Abstract—The existence of complex modes in electrically shielded suspended coupled microstrip lines has been studied extensively, and the results are presented. A rigorous full-wave spectral-domain approach (SDA) with a newly proposed and tested set of basis functions can efficiently and accurately determine the propagation characteristics of the dominant, higher order, and complex modes for planar or quasi-planar transmission lines. These basis functions are validated by comparing the convergence study of field solutions with those obtained by various sets of preconditioned bases and by the unconditioned subdomain ones. Excellent agreement is obtained for the propagation constants and the normalized complex longitudinal and transverse current distributions on conducting strips for the strongly coupled microstrip lines. This suggests that the proposed set of basis functions can be a viable candidate for the SDA in analyses of planar or quasi-planar transmission lines.

Under all the particular case studies of this paper, it is shown that the complex modes may exist in all the shielded suspended coupled microstrip lines, even when the substrate dielectric constant is low. Theoretical results for the fundamental, higher order, evanescent, and complex modes are presented for suspended coupled microstrip lines.

1. INTRODUCTION

PLANAR AND quasi-planar transmission lines are the most commonly used waveguides in microwave and millimeter-wave integrated circuits [1], [2]. Various applications such as impedance matching, filter, and coupler designs may inevitably introduce discontinuities into the passive circuits. Therefore the characterization of planar or quasi-planar transmission line discontinuities becomes an important task for the computer-aided design (CAD) of microwave and millimeter-wave integrated circuits [3]. For electrically shielded planar and quasi-planar transmission lines, many authors have reported that the fundamental, higher order, evanescent and complex modes may coexist [4]–[7]. As Omar and Schünemann reported in their work of finline analysis [4], complex modes and backward-wave modes may exist in any planar guiding structure with closed boundaries. Later the inhomogeneously and anisotropically filled waveguides [5] and the symmetric microstrip line [6], [7] were also found to support complex modes. Overlooking either one or both of a pair of complex modes in the waveguide discontinuity analysis may lead to an irregularity of circuit parameters in the entire frequency spectrum of interest [8].

So far two methods for obtaining the solutions of the complex modes for microstrip and slotlines (or finlines) have been reported. The singular integral equation (SIE) technique has been efficiently applied to analyze a symmetric finline [4] and a microstrip line [6]. In the literature [7], a discrete space-domain formulation has been used to calculate various modes for a symmetric microstrip line in closed structure.

Notice that all the reported solutions of the complex modes are for symmetric transmission lines. Besides the SIE and the discrete space-domain approach, the spectral-domain approach (SDA) is the most widely used method in analyzing millimeter-wave planar or quasi-planar transmission lines for the dominant and the first few higher order modes [4]. This paper extends the SDA to analyze the possible existence of complex modes in symmetric or asymmetric planar or quasi-planar transmission lines embedded in a layered dielectric medium. Should both longitudinal and transverse current distributions on each conducting strip be correctly obtained for any type of modal solution, the field solutions would be exact. One of the objectives of this paper is to obtain currents that are as close to being true as possible for any modal solution of general printed circuit lines. To achieve this, a newly proposed set of basis functions satisfying the criteria reported by Jansen [9] is derived from a two-dimensional electrostatic wedge problem.

Subsection II-A briefly describes the basic features of the SDA, and subsection II-B lists the proposed set of basis functions with detailed discussions. Subsection II-C qualitatively compares the numerical efficiency of the proposed bases with various types of preconditioned bases, namely 1) the sinusoidal type [10], [11], i.e., Fourier sine and cosine series divided by a term that corresponds to the edge singularity of an infinitely thin metal strip, and 2) the Chebyshev type [12], i.e., Chebyshev functions of the first and second kinds modified by the edge condition. These two sets of basis functions will be designated as S (sinusoidal) type and C (Chebyshev) type herein.

To test the accuracy of results obtained by using the proposed bases, subsection III-A performs a comparative convergence study of the complex modes obtained by
various types of basis functions for asymmetric strongly coupled microstrip lines. Because the entire-domain (or global) basis functions are preconditioned, it is necessary to test whether the results obtained by the proposed bases are the same as those obtained by the unconditioned ones, namely, the discretized or subdomain bases [13] (see also Section III). The subdomain bases are believed to be capable of representing true complex current distributions of single or coupled microstrip lines if each metal strip is partitioned into an adequate number of subsections. Subsection III-B compares the capability of representing true complex current distributions on the printed lines for various types of preconditioned bases and unconditioned subdomain bases. Subsection III-C presents the theoretical fundamental, higher order, evanescent, and complex modes of suspended coupled microstrip lines with different dielectric substrates.

II. FORMULATION

A. Spectral-Domain Approach

The SDA has been widely accepted for analyzing transmission lines, regardless of whether they are open or closed structures [14]. When analyzing the coupled microstrip lines embedded in a layered dielectric medium as shown in Fig. 1, the SDA is even more attractive if the concept of the immittance approach is invoked [15]. Conceptually the immittance approach, which combines network and field theories, provides much more physical insight than other techniques developed for microstrip and slotline analyses. For the structural geometry shown in Fig. 1, the SDA starts with the Fourier transform defined as follows:

\[ F(\Psi(x, y)) = \int_{-\infty}^{\infty} \Psi(x, y) \cdot e^{jnx} \, dx = \tilde{\Psi}(\alpha, y) \]  

(1)

By the immittance approach [15], the dyadic Green’s function \( \tilde{Z} \) can be derived as

\[
\begin{bmatrix}
\tilde{Z}_{zz}(\gamma) \\
\tilde{Z}_{zx}(\gamma) \\
\tilde{Z}_{xz}(\gamma) \\
\tilde{Z}_{xx}(\gamma)
\end{bmatrix}
\begin{bmatrix}
J_x \\
J_y
\end{bmatrix}
= 
\begin{bmatrix}
E_z \\
E_x
\end{bmatrix}
\]

(2)

where \( \gamma \) is the propagation constant to be determined and \( \exp(j\omega t - \gamma z) \) is assumed.

Next, the Galerkin procedure is applied to (2). The unknown surface current densities on each conducting strip are expanded by a complete set of basis functions. After matching the final boundary conditions imposed on the metal–dielectric interface, e.g., \( y = h_1 + h_2 \) in Fig. 1, a nonstandard eigenvalue problem can be formulated, namely

\[ \det[G_{p \times p}(\gamma)] = 0. \]  

(3)

The propagation constants \( \gamma \)'s are the roots of (3). These roots may stand for the fundamental, higher order, evanescent, or complex modes for the coupled microstrip lines. The integer subscript \( p \) stands for the order of the square matrix \( G \).

B. The Proposed Set of Basis Functions and Its Derivation

Motivated to obtain very accurate modal solutions, including the possible existence of complex modes, we seek to develop an alternative set of basis functions. Jansen [9] listed six criteria for obtaining basis functions used in the SDA, namely 1) edge condition, 2) twice continuous differentiability, 3) completeness, 4) integral relationship between longitudinal and transverse currents of a microstrip line, 5) ability to represent nearly true modal current distributions, and 6) capability of being Fourier transformed. Now consider the two-dimensional electrostatic wedge problem shown in Fig. 2, in which the complete solution for the surface charge distribution at point \( P|_{\theta = 0} \) can be expressed as [16]

\[ \sigma(\rho) = \sum_{n=1}^{\infty} a_n \rho^{(e \theta/\theta - 1)} \]  

(4)

where \( \rho \) is the distance measured from the corner, and the \( a_n \)'s are constants. For the infinitely thin microstrip lines shown in Fig. 1, \( \theta = 2\pi \). Let us consider the proposed basis functions for the current distributions on strip 1 of width \( W_1 \) in Fig. 1 (the derivation of those for the currents on strip 2 is identical). Since the dielectric substrates are nonmagnetic, the longitudinal current \( J_x(x) \) is
proportional to the charge distribution \( \sigma(x) \) under quasi-TEM conditions [17]. Letting the normalizing variable \( u_1 = 2/(X - S_j - W_1)/2W_1 \), and subsequently \( \rho^{n/2-1} = (W_1/2)^{1/2} - (1 \pm u_1)^{n/2-1} \) for \( n = 1, 2, \ldots \), we obtain \( J_{z1}(u_1) \) corresponding to the edge at \( u_1 = -1 \) as

\[
J_{z1}(u_1) = \sum_{n=1}^{\infty} b_{2n}(1 + u_1)^{n/2-1}
\]

and \( J_{z2}(u_1) \) corresponding to the edge at \( u_1 = 1 \) as

\[
J_{z2}(u_1) = \sum_{n=1}^{\infty} b_{2n}(1 - u_1)^{n/2-1}.
\]

In (5) and (6), the \( b_{2n}'s \) and \( 2n' \)s are constants and \(-1 < u_1 < 1\). The longitudinal current on strip 1, \( J_{z1}(u_1) \), is the sum of \( J_{z1}(u_1) \) and \( J_{z2}(u_1) \):

\[
J_{z1}(u_1) = J_{z1}(u_1) + J_{z2}(u_1)
\]

\[
= 2 \sum_{m=1}^{N_1} \sum_{n=1}^{N_1} c_{1mn} j_{zmn}(u_1).
\]

Notice that \( J_{z1}(u_1) \) obtained by extending \( J_{z1}(u_1) \) at \( u_1 = -1 + 1 \), is the sum of \( J_{z1}(u_1) \) and \( J_{z2}(u_1) \):

\[
J_{z1}(u_1) = (1 \pm u_1)^{n/2-1} - \sqrt{2}(1 \pm u_1)^{n/2+1/2}
\]

\[
+ \frac{1}{2}(1 \pm u_1)^{n/2-1}.
\]

where plus is for \( m = 1 \), minus is for \( m = 2 \), and \( n \) is the order of the basis functions. In (7), \( N_1 \) is the number of basis functions \( J_{z1}(u_1) \) used in the SDA for the expansion of \( J_{z2}(u_1) \). Notice that the edge condition of an infinitely thin strip is guaranteed by the asymptotic behavior of \( J_{z1}(u_1) \) at \( u_1 = -1 \) and \( 1 \) for \( m = 1 \) and 2, respectively. For example, as \( u_1 \) approaches \(-1 \), \( J_{z1}(u_1) \) prevails and the asymptotic behavior \( \delta^{1/2} \) is guaranteed. On the strip away from the edges the longitudinal current distribution is then modeled by (7) and (8). Notice also that criterion 3 is satisfied by (7) and (8). It is obvious that the bases in (8) satisfy criterion 2.

By integrating the following continuity equation imposed on the metal strip as is done in [17],

\[
\frac{\partial}{\partial x}(J_x(x)e^{-\gamma x}) + \frac{\partial}{\partial z}(J_z(x)e^{-\gamma z}) = -jw\sigma(x)e^{-\gamma x}
\]

where \( \sigma(x) \) has been defined in (4), we obtain

\[
J_{z1}(u_1) = J_{z1}(u_1) + J_{z2}(u_1)
\]

\[
= \sum_{m=1}^{N_1} \sum_{n=1}^{N_1} d_{1mn} j_{zmn}(u_1),
\]

and

\[
J_{z1mn}(u_1) = (1 \pm u_1)^{n/2-1} - \sqrt{2}(1 \pm u_1)^{n/2+1/2}
\]

\[
+ \frac{1}{2}(1 \pm u_1)^{n/2-1}.
\]

where the plus and the minus correspond to the subscript \( m = 1 \) and 2, respectively. Notice that \( J_{z1}(u_1) \) vanishes at the rate of \( \delta^{1/2} \) at \( u_1 = -1 \) and \( u_1 = 1 \). In addition, \( J_{z11}(u_1) = 0 \) and \( J_{z12}(u_1) = 0 \) for all \( n \). By doing so, \( J_{z1}(u_1) \) and \( J_{z2}(u_1) \) have no influence on the transverse current distribution at the edges \( u_1 = 1 \) and \( u_1 = -1 \), respectively. Although there are many possible ways of doing this, (7) and (8) and (10) and (11) are found to be capable of modeling true current distributions.

When \( n = 1 \) and 2, the Fourier transforms of \( (1 + u_1)^{n/2-1} \) can be readily derived, and the results are

\[
F[(1 + u_1)^{-1/2}] = e^{-j/n\kappa} W_1/2 \left[ F(x) + jF(2\kappa) \right] e^{j\alpha x}.
\]

(12)

\[
F[(1 + u_1)^{1/2}] = 1/n \cdot W_i \cdot \sin(\kappa) \cdot e^{j\alpha x}.
\]

(13)

where \( \kappa = (\pi W_1)/2 \), \( x = S_i + 1/2W_1 \), and the Fourier transform variable \( \alpha \). \( F(x) \) and \( F(2\kappa) \) are respectively the Fresnel cosine and sine integrals, defined by

\[
F(\xi) = \int_0^\infty u^{1/2} \cdot e^{j\alpha u} du.
\]

(14)

By definition and by integrating by parts, \( F[(1 + u_1)^{n/2-1}] \) with \( n > 3 \) can be expressed as

\[
F[(1 + u_1)^{n/2-1}] = -j \frac{2\kappa}{W_1} \left( 2^{n/2} \cdot e^{\alpha x} \cdot W_i \cdot e^{j\alpha x} \right)
\]

\[
- (n - 2) \cdot F[(1 + u_1)^{n/2-1}].
\]

(15)

Since \( F[(1 + u_1)^{n/2-1}] \) is satisfied by (7) and (8) and (10) are well suited to determine the current distributions on the asymmetric coupled microstrip lines in Fig. 1. Even in the case of a symmetric structure, e.g., a symmetric microstrip line, they automatically result in both even and odd modes of solutions.

C. Other Bases Commonly Used for Analyzing Asymmetric Structures by the SDA or Space-Domain Technique

Jansen [10] chose a complete set of basis functions satisfying the edge condition term by term to compute the characteristics of both shielded and open planar microwave and millimeter-wave transmission lines. Later, Schmidt et al. [11] used only two expansion terms of this set of basis functions to obtain both the propagation constant and the characteristic impedance with 0.5% accuracy for an arbitrarily located unilateral finline. Kobayashi et al. formulated closed-form expressions to approximate the unknown longitudinal and transverse current distributions on microstrip line [19] and coupled microstrip lines [20]. Both sets of closed-form expressions
have resulted in accurate dominant-mode solutions when they are employed in the SDA. However, the mathematical expressions of the basis functions for the coupled microstrip lines are complicated [20]. Recently, Faché and De Zutter [13], [21] analyzed single and coupled microstrip lines in the space domain using the method of moments to discretize each current component. Tripathi and Lee [12] used Chebyshev polynomials of the first and second kinds modified by the edge condition to compute the dispersive characteristics of multiple coupled line structures in an inhomogeneous medium.

One of the most advantageous features of the SDA is the fact that the preconditioned basis functions do not have to be very accurate to obtain a moderately accurate propagation constant for practical applications [19]. An insufficient number of basis functions and inaccurate bases, however, will result in poor SDA solutions [19], [20]. This is due to the fact that criterion 5 in subsection II-B is violated.

To verify the fact that the proposed set of bases is capable of modeling true currents, i.e., criterion 5, we incorporate the SDA basis functions employed in [10]–[12] into the analysis of asymmetric coupled microstrip lines for comparative study. The Fourier transforms for these sets of basis functions are listed here for reference.

1) Sinusoidal (S) Type [10], [11]:

\[
\tilde{f}_{\text{S},n}(a) = \frac{\pi}{2} \cdot W \cdot \left[ B_0(aW + p_n) \cdot e^{ip_n} + B_0(aW - p_n) \cdot e^{-ip_n} \right] \quad (16)
\]

\[
\tilde{f}_{\text{S},n}(a) = -j \frac{\pi}{2} \cdot W \cdot \left[ B_0(aW + p_n) \cdot e^{ip_n} - B_0(aW - p_n) \cdot e^{-ip_n} \right] \quad (17)
\]

where \( p_n = (n/2)\pi \).

2) Chebyshev (C) Type [12]:

\[
\tilde{f}_{\text{C},n}(a) = (j)^n \cdot \pi \cdot W \cdot B_n(aW) \quad (18)
\]

\[
\tilde{f}_{\text{C},n}(a) = (j)^n \cdot \pi \cdot W \cdot (n+1) \cdot B_n(aW)/(aW) \quad (19)
\]

In (16) through (19), the center of the strip is located at \( x = 0 \). \( B_n \) is the \( n \)th-order Bessel function of the first kind, \( n = 0, 1, 2, \cdots, \) and \( W \) is the half-width of the strip.

To compare the numerical efficiencies of the three sets of global or preconditioned bases, we must compare the CPU times used for setting up the determinantal matrix \( G \) in (3). For the proposed bases, a) \( F[(1 + u_1)^{y/2 - 1}] \), for \( n \geq 3 \), can be obtained by the application of recursive formula (15); b) \( \tilde{f}_{\text{S},1,\text{m}}(a) = \tilde{f}_{\text{S},1,\text{m}}(a) \), \( p = n + 2 \) for \( m = 1 \) or 2, (6) and (11); and c) \( \tilde{f}_{\text{C},1,\text{m}}(a) \) and \( \tilde{f}_{\text{C},1,\text{m}}(a) \) are the complex conjugates of \( \tilde{f}_{\text{C},1,\text{m}}(a) \) and \( \tilde{f}_{\text{C},1,\text{m}}(a) \), respectively, for \( n = 1, 2, \cdots, N_\theta \) (subsection II-B). Therefore the most time-consuming part is the evaluation of Fresnel integrals. The computations of \( \tilde{f}_{\text{C},1,\text{m}}(a) \) and \( \tilde{f}_{\text{C},1,\text{m}}(a) \) invoke simple algebraic computations on the stored data. Our experiences show that the CPU time of the SDA program using bases of the S type is nearly 10% more than that using bases of the C type or the bases proposed for implementing a determinantal matrix of size \( 32 \times 32 \).

### III. Results

The legitimacy of the proposed bases will be tested against the existing ones, namely, the preconditioned S and C types and the unconditioned subdomain bases. We choose a pair of asymmetric strongly coupled microstrip lines (Fig. 1) with aspect ratios of \( S_1:W_1:S_2:W_2:S_3 = 89.5:20:1:40:49.5 \) as a test case. This can be a difficult situation for the SDA program to obtain an accurately converged result for any modal solution. Due to the strong coupling between the adjacent edges, a sufficient number of bases and a sufficiently large number of spectral terms are required to obtain very accurate results.

Similar experience was reported in [10], where a square matrix of order 20 and \( 10^3 \) spectral summation terms were used to obtain accurate electromagnetic field solutions for a narrow microstrip with a shielding/line width ratio of 30. Here, a square matrix order from 32 to 40 (16 to 20 for one strip) and a number of spectral terms \( N \) from \( 10^4 \) to \( 10^5 \) are used in the SDA program to perform the comparative convergence study of various types of bases.

The discretized, or subdomain, bases [13] are also incorporated into the SDA to verify the solutions. Each conducting strip is partitioned into \( M \) intervals of equal width, and the modeling of current distributions in each interval is similar to that of [13] except that the first three terms of (5) and (6) are used to account for the edge condition in the outermost intervals. These unconditioned subdomain bases enforced with the edge condition are simply called the unconditioned bases or the subdomain bases throughout the paper.

The results reported herein are obtained by using the RM/FORTRAN version 2.4 on an IBM personal computer or the VAX/VMS FORTRAN on a VAX-3200 workstation. Both result in the same answers for all the particular case studies presented here. Since one of the goals of this work is to obtain modal solutions that are as accurate as possible, faster converging algorithms, such as the technique reported in [7] are not used. No approximation is made except for the truncation on the finite number of spectral summation terms \( N \). All the variables in the SDA program are declared as double precision to maintain the best accuracy.

#### A. Convergence Study of the Modal Solution of the Strongly Coupled Microstrip Lines

Table I presents the solution of the complex mode of the asymmetric strongly coupled microstrip lines analyzed at 150 GHz. Two aspects of the convergence study will be discussed, namely the number of spectral terms \( N \) and the number of basis functions \( N_\theta \). The bases adopted in Table I fall into two classes, namely preconditioned bases and unconditioned subdomain bases.
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The nearly converged results obtained by the unconditioned subdomain bases have the closest agreement with those obtained by the preconditioned bases, i.e., the solutions obtained by the subdomain bases agree with those by the S type, C type, and proposed bases to $7.4 \times 10^{-4}\%$ ($3.4 \times 10^{-5}\%$), $7.7 \times 10^{-5}\%$ ($7.8 \times 10^{-6}\%$), and $2.8 \times 10^{-4}\%$ ($5.1 \times 10^{-5}\%$) accuracy for $\alpha$ ($\beta$), respectively. Compared with bases of the S and C types the solutions obtained by the proposed bases have the closest agreement with those obtained by the unconditioned subdomain bases.

We do not know yet which set of basis functions results in the closest solution to the true current. Thus the next subsection plots the complex modal $J_1$ and $J_2$ current distributions for the comparative study.

B. Current Distributions on Strongly Coupled Microstrips: Capability of Modeling True Current Distributions — Criterion 5

The preconditioned basis functions in the SDA are required in order to represent the true modal current distributions. For the particular case of the asymmetric strongly coupled microstrip lines discussed in subsection III-A, the current distributions for the complex modes obtained by the proposed bases may nearly converge by using $N_b = 4$ and $N = 5 \times 10^4$, whereas bases of the S type and the C type require the use of $N_b = 10$ and $N = 5 \times 10^4$.

Parts (a) through (d) of Fig. 3 illustrate the magnitude and phase of the normalized $J_1$ and $J_2$ on strips 1 and 2 for one of the complex modes. All the current components are normalized by the total longitudinal current on strip 1, i.e., $I_L$. Using the same structural parameters and the same complex mode in Table I, the results obtained by the unconditioned subdomain bases and various preconditioned bases are presented. The number of spectral terms $N$ and the number of intervals $M$ used for the case of subdomain bases are $10^4$ and 40, respectively.

As shown in parts (a) through (d) of Fig. 3, the normalized current distributions obtained by the S type and C type bases are oscillatory. Not shown in these figures is the fact that the amplitudes of the oscillations surrounding the plots computed on the basis of the proposed bases decrease as $N_b$ increases from 6 to 10. The current distributions presented for the S and C type bases are those for $N_b = 10$. In an average sense, all the bases result in close agreement for the normalized $J_1$ and $J_2$ components; however, the normalized $J_1$ and $J_2$ obtained by bases of the C type deviate appreciably from those obtained by the others. Notice that the longitudinal ($J_1$ or $J_2$) and transverse ($J_1$ or $J_2$) currents of the complex mode are comparable in magnitude while the longitudinal

### Table I

**Convergence Study of the Normalized Propagation Constant of a Complex Mode of the Suspended Asymmetric Strongly Coupled Microstrip Lines.**

<table>
<thead>
<tr>
<th>$N_b$</th>
<th>$N = 10^4$</th>
<th>$N = 5 \times 10^4$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\gamma/k_0 = a/k_0 + j\beta/k_0$</td>
</tr>
<tr>
<td>S type*</td>
<td>6</td>
<td>1.5454488 + 0.01898261</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>1.5455237 + 0.01905369</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>1.5455128 + 0.01901245</td>
</tr>
<tr>
<td>C type*</td>
<td>6</td>
<td>1.5457093 + 0.01916921</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>1.5456974 + 0.01916724</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>1.5456974 + 0.01916724</td>
</tr>
<tr>
<td>Proposed bases*</td>
<td>3</td>
<td>1.5444925 + 0.01609656</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>1.5444999 + 0.01601750</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>1.5444999 + 0.01601750</td>
</tr>
<tr>
<td>Subdomain bases M*</td>
<td>20</td>
<td>1.5450036 + 0.01918927</td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>1.5449991 + 0.01918936</td>
</tr>
<tr>
<td>N</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Subdomain bases M</td>
<td>20</td>
<td>1.5445107 + 0.01901548</td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>1.5445062 + 0.01901672</td>
</tr>
</tbody>
</table>

Structural parameters: $2a = 2.54 \text{ mm}$, $S = 1.13665 \text{ mm}$, $W = 0.254 \text{ mm}$, $S_x = 0.0127 \text{ mm}$, $W_b = 0.508 \text{ mm}$, $S_y = 0.26865 \text{ mm}$, $h = k_x = 0.254 \text{ mm}$, $h_x = 0.762 \text{ mm}$, $\epsilon_r = \epsilon_x = 1$, $\epsilon_z = 2.2$; frequency $= 150 \text{ GHz}$.

*These sets of bases belong to the class of preconditioned or entire-domain or global basis functions. The S and C type bases are for sinusoidal [10], [11] and Chebyshev [12] types of bases, respectively.

*The number of the basis functions used to expand $J_1$ or $J_2$ on each conducting strip. The matrix size for the proposed set of bases is $8N_b \times 8N_b$ and for the S or C type of bases the matrix size is $4N_b \times 4N_b$.

*Normalized complex propagation constant, $k_0$ being the wavenumber in free space.

$M$ is the number of equally partitioned intervals of each conducting strip. The first three edge terms in (5) and (6) are incorporated into the subdomain bases for both $J_1$ and $J_2$ at the outermost intervals. The matrix size is $(4(M + 3)) \times (4(M + 3))$.
c. Fundamental, Higher Order, Evanescent, and Complex Modes in Suspended Coupled Microstrip Lines

Some of the important features of the complex modes and the backward-wave modes of planar transmission lines were thoroughly discussed in [4]. Here, theoretical results for various types of modes of the suspended coupled microstrip lines are presented for the first time. For all the modal solutions shown later, the normalized propagation constant \( \gamma / k_0 (\alpha / k_0 + j \beta / k_0) \) is plotted against frequency. The evanescent modes are in the lower parts of the figures. For one of a pair of complex modes, the normalized phase constant \( \beta / k_0 \) and the normalized attenuation constant \( \alpha / k_0 \) are represented by a dotted line and a dashed line, respectively. Since the convergence study of the modal solutions has been established in subsections III-A and III-B, the search of the modal...
ward-wave(2,8),(996,992) and complex mode regions are designated as 

\[ \varepsilon_r = 2.2, \] 

the electric constant of the suspended substrate is 2.2, and Notice that these two regions of complex modes have 

end, the dominant mode is designated as mode 1, the first 

pled microstrip lines, respectively. At the high-frequency 

dispersion characteristics of the particular suspended cou-

sets of complex modes. One ranges from under 

Fig. 4. Normalized propagation constant \( \gamma/k_0 = a/k_0 + j\beta/k_0 \) of 

the suspended coupled microstrip lines versus frequency. Structural 

parameters: \( 2a = 2.54 \text{ mm}, S_1 = S_2 = 1.1049 \text{ mm}, W_1 = W_2 = 0.127 \text{ mm}, \) 

\( S_3 = 0.0762 \text{ mm}, 3b_1 = b_2 = 0.361 \text{ mm}, a_1 = 0.762 \text{ mm}; \) \( \varepsilon_r = \varepsilon_r = 1, \) 

\( \varepsilon_r = 2.2. \) (a) Odd-mode excitation. (b) Even-mode excitation.

solutions is carried out by using the proposed basis func-

tions with \( N_b = 3. \) 

Parts (a) and (b) of Fig. 4 plot the odd- and even-mode 

dispersion characteristics of the particular suspended cou-

pled microstrip lines, respectively. At the high-frequency 

end, the dominant mode is designated as mode 1, the first 

higher order mode as mode 2, and so on. Reading from 

the right of each figure, the primed number denotes the 

mode coming out of the complex mode region. The back-

ward-wave and complex mode regions are designated as 

\( b_2 \) and \( c_4 \), respectively. The subscript \( k \) denotes different 

regions for the backward-wave and complex modes in the 

figure. The \( W \)-band (75–110 GHz) waveguide housing of 

2.54×1.27 \text{ mm}^2 \) (100×50 mil²) is used. The relative di-

electric constant of the suspended substrate is 2.2, and 

the thickness is 15 mils. 

The odd-mode case of Fig. 4(a) consists of two distinct 

sets of complex modes. One ranges from under 50 GHz to 

138 GHz, the other from 73 GHz to beyond 150 GHz. 

Notice that these two regions of complex modes have 

nearly the same values of imaginary \( (\beta) \) parts. In the case 

of the even mode, Fig. 4(b) indicates that the first higher 

order mode has a cutoff frequency near 55 GHz, which is 

the reduction of the cutoff frequency of an air-filled 

\( W \)-band waveguide (59 GHz) due to the presence of the 
dielectric layer. The first pair of complex modes is beyond 

130 GHz. 

Keeping the same physical structure as used in Fig. 4 

except for an increase in the relative dielectric constant 

from 2.2 to 10, the odd-mode case of Fig. 5(a) has four 

sets of backward-wave modes and nine regions of complex 

modes, and the modal solutions become much more com-
plicated than those in Fig. 4(a). Notice that mode 1 and 

mode 2, and mode 6 and mode 7 do not intersect as 

shown in the zoomed windows. Similarly, Fig. 5(b) of the 
even-mode case has at least two and six sets of backward-

wave and complex modes, respectively, in the frequency 
spectrum plotted.

Through the particular case studies investigated, a few 

observations from the modal solutions with complex 

modes are made: 1) the complex modes and the back-

ward-wave modes may start to appear as the second and 

the third higher order modes (See Fig. 5); 2) the complex 

modes may start to appear in the third and fourth higher 

order modes even when a low-dielectric-constant sub-

strate is used (See Fig. 4(a)); 3) the complex modes may 

spread entirely beyond the frequency spectrum of interest 

for certain suspended coupled microstrip lines (See Fig. 

4(a)); 4) the complex modes may repeatedly occur within 

a certain frequency band (See Fig. 5(a)); and 5) the 

above-cutoff higher order modes may result in the gener-

ation of complex modes (See Fig. 5(b)). 

Therefore, for the particular case study of the sus-

pended coupled microstrip lines, it is important to take 

into account the existence of the complex modes in formu-

lating the discontinuity problem.

IV. Conclusion

An extensive convergence study of the full-wave spec-
tral-domain analysis (SDA) of quasi-planar suspended 
coupled microstrip lines is presented. It validates the use 
of the proposed set of bases incorporated into the SDA 
program. In particular, the case of the suspended strongly 
coupled microstrip lines is investigated thoroughly. The 
comparative analyses of the normalized current distribu-
tions, which are obtained by the proposed bases and other 
commonly used preconditioned and unconditioned bases, 
dicate that the solutions based on the proposed bases 
exhibit the smoothest current distributions for the com-
plex modes and have excellent agreement with those 
obtained by the unconditioned subdomain bases. Thus 
this paper provides a viable set of basis functions for the 
SDA in analyzing transmission lines of planar or quasi-
planar structures.

Of more importance is the fact that the suspended 
coupled microstrip lines may have complex modes which 
start to appear as the first few higher order modes even 
when a substrate of low dielectric constant \( (\varepsilon_r = 2.2) \) is
Fig. 5. Normalized propagation constant $\gamma/k_0 = \alpha/k_0 + j\beta/k_0$ of the suspended coupled microstrip lines versus frequency. Structural parameters: $2a = 2.54$ mm, $S_1 = S_2 = 1.1049$ mm, $W_1 = W_2 = 0.127$ mm, $S_3 = 0.0762$ mm, $h_1 = h_2 = 0.381$ mm, $h = 0.762$ mm; $\varepsilon_1 = \varepsilon_2 = 1$, $\varepsilon_3 = 10$. (a) Odd-mode excitation. (b) Even mode excitation.
used. In contrast to the suspended coupled microstrip lines, the symmetric finline may have complex modes at fairly higher order modes, say the eighth higher order mode [8]. Thus the characterization of the discontinuity problem of the suspended coupled microstrip lines would have inaccurate results if the effects of the complex modes were not included in the analysis. The results presented in this paper may prove useful for understanding the operation of microwave or millimeter-wave circuits integrated by suspended coupled microstrip lines involving discontinuities.

REFERENCES


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