Effective boson calculations on Sm isotopes

This content has been downloaded from IOPscience. Please scroll down to see the full text.
(http://iopscience.iop.org/0305-4616/12/7/003)

View the table of contents for this issue, or go to the journal homepage for more

Download details:
IP Address: 140.113.38.11
This content was downloaded on 28/04/2014 at 20:40

Please note that terms and conditions apply.
LETTER TO THE EDITOR

Effective boson calculations on Sm isotopes

S T Hsieh†, H C Chiang†, M M King Yen‡ and D S Chuug§
† Department of Physics, National Tsing Hua University, Hsinchu, Taiwan
‡ Department of Nuclear Engineering, National Tsing Hua University, Hsinchu, Taiwan
§ Department of Electrophysics, National Chiao Tung University, Hsinchu, Taiwan

Received 3 March 1986

Abstract. Effective boson numbers are introduced in IBA-1 calculations to simulate the partial subshell closure effects at \( Z = 64 \). The energy levels, wavefunctions and \( B(E2) \) values of Sm isotopes are calculated and compared with the experimental data. It is found that the agreement with the experimental data can be improved by introducing effective boson numbers in the calculations.

From a microscopic point of view, the bosons of the interacting boson model (Arima and Iachello 1976, 1978, 1979, Scholten et al 1978) are considered as collective pairs of fermions. Therefore, the number of bosons for a specific nucleus is counted as half of the number of particles (or holes, whichever is less) in the valence shell. This interpretation becomes ambiguous when there is some subshell closure. For example, it was shown that the \( Z = 64 \) subshell had significant effects for \( N < 88 \) transitional nuclei (Gill et al 1982). Therefore, for those nuclei with a proton number near \( Z = 64 \), the counting of the number of bosons becomes ambiguous. Using two counting schemes; one which counts the proton–boson number from the \( Z = 50 \) closed shell and one which counts the proton–boson number from the \( Z = 64 \) subshell, Wolf et al (1983) calculated the magnetic moments of the first \( 2^+ \) states of the transitional nuclei Ba, Nd, Sm and Gd in the framework of \( \text{IBA-2} \) (Wolf et al 1983, Arima et al 1977, Otsuka et al 1978). It was interesting to note that for those nuclei with \( N < 88 \), the counting scheme from the \( Z = 64 \) subshell yielded better agreement with the experimental data (Wolf et al 1983).

By treating the s and d boson as the shell-model S and D pair states, Scholten proposed a method to calculate the number of ‘effective bosons’ in a microscopic model (Scholten 1983a). It is interesting to see that when the number of effective bosons is calculated for the 50–82 major shell by this method, it does yield a minimum value at \( Z = 64 \). However, the minimum value is about 2.4 instead of 0 as required by a full closure structure. This means that there is a considerable reduction of the shell closure effect at \( Z = 64 \).

Federman and Pittel (1979) and Federman et al (1979) studied the shape transition of \( \text{Zr} \) and \( \text{Mo} \) isotopes in the framework of a shell model. It was found that the isospin \( T = 0 \) component of the n–p interaction is responsible for the onset of deformation. In a parallel argument, it is believed that the onset of deformation for \( \text{Ba}–\text{Gd} \) nuclei with \( N \geq 90 \) is due to the same type of n–p interaction, which in this case comes mainly from the spin–orbit partners \( 1h_{9/2} \) (neutron) and \( 1h_{11/2} \) (proton) orbits. However, it is well known that the shell-model single-particle level spacings are dependent on the model space. Therefore, the sudden disappearance of the \( Z = 64 \) subshell for \( N \leq 88 \) nuclei seems to be oversimplified in
Letter to the Editor

the sense that it does not take the effects of partial closure and the smooth neutron number dependence into account. Also, Scholten's calculation (1983a) ignored the neutron number dependence. Hence, it will be interesting to see the effects of a more flexible counting scheme for effective bosons. In this Letter we report some results in this approach.

In the calculation we choose Sm isotopes with \( N = 86-96 \) as examples because abundant experimental data are available for these isotopes. The number of neutron–bosons, \( \mathcal{N}_n \), is counted as \( \frac{1}{2}(N - 82) \) where \( N \) is the number of neutrons as usual. For the number of proton–bosons, \( \mathcal{N}_p \), we relax all shell closure restrictions but maintain the only requirement that they must be integers. The total number of bosons \( \mathcal{N}_T \) is then counted as \( \mathcal{N}_n + \mathcal{N}_p \). Once we have chosen a set of \( \mathcal{N}_T \)'s for the Sm isotopes, we may calculate their energy levels in the framework of IBA-1.

It was found that two sets of choices yielded better agreement with the experimental data than all other conceivable choices. One set of the effective proton-boson numbers is \( \mathcal{N}_p = 4 \) for \( ^{148}\text{Sm} \), \( \mathcal{N}_p = 5 \) for \( ^{150}\text{Sm} \) and \( \mathcal{N}_p = 6 \) for all other Sm isotopes (denoted as set B hereafter); the other set is \( \mathcal{N}_p = 2 \) for \( ^{148}\text{Sm} \), \( \mathcal{N}_p = 4 \) for \( ^{150}\text{Sm} \) and \( \mathcal{N}_p = 6 \) for all other Sm isotopes (denoted as set C hereafter). In order to see the effects introduced by the effective boson calculations, we also compared the results of set B and set C calculations with the conventional choice in which all \( \mathcal{N}_p \)'s are counted as 6 (denoted as set A hereafter). It is worth noting that both set B and set C count the same \( \mathcal{N}_p \) as set A for \( N > 90 \) Sm isotopes. This is consistent with the disappearance of the \( Z = 64 \) subshell for such isotopes. For \( N = 88 \) and 86 isotopes, the effective \( \mathcal{N}_p \) of both sets B and C reduced linearly from 6.

The ground band energy levels of the Sm isotopes are shown in figure 1. It is clear from figure 1 that the energy spectra are very similar for \( N > 90 \) Sm isotopes and the energy values for each of the specific \( J \) states becomes quite flat. For \( N = 88 \) and 86 isotopes, the energy values for each specific \( J \) state rise quite quickly and the energy level spacings also increase quite rapidly. This is consistent with the onset of the \( Z = 64 \) subshell effects. Also, the energy values for each specific \( J \) state fall linearly as the neutron number increases from 86 to 90. Therefore, the linear variation of the effective proton–boson number found in sets B and C seems to be quite reasonable. The subsequent discussions will be restricted to the calculations in these two sets of effective bosons and will compare them with the results of the set A calculations. In the calculation of energy levels, the most general Hamiltonian with nine parameters of IBA-1 was used (Arima and Iachello 1976):\[ H = H(\epsilon_s, \epsilon_d, \alpha, \beta, \gamma, u_0, u_2, \tilde{\nu}_0, \tilde{\nu}_2). \] (1)

For set A and set B calculations, 63 reliable energy levels were included in the least-squares fittings. For the set C calculation, the \( 10^+ \) state of \( ^{148}\text{Sm} \) (\( \mathcal{N}_T = 4 \) and \( 16^+ \) state of \( ^{150}\text{Sm} \) (\( \mathcal{N}_T = 7 \) are already outside the available model space. Therefore these states have to be removed in the fittings and they leave only 61 states in the least-squares fittings. The resulting interaction parameters and the overall root-mean-square deviations are listed in table 1. It is well known that not all parameters in the Hamiltonian are linearly independent. Also in the fittings, it was found that the parameters \( \gamma \) and \( u_0 \) were quite small and the quality of fittings did not depend greatly on these parameters. Therefore, we set these parameters to zero. The single s-boson energy \( \epsilon_s \) was set to zero as usual. It is interesting to note that the parameters of the calculations for set B and set C are quite similar. For the set A calculation the parameters of \( \epsilon_d, \alpha \) and \( u_2 \) are relatively large. These are consistent with those of a previous calculation in which it was shown that bigger magnitudes of \( \epsilon_d, \alpha \) and \( u_2 \) were needed in order to fit the experimental data by a mass-number-independent Hamiltonian (King et al 1984). However, it is interesting to mention that for a given nucleus with a specific \( N \) value, some parameters can be grouped together...
The general trend of the ground band energy levels of the Sm isotopes. A change of general behaviour is evident for $N = 88$ to $N = 90$ transition.

To form another parameter. If we calculate the so called renormalised single d-boson energy (Castanos et al 1982)

$$
\tilde{\epsilon} = \epsilon_d - u_0(N - 1) + \frac{1}{\sqrt{5}}u_2(N - 1),
$$

the resulting values of $\tilde{\epsilon}$ for the calculations of sets A, B and C are all quite similar when the corresponding effective boson numbers are inserted into equation (2). This serves to explain why very different values of $\epsilon_d$ can produce energy spectra that do not differ very much from each other. The calculated energy spectra of $^{150}$Sm corresponding to the three

<table>
<thead>
<tr>
<th>$\epsilon_s$</th>
<th>$\epsilon_d$</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$\gamma$</th>
<th>$u_0$</th>
<th>$u_2$</th>
<th>$\tilde{\epsilon}_2$</th>
<th>$\tilde{\epsilon}_0$</th>
<th>Overall RMSD (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A 0</td>
<td>1.9334</td>
<td>-0.3441</td>
<td>-0.0201</td>
<td>0 0</td>
<td>-0.4127</td>
<td>-0.1257</td>
<td>-0.0819</td>
<td>0.110</td>
<td></td>
</tr>
<tr>
<td>B 0</td>
<td>1.0688</td>
<td>-0.1613</td>
<td>-0.0201</td>
<td>0 0</td>
<td>-0.1716</td>
<td>-0.1397</td>
<td>-0.0854</td>
<td>0.096</td>
<td></td>
</tr>
<tr>
<td>C 0</td>
<td>0.7899</td>
<td>-0.1013</td>
<td>-0.0180</td>
<td>0 0</td>
<td>-0.1045</td>
<td>-0.1461</td>
<td>-0.0931</td>
<td>0.082</td>
<td></td>
</tr>
</tbody>
</table>

Figure 1. The general trend of the ground band energy levels of the Sm isotopes. A change of general behaviour is evident for $N = 88$ to $N = 90$ transition.
The calculated and experimental energy levels of $^{150}$Sm. Some higher states that were not included in the least-squares fittings were not shown. The experimental data were adopted from Sakai (1984).

sets of boson numbers together with the experimental data are shown in figure 2 for comparison. This is a typical result for all Sm isotopes. In the figure different quasibands are separated in order to have a clear comparison. It is seen that sets B and C yielded slightly better agreement with the experimental data for the ground band and the first $\beta$ band while set A gives slightly better results for the first $\gamma$ band. In the set A calculation there is some incompatibility between the low spin and high spin states. This tendency is removed in the calculations of sets B and C.

The wavefunctions of $^{148}$Sm and $^{150}$Sm obtained from these three calculations show different intensity distributions. The wavefunctions obtained in the set A calculation are the most dispersed ones and for the sets B and C calculations the wavefunctions become more and more concentrated. In table 2 we display the intensities of the most dominant configurations for some states of $^{148}$Sm. The reduction in dispersion in passing from set A to set C calculation is quite clear. It is well known that the wavefunctions will be more concentrated as one passes from the rotational to the vibrational nuclei. Therefore we are

Table 2. Dominant configuration intensities for the low-lying states of $^{148}$Sm. The total intensity for all configurations is normalised to 1.

<table>
<thead>
<tr>
<th></th>
<th>$0_1^+$</th>
<th>$0_2^+$</th>
<th>$2_1^+$</th>
<th>$2_2^+$</th>
<th>$3_1^+$</th>
<th>$4_1^+$</th>
<th>$4_2^+$</th>
<th>$6_1^+$</th>
<th>$8_1^+$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.845</td>
<td>0.480</td>
<td>0.738</td>
<td>0.572</td>
<td>0.878</td>
<td>0.673</td>
<td>0.387</td>
<td>0.655</td>
<td>0.674</td>
</tr>
<tr>
<td>B</td>
<td>0.919</td>
<td>0.668</td>
<td>0.842</td>
<td>0.718</td>
<td>0.944</td>
<td>0.786</td>
<td>0.544</td>
<td>0.775</td>
<td>0.812</td>
</tr>
<tr>
<td>C</td>
<td>0.959</td>
<td>0.856</td>
<td>0.927</td>
<td>0.892</td>
<td>1.000</td>
<td>0.915</td>
<td>0.851</td>
<td>0.942</td>
<td>1.000</td>
</tr>
</tbody>
</table>
Table 3. The best fitted values of $\alpha$ and $\beta$ and the overall root-mean-square deviations for $B(E2)$ values for three calculations.

<table>
<thead>
<tr>
<th></th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>Overall RMSD ($e^2b^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.157</td>
<td>0.557</td>
<td>0.098</td>
</tr>
<tr>
<td>B</td>
<td>0.153</td>
<td>0.681</td>
<td>0.092</td>
</tr>
<tr>
<td>C</td>
<td>0.150</td>
<td>0.760</td>
<td>0.072</td>
</tr>
</tbody>
</table>

on the right lines in using effective bosons in the calculations when simulating the partial subshell closure effects.

For the Sm isotopes, there are quite abundant experimental $B(E2)$ data. Hence, it will be also helpful to make a general comparison on the $B(E2)$ values for the three calculations. In the calculation, the general form of the E2 operator was used:

$$T^{(2)} = \alpha[(d^+ \times $\bar{s}$ + s^+ \times \bar{d})^{(2)} + \beta(d^+ \times \bar{d})^{(2)}].$$  (3)

The two parameters $\alpha$ and $\beta$ were determined directly from least-squares fittings to 39 experimental data values of $B(E2)$ for Sm isotopes. The resulting best fitted values of $\alpha$ and $\beta$ and the overall root-mean-square deviations for $B(E2)$ values are summarised in Table 3.

Figure 3. Calculated and experimental $B(E2)$ values for the $2^+_1 \rightarrow 0^+_1$ transitions of the Sm isotopes. $\times$, set A; $\Delta$, set B; $\Box$, set C; $\bullet$, experimental. The experimental data are adopted from Tamura (1979) and Gupta (1983) and sources cited therein.
Figure 4. Calculated and experimental $B(E2)$ values for the $4^+_1 \rightarrow 2^+_1$ transitions of the Sm isotopes. The same symbolism is used as in figure 3. The experimental data are adopted from Tamura (1979) and Gupta (1983) and the sources cited therein.

It is interesting to note that the calculated $\alpha$ values are similar and agree approximately with those obtained in previous calculations (Scholten 1983b, Bijker et al 1980). The resulting values of $\beta$ are also similar to each other with a slight linear change passing from the set A to the set C calculation. However, we would like to mention that in the fittings it was found that the quality of fittings did not depend very sensitively on the values of $\beta$. An extensive analysis on the $B(E2)$ branching ratios will be difficult since the magnitudes of most experimental data are small and the experimental errors will introduce ambiguities in the branching ratios that are too big to prevent us from reaching any significant conclusion. In figures 3 and 4 the experimental and calculated $B(E2)$ values for the $2^+_1 \rightarrow 0^+_1$ and $4^+_1 \rightarrow 2^+_1$ transitions are shown for comparison. From the figures we can see that, on the whole, the agreements with the experimental data improve from the set A calculation to the set C calculation. This tendency is also revealed in the $B(E2)$ values of other transitions.
In summary, the existence of partial subshell effects causes ambiguities in counting boson numbers in the IBA type calculations. Lacking a precise microscopic counting scheme, we may try to simulate these effects by introducing effective boson numbers in the phenomenological calculations. The results reported in this Letter suggest that this is an encouraging approach. To explore its implications in more detail, more extensive applications on other nuclei are needed. Also, it will be helpful to study the foundation of this approach in a more microscopic model, such as the shell model which can manifest the effects of n–p interactions explicitly. Such an analysis, though much more tedious, will provide more decisive conclusions.

The authors wish to thank Professor H T Chen for helpful discussions. One of us (STH) wishes to thank Professor W B Walters for stimulating discussions. This work is supported by the National Science Council, Republic of China, Grants NSC74-0201-M007a-05 and NSC74-0201-M009a-02.

References

—— 1978 Ann. Phys., NY 111 201
—— 1979 Ann. Phys., NY 123 468
Sakai M 1984 Tables of Members of Quasi-Bands (Tokyo: Institute for Nuclear Study)
Scholten O 1983a Phys. Lett. 127B 144
—— 1983b PhD Thesis Cyclotron Laboratory, Michigan State University
Tamura T 1979 Phys. Rev. C 20 307