verse resonance condition, we know that the sum of the immitances across $X-X$ must equal zero, and can obtain the following equations:

(a) For odd $TE_{m0}$ modes:

$$B_{2Z,b} = \frac{b - d - t}{\tan \left( \frac{\pi (a - w)}{\lambda_c} \right)} \cdot d \cdot \frac{B_{2Z,d} + b \tan \left( \frac{\pi x}{\lambda_c} \right)}{\tan \left( \frac{\pi (a - s)}{\lambda_c} \right)} = 0 \quad (1)$$

(b) For even $TE_{m0}$ modes:

$$B_{2Z,b} = \frac{b - d - t}{\tan \left( \frac{\pi (a - w)}{\lambda_c} \right)} \cdot d \cdot \frac{B_{2Z,d} + b \tan \left( \frac{\pi x}{\lambda_c} \right)}{\tan \left( \frac{\pi (a - s)}{\lambda_c} \right)} = 0 \quad (2)$$

The discontinuity susceptance terms $B_{2Z,b}$ and $B_{2Z,d}$ are obtainable from Reference 3.

A similar procedure was carried out for the double T-septum guide (DTSG); the equivalent circuit and equations obtained are omitted.

Numerical results: Numerical results for the cutoff wavelength of the dominant $TE_{m0}$ mode and bandwidth characteristics for the STSG are shown in Figs. 3 and 4, respectively. Results obtained by the Ritz-Galerkin technique and the transmission line modelling method are also included in Figs. 3 and 4; we can see that they are in good agreement.

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References

**SIMPLE IMPLEMENTATION OF LOAD-SHARING BANYAN NETWORK AND ITS THROUGHPUT PERFORMANCE**

**Fig. 2 Equivalent circuit of single T-septum guide**

**Fig. 3 Normalised cutoff wavelength of dominant mode in STSG**

$$\lambda_{n0} = 0.45, w/a = 0.1, \phi = 0.05$$

○ Reference 1 --- Reference 2

**Fig. 4 Bandwidth characteristics of STSG**

$$\lambda_{n0} = 0.45, w/a = 0.1, \phi = 0.05$$

○ Reference 1 --- Reference 2

**Introduction:** Because of the properties such as self-routing and potential VLSI implementation, banyan networks are widely considered to construct the switching fabric in communication networks and to interconnect processing elements and memory modules in multiprocessor systems. In reality, it was proved that banyan networks are more cost-effective than crossbars for large systems. Unfortunately, banyan networks are blocking, meaning that multiple connection requests between arbitrary pairs of inputs and outputs may not be granted simultaneously. As a result, the performance for a large banyan network may not be acceptable.

Lea proposed the load-sharing concept to improve the performance of banyan networks. However, no implementation was considered. In this letter we propose a simple implementation based on two basic cells, the sorting cell and the routing cell. The sorting cell is nothing but a bitonic sorter with two inputs, and the routing cell is a usual 2 → 2 switching cell of a banyan network. The throughput performance of the proposed implementation is analysed under the uniform traffic model.

**Implementation:** A three-stage load-sharing banyan network is illustrated in Fig. 1. It is noted that the connection pattern between stages shown in Fig. 1 is for the baseline network. However, since the baseline network is topologically equivalent to the banyan network, we do not distinguish between these two terms in this letter. Basically, a load-sharing banyan network is constructed by inverting sorting cells into a banyan network. For the n-stage load-sharing banyan network we propose, the routing cells in each stage, except the last one, are partitioned into $2^{n-1}$ groups so that each group consists of two routing cells. If we use binary sequences of length $n - 1$
to represent, from top to bottom, the routing cells in each stage, then two routing cells in stage \( i \) \((1 \leq i \leq n - 1)\) are in the same group and their representations differ only in the last bit. It is not hard to see that two routing cells belonging to the same group share their loads. Moreover, our implementation is simple because the connections inside the two building blocks are both bit-controlled and hence high-speed switching is achievable. Finally, since the connection patterns of the sorting cell and the routing cell of the load-sharing banyan network are the same, any single fault can be easily diagnosed by the techniques developed in Reference 5.

We now analyze the throughput performance of our proposed load-sharing banyan network under the uniform traffic model. By uniform traffic model, it is meant that all the inlets have independent and identical input rates and each outlet is equally likely to be the destination of any connection request. Consider a pair of routing cells of the same group in stage \( k \). Let \( h(k) = \{h_0(k), h_1(k), \ldots, h_{n-1}(k)\} \), \( 0 \leq k \leq n - 1 \), denote the probability distribution of the upper (and the lower) input links, i.e. \( h_i(k) \), \( 0 \leq i \leq n - 1 \), represents the probability that the upper (or lower) input link \( i \) receives totally \( i \) active connection requests at the beginning of a cycle. Given the input rate \( \rho \), we clearly have \( h_j(0) = (1 - \rho)^j \), \( h_j(n - 1) = 2(n - 1) - 2j \rho \) and \( h_j(i) = \rho^i \). Let \( S(\rho) \) denote the average throughput of an \( n \)-stage load-sharing banyan network with input rate \( \rho \). Then \( S(\rho) \) can be computed by the following iterative algorithm:

**Step 1:** Do \( k = 0 \), \( n - 2 \)

\[
\begin{align*}
S(\rho) &= 1/2[S(n - 1) + 3/2 h_0(n - 1)]
\end{align*}
\]

\[
\begin{align*}
h_k(k + 1) &= \sum_{j=0}^{n-1} h_j(k) h_j(k + 1) \rho^{j+1} \\
h_k(k) &= \sum_{j=0}^{n-1} h_j(k) h_{j+1}(k + 1) \\
h_k(k+1) &= \left[ \sum_{j=0}^{n-1} \sum_{l=j}^{n-1} h_j(k) h_{j-l}(k + 1) \rho^l \right]
\end{align*}
\]

**Step 2:** Compute

To analyse the average throughput of the load-sharing banyan network we propose, the four input links (to sorting cells) of a pair of routing cells of the same group should be considered together. For convenience, the two input links of the upper (or lower) sorting cell are called the upper (lower) input links of a pair of routing cells. Similarly, the four output links of a pair of routing cells are also partitioned into two groups so that the two upper (or lower) output links, one from each routing cell, are called the upper (lower) output links. It is clear that the upper (lower) output links of a pair of routing cells in stage \( k \) \((1 \leq k \leq n - 2)\) are the upper or lower input links of another pair of routing cells in stage \( k + 1 \). Besides, blocking occurs only when three or four active connection requests received by the four input links are intended to be routed simultaneously to the upper or the lower output links.

Fig. 1 shows the average throughputs of a pure banyan network, a load-sharing banyan network and a crossbar switch for 64 inlets and outlets. By sharing the loads of a pair of routing cells in each stage, the average throughput of the banyan network is increased by more than 20% for \( p > 0.4 \). Fig. 2 shows the curves of the maximal achievable throughput against number of stages for the same three networks. When \( n \geq 6 \), load-sharing results in more than 23% improvement.

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**Throughput performance:** To analyse the average throughput of the load-sharing banyan network we propose, the four input links (to sorting cells) of a pair of routing cells of the same group should be considered together. For convenience, the two input links of the upper (or lower) sorting cell are called the upper (lower) input links of a pair of routing cells. Similarly, the four output links of a pair of routing cells are also partitioned into two groups so that the two upper (or lower) output links, one from each routing cell, are called the upper (lower) output links. It is clear that the upper (lower) output links of a pair of routing cells in stage \( k \) \((1 \leq k \leq n - 2)\) are the upper or lower input links of another pair of routing cells in stage \( k + 1 \). Besides, blocking occurs only when three or four active connection requests received by the four input links are intended to be routed simultaneously to the upper or the lower output links.

We now analyse the throughput performance of our proposed load-sharing banyan network under the uniform traffic model. By uniform traffic model, it is meant that all the inlets have independent and identical input rates and each outlet is equally likely to be the destination of any connection request. Consider a pair of routing cells of the same group in stage \( k \). Let \( h(k) = \{h_0(k), h_1(k), \ldots, h_{n-1}(k)\} \), \( 1 \leq k \leq n - 1 \), denote the probability distribution of the upper (and the lower) input links, i.e. \( h_i(k) \), \( 1 \leq i \leq n - 1 \), represents the probability that the upper (or lower) input link \( i \) receives totally \( i \) active connection requests at the beginning of a cycle. Given the input rate \( \rho \), we clearly have \( h_j(0) = (1 - \rho)^j \), \( h_j(n - 1) = 2(n - 1) - 2j \rho \) and \( h_j(i) = \rho^i \). Let \( S(\rho) \) denote the average throughput of an \( n \)-stage load-sharing banyan network with input rate \( \rho \). Then \( S(\rho) \) can be computed by the following iterative algorithm:

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**Conclusion:** We propose in this letter an implementation of a load-sharing banyan network which requires simple management and is easy to diagnose. The complexity of our proposed load-sharing banyan network is about twice of that of a pure banyan network, since the complexity of a sorting cell is roughly the same as that of a routing cell. Our implementation can be easily extended so that more routing cells in each stage form a group and share their loads to further improve the performance of banyan networks.

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