Sliding Abrikosov lattice in a superconductor with a regular array of artificial pinning centers: AC conductivity and criticality at small frequencies

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ABSTRACT

Dynamics of the flux lattice in the mixed state of strongly type-II superconductor near the upper critical field subjected to AC field and interacting with a periodic array of short range pinning centers is considered. The superconductor in a magnetic field in the absence of thermal fluctuations on is described by the time-dependent Ginzburg–Landau equations. For a special case of the δ-function shaped pinning centers and for pinning array commensurate with the Abrikosov lattice (so that vortices outnumber pinning centers) an analytic expression for the AC conductivity is obtained. It is found that below a certain critical pinning strength and for sufficiently low frequencies there exists a sliding Abrikosov lattice, which vibrates nearly uniformly despite interactions with the pinning centers. At very small frequencies the conductivity diverges at the critical pinning strength.

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The great interest in the problem of magnetic flux pinning in type-II superconductors stems from its relevance to technological applications as well as with its implications to the general problem of complex nonlinear dynamics with tunable parameters. An important challenge in applications of type-II superconductors is in achieving optimal critical currents under given magnetic fields. This requires preventing depinning of Abrikosov vortices during formation of the resistive state under the applied current. Recently there have been advances in the study of vortex pinning by fabricating periodic arrays of pinning sites where each pinning site may be either magnetic or normal inclusion effectively trapping vortices. Pinning arrays with triangular, square, and rectangular geometries have been fabricated using either microholes or blind holes arrays of magnetic dots and periodic array of columnar defects [1]. The resulting critical current is enhanced when vortex lattice is commensurate with the periodic array of pinning sites. In addition this system is a convenient experimental tool to study the general problem of interacting periodic system moving in periodical potential like dislocations in crystals or charge density waves. Theory of dynamics of the pinned vortex matter by a random distribution of pins is very complicated. However in the absence of significant thermal fluctuations, the problem simplifies considerably. It was studied theoretically, mostly in 2D systems, using either numerical methods within a model of interacting points-like particles representing vortices subject to pinning potential and driving force [2] or within the framework of elasticity theory, in which the vortex matter is treated as an elastic manifold subject to both the pinning stress and a driving force [3].

Theory of the Abrikosov lattice subjected to an AC field and periodic pinning is simpler, but so far has been treated either numerically using molecular dynamics approach or by means of the elastic manifold approach in London approximation. On the other hand this approach completely ignores the contribution of the vortex cores essentially important when the distances between vortices and artificial pinning sites are not much larger than the size of the coherence length. In fact there is still no an analytical theory describing AC properties of a type-II superconductor with periodic pinning array subjected to a strong magnetic field. Here we present a theory of AC conductivity in the time-dependent Ginzburg–Landau (TDGL) approximation describing superconductor in a strong magnetic field. In the absence of thermal fluctuations, an exact solution for the linear response in the case of a δ-function model for the periodical array of the pinning centers in which it is commensurate with the Abrikosov lattice (vortices outnumber pinning centers) is obtained for the first time.

Let us consider a type-II superconductor under a constant external magnetic field H parallel to a system of pinning centers directed along z axis and carrying electric current along the y axis, see Fig. 1.

The simplest relaxation dynamics of a superconductor in the presence of electric field is described by TDGL [4] and Maxwell equations (in the dimensionless variables)
Eq. (1) allows to relate the dynamic conductivity in the superconductor with the Green function \( G \) of the quantum mechanical Hamiltonian \( \hat{H}_p \) of a charged particle in magnetic field in the presence of periodic potential. Representing Green function in the integral form, one obtains the Dyson equation

\[
G(\mathbf{r}, \mathbf{r}', \omega) = G_{cl}(\mathbf{r}, \mathbf{r}', \omega) - U_0 \sum \sigma(\mathbf{r}_n, \omega) G_{cl}(\mathbf{r}_n, \mathbf{r}', \omega)
\]

where \( G_{cl}(\mathbf{r}, \mathbf{r}', \omega) \) is the Green function of the clean superconductor

\[
g_{cl}(\mathbf{r}, \mathbf{r}', \omega) = e^{i \Phi(\mathbf{r}')} g_{cl}(\mathbf{r}, \mathbf{r}', t)
\]

\[
C(t) = \frac{\bar{e}}{\bar{e}^2 \sinh^{-1}(\bar{e}/\eta)}
\]

In particular at pinning points \( \mathbf{r} = \mathbf{r}_n \) assuming commensurability with the vortex lattice, one obtains

\[
G(\mathbf{r}_n, \mathbf{r}, \omega) = \sum \sigma(\mathbf{r}_n, \omega) G_{cl}(\mathbf{r}_n, \mathbf{r}, \omega)
\]

where a symmetric matrix \( M_{\sigma}(\omega) \) is defined by

\[
M_{\sigma}(\omega) = \frac{1}{2 \pi} \int_{k_1}^{k_2} e^{i \Phi(\mathbf{r})} \Pi_{\sigma, \omega}
\]

\[
\Pi_{\sigma, \omega} = \frac{1}{2 \pi} \int_{k_1}^{k_2} \frac{e^{i \Phi(\mathbf{r})} \Pi_{\sigma, \omega}}{\bar{e}^2 \sinh^{-1}(\bar{e}/\eta)}
\]

Substituting it into the expression for full GF with arbitrary positions one obtains

\[
G(\mathbf{r}, \mathbf{r}', \omega) = G_{cl}(\mathbf{r}, \mathbf{r}', \omega) - U_0 \sum \sigma(\mathbf{r}_n, \omega) K(\mathbf{r}, \mathbf{r}_n, \mathbf{r}', \omega)
\]

\[
K(\mathbf{r}, \mathbf{r}_n, \mathbf{r}', \omega) = G_{cl}(\mathbf{r}, \mathbf{r}_n, \omega) G_{cl}(\mathbf{r}_n, \mathbf{r}', \omega)
\]

To determine the operator GF for operator \( \hat{H} \) one has to subtract the constant \( u_0 \) from \( \hat{H}_p \). In the \( \omega \) space such transformation is equivalent to a shift of frequency by the imaginary number \( i \omega \) in the GF. Substituting the full GF into expression for conductivity one obtains two contributions in terms of “clean” GF:

\[
\sigma(\omega) = \sigma(\omega) + \sigma(\omega)
\]

\[
\sigma(\omega) = -\frac{2}{L_{xy}^4} \int \frac{d^2 \phi}{d^2 \phi} \int \frac{d^2 \phi}{d^2 \phi} G_{cl}(\mathbf{r}, \mathbf{r}', \omega + iu_0)
\]

\[
\sigma(\omega) = \frac{2U}{L_{xy}^4} \sum \sigma(\omega) \Sigma(\omega) \int \frac{d^2 \phi}{d^2 \phi} \Pi_{\sigma, \omega, \omega} + u_0)
\]

where

\[
\Sigma(\omega) = \frac{1}{\sqrt{4 \pi L_{xy}^2 \phi}} \int \frac{d^2 \phi}{d^2 \phi} G_{cl}(\mathbf{r}, \mathbf{r}, \omega + iu_0)
\]

\[
\Sigma(\omega) = -\frac{1}{\sqrt{4 \pi L_{xy}^2 \phi}} \int \frac{d^2 \phi}{d^2 \phi} G_{cl}(\mathbf{r}, \mathbf{r}, \omega + iu_0)
\]

Integrations result in:

\[
\sigma(\omega) = \sigma_{HF} \left[ 1 + \frac{3.75U_0 h (\omega + h - u_0)^{-1}}{\bar{e}^2 \sinh^{-1}(\bar{e}/\eta)} \right]
\]

\[
\sigma_{HF} = \frac{a_b}{\bar{e}_0} \frac{1}{\bar{e}_0^2 \sinh^{-1}(\bar{e}/\eta)} \theta(X) = \log \left( \frac{K_{\max}^2}{2h} \right) - \Psi(X)
\]

Here \( \Psi(X) \) is digamma function and \( K_{\max} \) is maximal vector of the reciprocal pinning centers’ lattice.

Let us consider some limiting cases important for experiment.
(i) No pinning \( U_0 = 0 \)

\[
\sigma_{ff} = \frac{\alpha_b}{\beta_h} \frac{1}{\omega + h/h_0} \tag{16}
\]

If \( \omega = 0 \) when Eq. (18) gives a well known Bardeen–Stephen result for flux flow conductivity.

(ii) Criticality near the critical pinning strength \( h = u_0 \) for small frequency. This means that the vortex lattice is pinned and electric field cannot penetrate the superconductor despite persistent current flow in it at least when the current is not large. In this case the real part of the conductivity diverges. Near this line the conductivity reads (see Figs. 2 and 3):

\[
\sigma_r(\omega \to 0) \approx \frac{\alpha_b}{\sum \pi n_p \beta_h} \frac{1}{U_0 - U_0} \tag{17}
\]

where \( U_0 = (2\pi n_p)^{-1} \). Therefore the pinning strength is only factor determining the transition into the pinned state. The critical value is independent of the magnetic induction.

(iii) AC conductivity at the critical line \( (u = u_c) \)

In this case the conductivity at small frequencies \( \omega \ll h \) has only the imaginary part

\[
\sigma_i \approx \frac{\alpha_b}{\beta_h} \frac{1}{\omega^2} \tag{18}
\]

(iv) AC for subcritical pinning strength

In this case \( u \ll 1 \) and AC conductivity reads

\[
\sigma_r = \frac{1}{(i\omega + h)(u_0 - h) + u(1 + \frac{\omega}{v})} \tag{17}
\]

where the second term in this expression describes pinning correction to usual Bardeen–Stephen conductivity.

In summary, we developed the theory of AC conductivity for a superconductor with periodic pinning array in the Ginzburg–Landau approximation and predicted that above some pinning strength, the AC conductivity in the limit of small frequency shows typical for ideal superconductor behavior.

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References