not all the possible ones: in fact, the fade durations for which
the attenuation does not reach $S + dS$ dB are excluded.

The loss of memory is also observable in the scattergram
$|R|$ against $D$ shown in Fig. 3, for $S = 2$ dB. This behaviour is
also confirmed by the joint analysis of successive durations and intrafades on the same database, discussed in Reference 4.

Note that, in any case, rates are not independent of the fade
duration as, obviously, each rate and their sum cannot exceed
the corresponding fade duration.

Similar behaviour is also found for higher thresholds.

Table 3 gives the parameters of logarithmic regression lines
between $R$ and $D$ for the four thresholds.

Conclusions: The joint analysis of positive and negative rates of change and the in-between duration of fade shows that
medium-to-low correlation exists between any two variables.
Positive and negative rates of change, conditioned to the threshold, are statistically alike and are well described by log-
normal distributions.

E. MATRICCIANI
M. MAURI
A. PARABONI
Dipartimento di Elettronica
Politecnico di Milano
Piazza Leonardo Da Vinci 32
20133 Milano, Italy

References
1 MATRICCIANI, E.: 'Rate of change of signal attenuation from Sirio at
2 MATRICCIANI, E.: 'Effects of filtering on rate of change of rain-
induced attenuation', ibid., 1980, 16, pp. 377–378
3 CARASSA, F., MAURI, M., and PARABONI, A.: 'Attenuation statistics in
Italy as measured by the Sirio satellite during its five year lifetime',
4 MATRICCIANI, E., and MAURI, M.: 'Rain attenuation successive fade
durations and time intervals between fades in a satellite-earth link',

PERFORMANCE BOUNDS OF RATE-1/4 CONVOLUTIONAL CODES WITH QPSK ON RAYLEIGH FADING CHANNEL

An analytic expression for the bit error probability upper
bounds of rate-1/4 convolutional codes in conjunction with
QPSK modulation and maximum-likelihood Viterbi decoding
on the fully interleaved Rayleigh fading channel is pre-
sentated. The given expression is evaluated numerically for
selected rate-1/4 optimum convolutional codes together with
QPSK.

Introduction: The performance of rate-1/4 short-constraint-
length convolutional codes in conjunction with BPSK modu-
lation and various kinds of maximum-likelihood (ML) Viterbi
decoding on Rician-Rayleigh fading channels has been dis-
cussed in detail.1–3 It has been found that the ML Viterbi
decoder which makes perfect channel measurements, channel
amplitude and phase gives the best performance. However,
with BPSK modulation the code redundancy introduced by
convolutional coding represents a bandwidth sacrifice in that
the ratio of required bandwidth to information rate is
increased. To avoid this sacrifice in bandwidth efficiency and
also provide effective error protection, QPSK modulation
could be used instead. This letter is concered with the bit error
probability performance of rate-1/4 convolutional codes in con-
junction with QPSK modulation and ML Viterbi decoding
which makes perfect channel measurements on the fully inter-
leaved Rayleigh fading channel. An analytical expression for
the bit error probability upper bounds for rate-1/4 optimum convolutional codes with constraint lengths $k = 3$ to 7
together with QPSK are evaluated.

System model: The system under consideration is shown in
Fig. 1. To compute the error probability the channel is rep-
resented by a discrete-time model where the $i$th input is given by

$$x_i = \exp(j\theta) \quad e \in \{0, \pi/2, \pi, 3\pi/2\}$$

\[ \text{(1)} \]
and the corresponding normalized matched filter output after deinterleaving is

\[ y_i = \sqrt{(E_b/N_0)} a_i x_i + N_i \]  

where \( \{N_i\} \) is an independent identically distributed (IID) noise sequence of complex Gaussian variates with zero mean and unit variance. \( \{a_i\} \) with \( a_i \) representing the value of the amplitude fading process \( a(t) \) throughout the \( i \)th signalling interval, is an IID sequence of random variates with distribution

\[ f(a_i) = 2a_i e^{-a_i^2} \]  

and \( E_b/N_0 \) is the received signal/noise ratio (SNR) per received symbol, which equals the received SNR per bit, \( E_b/N_0 \), in this case, where \( E_b \) is the energy per input bit. In addition to \( y_i \), the Viterbi decoder is also supplied with the channel amplitude \( a_i \) from a channel estimator.

**Pairwise error probability:** The pairwise error probability between the transmitted sequence \( x = \{x_i\} \) and the estimated sequence \( \hat{x} = \{\hat{x}_i\} \) is now developed. Given the received sequences \( y = \{y_i\} \) and \( a = \{a_i\} \), the ML decision rule for the Viterbi decoder which makes perfect amplitude measurement implies that

\[ P(y|\hat{x}, a) < P(y|x, a) \text{ when } \hat{x} \neq x \]  

Since \( \{x_i\} \), \( \{\hat{x}_i\} \) and \( \{a_i\} \) are IID sequences, eqn. 4 can be manipulated to give

\[ \sum a_i y(x_i - \hat{x}_i)^* + a_i y(x_i - \hat{x}_i) < 0 \]  

where * denotes the complex conjugate. Without loss of generality we may assume that, in the transmitted sequence \( x \), \( x_i = +1 \) for all \( i \) (corresponding to the all-zero message). If \( \hat{x} \) differs from \( x \) in exactly \( k \) symbols, and there are \( k_1 \) symbols with value \( \pm j \) and \( k_2 \) symbols with value \(-1\), then eqn. 5 is equivalent to

\[ U = \sum a_i a_i^* \sqrt{(E_b/N_0)} a_i + \text{Re}(N_i) + \text{Im}(N_i) \]  

\[ + 2 \sum a_m a_m^* \sqrt{(E_b/N_0)} a_m + \text{Re}(N_m) + \text{Im}(N_m) < 0 \]  

where \( \text{Re}(N_i) \) and \( \text{Im}(N_i) \) are the quadrature components of complex noise \( N_i \) described in eqn. 2. Making use of the inequality \( Q(x) \leq \frac{1}{2} \exp(-x^2/2) \), we have the conditional pairwise error probability

\[ P(k_1, k_2)(E|a) = P(U < 0) \]  

\[ = Q\left( \sqrt{\frac{E_b}{N_0}} \sum a_i^2 + 2 \sum a_m^2 \right) \]  

\[ \leq \frac{1}{2} \exp \left( - \frac{E_b}{2N_0} \sum a_i^2 \right) \exp \left( - \frac{E_b}{N_0} \sum a_m^2 \right) \]  

where

\[ Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-w^2/2} \, dw \]  

Since \( a_i \) is Rayleigh-distributed, \( q = \sum a_i^2 \) is a \( \chi^2 \)-distributed random variable. Thus

\[ E[\exp(wq^k)] = \frac{1}{(1-w)^{k+1}} \]  

The unconditional pairwise error probability hence becomes

\[ P_{k_1, k_2}(E) = E[Q(k_1, k_2)(E|a)] \]  

\[ \leq \frac{1}{2} \left( 1 + \frac{E_b}{2N_0} \right)^{k_1} \left( \frac{1}{1 + E_b/N_0} \right)^{k_2} \]  

**Performance upper bound:** At this point the generating function union bounding approach initiated by Viterbi is applied to derive the bit error probability upper bound. For a convolutional code with QPSK the augmented generating function denoted by \( P(D_1, D_2, I) \) can be obtained in the conventional manner, except that the weight label of the branches on the state diagram should be properly modified. For illustrative purposes, an example is given in Fig. 2. In terms of \( T(D_1, D_2, I) \),

\[ T(D_1, D_2, I) = S_D/S_0 = \frac{D_1 D_2 I}{1 - 2D_1 I} \]
D_{i, l}) the bit error probability \( P_b \) for rate-\( \frac{1}{2} \) convolutional codes with QPSK modulation is upper-bounded by

\[
P_b \leq \frac{1}{2} \frac{\partial T(D_{i, l}, D_{i, l})}{\partial l} \bigg|_{l=1} = \frac{1}{1 + E_b/2N_0} D_{l, l} = \frac{1}{1 + E_b/N_0}
\]

This expression for \( P_b \) is numerically evaluated and is plotted in Fig. 3 for the selected rate-\( \frac{1}{2} \) optimum convolutional codes with constraint lengths \( k = 3 \) to 7 and with Gray mapping into QPSK.

**Conclusion:** For rate-\( \frac{1}{2} \) convolutional codes with QPSK on a Rayleigh fading channel, the derived analytic expression is a weak upper bound on the bit error probability. However, the approach utilized here can be extended to convolutional codes with M-ary PSK modulation on Rician fading channels. A generalised expression for the tight bit error probability upper bounds is under investigation.

YUH-LONG CHEN

CHE-HO WEI

Institute of Electronics
National Chiao Tung University
Hsin-Chu, Taiwan, Republic of China

**References**


**COAXIALLY MOUNTED 67 GHz BANDWIDTH InGaAs PIN PHOTODIODE**

*Indexing terms: Photoelectric devices, Photodiodes*

We report high-speed measurements of an InGaAs PIN photodiode in a millimetre-wave coaxial mount. The photodiode has an impulse response (including diode, parasitics and connector) of 9 ps FWHM and a 3 dB bandwidth of 67 GHz. This is the largest bandwidth reported to date for a photodiode in a coaxial mount with a connector.

Wideband photodiodes are important components for multigigabit/second optical systems operating in the 0-8 \( \mu \)m and 1-3 \( \mu \)m to 1-55 \( \mu \)m wavelength ranges. To attain high-speed performance, it is necessary that the active region of the photodiode have a small area (to minimise capacitance) and be thin (to minimise carrier transit times). In addition, it is necessary that electrical parasitics associated with the device package be kept small. To this end, the chip is usually mounted in a microstrip, coaxial or waveguide fixture, preferably with some form of connector for convenient connection to test instrumentation. The highest bandwidth reported to date for an InGaAs PIN detector, with a 50 \( \Omega \) coaxial connector and an internal biasing circuit, is 36 GHz. A similar device, in a 25 \( \Omega \) hybrid circuit, has shown an impulse response of 10 ps FWHM. Responses at frequencies to 60 GHz have been achieved using waveguide test fixtures, but separate photodiodes and waveguides are required to cover the full 26-5 GHz to 60 GHz measurement range.

Letter reports bandwidth measurements of a PIN photodiode in a coaxial mount. The photodiode shows a response extending from DC to millimetre-wave frequencies, with a 3 dB bandwidth of 67 GHz and an impulse response of 9 ps FWHM. This is the fastest response reported to date for a photodiode in a coaxial mount with a connector.

The PIN photodiode used here is a back-illuminated mesa structure with an active area of approximately 150 \( \mu \)m\(^2\) and an intrinsic layer thickness of approximately 0-5 \( \mu \)m. Further details of the photodiode structure are given in Reference 3. The chip was mounted on a small, low-inductance L-shaped bracket provided with a hole to allow back-illumination of the intrinsic region. The mesa contact on the chip was connected directly via a short (250 \( \mu \)m) length of low-inductance gold mesh to the centre pin of a commercial coaxial-to-microstrip transition (Wiltron K-connector). The chip was from the same wafer as those of References 4–6, and the mounting was nominally identical to that of Reference 4.

The electrical parasitics of the photodiode were determined from a series of network analyser reflection coefficient measurements at frequencies up to 10 GHz. These measurements were carried out using (i) a bare K-connector and (ii) a K-connector with the photodiode mounted. From these measurements, the parasitic capacitance shunting the depletion region was determined to be 0-05 pF at 5 V reverse bias, in agreement with previous estimates for a similar device. The series inductance of the gold mesh is approximately 0-2 nH.

The response of the photodiode was measured at a wavelength of 1-3 \( \mu \)m by electro-optic sampling in GaAs. The sampler comprised a 50 \( \Omega \) microstrip line deposited on a semi-insulating GaAs substrate and mounted in a package which was equipped with K-connectors of opposite sex to the photodiode package. The photodiode was connected directly to one end of the sampler, while the other end of the sampler was terminated with a bias tee and a 50 \( \Omega \) load. Note that this experimental arrangement gives the response of the diode, parasitics and coaxial connector, but enables the response of the bias network to be excluded. We believe that this is a major factor responsible for the improvement in our measured bandwidth over that reported in Reference 4.

A mode-locked 1-3 \( \mu \)m InGaAsP injection laser operating at a repetition rate of 3-7 GHz was used both to pump the photodiode and to probe the electro-optic sampler. An important feature of the measurement system is that the effect of the nonzero pulsewidth of the mode-locked laser can be deconvoluted from the measured data if the intensity autocorrelation function of the laser is known. In the present work, the autocorrelation was measured by conventional second-harmonic generation techniques, using a nonlinear LiIO\(_3\) crystal. The measured autocorrelation width was 17 ps FWHM. The uncertainty in the deconvolved photodiode response was approximately ±2 ps, due to the finite signal/noise ratio of the measurement. Further details of the measurement technique are presented in Reference 7.

Fig. 1 shows the measured impulse response of the photodiode when biased at 5 V. The broken curve is the response as obtained directly from electro-optic sampling, while the solid curve is the photodiode response after deconvolving the auto-