Alternatively, the matrix used in the paper is defined as
\[ T(m,n) = \begin{bmatrix} M_0 & M_1 & \cdots & M_{m-1} \\ M_1 & M_2 & \cdots & M_n \\ \vdots & \vdots & \ddots & \vdots \\ M_{m-1} & M_m & \cdots & M_{m+n-2} \end{bmatrix} \]

where
\[ \gamma(s) = \sum_{i=0}^{r} b_i s^i, \quad b_r = 1; \]


\[ H(m,n) = [H_1(m,n)H_2(m,n)\cdots H_p(m,n)] \]

\[ H_i(m,n) = \begin{bmatrix} m_{i0} & m_{i1} & \cdots & m_{im-1} \\ m_{i1} & m_{i2} & \cdots & m_{mn} \\ \vdots & \vdots & \ddots & \vdots \\ m_{im-1} & m_{in} & \cdots & m_{im+n-2} \end{bmatrix} \]

Proof of Theorem 1: Observe from the definition of \( M_n \), (22), that \( H(m,n) \) can be obtained from \( T(m,n) \), (2), through column interchanges. This implies that
\[ \rho[H(m,n)] = \rho[T(m,n)]. \]

This allows \( T(m,n) \) to be used in place of \( H(m,n) \) in the proof of this theorem.

Next, recall the well-known fact that
\[ \rho[T(m,n)] = \rho[T(r,r)], \quad m,n \geq r \]
\[ = n \]

where \( r \) equals the degree of least common denominator of \( G(s) \) and \( n \) equals the dimension of minimal realization. For the general case of multi-input multi-output systems, use of the foregoing lemma allows one to write
\[ n \geq r. \]

The equality in (24) holds in the special case being considered since an explicit realization of order \( r \) may be obtained by letting
\[ A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ -b_0 & -b_1 & -b_2 & \cdots & -b_{r-1} \end{bmatrix} \]

By considering a multi-output single-input system and repeating the foregoing procedure, the second part of Theorem 1 can be proved.

Reference


Author's Reply

C.-T. CHEN

It seems that Gupta and Fairman have missed the main contribution of the paper. Theorem 1 is established without resorting to any result in irreducible realization. It is clear that, after establishing irreducible realization, Theorem 1 can then be reduced. It seems, however, that it is more logical to establish first properties of Hankel matrices and then to establish irreducible realization.

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