that the detected location and the recomputed shape of the film box in Fig. 8(k) are quite close to the original location and shape in Fig. 8(e).

It takes about 10 min on an average to process an input image. The speed can be reduced if the resolution of the HCS space is reduced.

6. CONCLUSIONS

A new version of the GHT, called PTIGHT, has been proposed, which can be used to detect and locate a template planar shape in a perspective input image with an unknown viewpoint. The new transform is perspective transformation invariant. In the proposed PTIGHT process, a PR-table instead of the R-table of the conventional GHT is constructed first. The PR-table contains all perspective transformation information of the template shape. Constructing the PR-table is time-consuming but it can be done in advance before the PTIGHT is performed. The cell value incrementation strategy of the PTIGHT is similar to that of the conventional GHT. It only needs a 2D HCS so that the storage and computation requirements can be effectively reduced. After detecting the peaks in the HCS whose values are larger than a threshold value as the candidate shape locations, the perspective transformation parameters of the optimal
candidate are retrieved by using the proposed inverse PTIGHT. From the experimental results it is seen that the PTIGHT has good potential for practical applications. How to extend the PTIGHT to gray-scale and color images is worth further research.

REFERENCES


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