Optimum multicast of multimedia streams

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Scope and purpose

As multimedia service becomes more widely used through computer networks, conserving network resources becomes increasingly important. Multicast communications can save network bandwidths when delivering data to multiple destinations. Therefore, many researchers have given much attention to multicast routing problems. Most of the multicast routing problems only consider a single multicast session. However, in the real world, several multicast sessions will be broadcast, simultaneously. These multicast sessions will contend for the limited resources (such as bandwidth) of networks. This creates a new network optimization problem which is different from any other multicast routing problem. In this research, we studied an optimal video distribution problem which arises from video on demand (VOD) systems with multiple multicast sessions. We believe that the results are useful for improving video distribution methods in multimedia networks. In addition, this study would be applicable on the field of network flows, especially multicommodity flows.

Abstract

In a VOD system, multicast is a preferred method for saving network bandwidth. That is, customers requesting the same video program over a small time interval can be arranged in a multicast tree (group), and

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then the video server sends a video stream via this tree to customers. In this research, we considered how to arrange a set of multicast trees such that the number of customers served is maximized and the link capacity constraint is maintained. For a directed acyclic graph (DAG), we proposed a branch-and-bound algorithm to solve this problem and the solution method is illustrated by example. Next, we extended the algorithm to find an approximate solution for a general graph case. It is shown, through simulations on randomly generated graphs that the solution for our approximation method is very close to the optimal solution. © 1999 Elsevier Science Ltd. All rights reserved.

Keywords: Integer programming; Branch-and-bound algorithm; Network optimization; Multicast routing

1. Introduction

The demand for multimedia (such as video or audio) transmissions is increasing rapidly and the transmission of multimedia is time sensitive. In order to maintain quality of service (QoS), networks have to allocate enough bandwidth for transmitting multimedia data. Two types of multimedia transmissions are usually used. They are point-to-point transmission and point-to-multipoint transmission. The point-to-multipoint transmission, known as multicast, occurs when the multimedia data is to be delivered to a subset of nodes in the network. Broadcast is a special case of multicast in which the data is to be delivered to a set that includes all the network nodes.

In a multicast communication, the source sends identical data to all destinations. Since the data can be duplicated at switching nodes (the intermediate nodes), it is not necessary for the source node to send separate copies to all destinations. Thus, a good multicasting path can help to reduce the number of redundant streams flowing on the network. For example, consider a VOD system to which several users may subscribe an identical multimedia stream over a small time interval. Because the transmission of the video stream consumes the high bandwidth, it is a good idea for video server to collect the subscription information and then find a multicast tree to transmit a copy of the subscribed video stream to all subscribers.

Usually, two types of objective functions are considered in solving multicast transmission problems. One function minimizes the transmission cost. The other function minimizes transmission delay. For unconstrained case, a least cost multicasting path finding problem is known as the Steiner tree problem. In this problem, we are given a subset \( S \subseteq N \) of nodes, called member nodes, and we wish to determine the minimum cost tree that has to contain all of the member nodes in \( S \). The Steiner tree problem is known to be NP-complete [1]. Several exponential time algorithms have been developed for finding exact solution. Among these works, Hakimi [2] proposes a spanning tree enumeration algorithm that runs in \( O(k^22^{n-k} + n^3) \) time, where \( n = |N| \), and \( k = |S| \). Levin [3] proposes a dynamic programming approach algorithm that runs in \( O(3^kn + 2^kn^2 + k^2n) \) time. Other exponential time algorithms have also appeared without time complexity analysis. Beasley [4] gives a Lagrangean relaxation algorithm. Another 0–1 linear programming algorithm in a slightly different version is given by Wong [5]. Also, there are many heuristic algorithms [6–8] that have been presented for solving Steiner tree problem. For detail description and more lectures on both exact and heuristic methods, one can refer two survey papers, Hwang and Richards [9] and Winter [10]. On the other hand, if the goal is to minimize the delay, a shortest path tree is the solution [11–14]. The problem can be solved by finding the
shortest paths from the source node to all of the destination nodes and then merging these paths into a single multicast tree. For example, in Fig. 1a, node $s$ is the source node and nodes $v_2$, $v_4$ and $v_5$ are the destination nodes. Each link is associated with a nonnegative integer to represent its transmission delay. Then the optimal solution for this problem (see Fig. 1c) can be obtained by merging three shortest paths, which are shown in Fig. 1b, into a tree.

In previous research, most of the multicast problems only considered a single multicast session. In the single multicast session, the major concern is how to find a minimum cost Steiner tree. Then use this tree to transmit the same data to every multicast member. However, in the real world, several multicast sessions will happen, simultaneously. For example, if there are a lot of video programs available in a VOD system for subscriptions, the server will establish several multicast sessions (i.e., each video has a multicast session) to transmit the video stream to customers. Note that the amounts of network resources are fixed. When multiple multicast sessions are setup simultaneously, they will contend for these network resources. If the resources are large enough, all multicast sessions can be setup. Otherwise, some of the sessions will fail. Therefore, the important issue is how to allocate network resources to each multicast session. The objective function for multisession problem can be the minimization of the total cost or the maximization of the number of VOD customers served. Note that for a VOD system, which provides pay-movies, the more number of customers served means the more revenues earned.

Fig. 1. An example for solving multicast transmission problem, which the objective function is to minimize the transmission delay.
In this research, we considered a VOD system with multiple multicast sessions. The problem was to find a set of multicast trees such that the number of VOD customers served was maximized under the bandwidth constraint. For the directed acyclic network, a branch-and-bound method was proposed to solve this problem. The solution method can be modified to solve the general graph case.

The remainder of this paper is organized as follows. Section 2 states the formulation for this problem. Section 3 presents a branch-and-bound method that can solve this problem over a directed acyclic network. Section 4 presents a DAG's based heuristic algorithm to solve the general graph using a modified branch-and-bound method. In Section 5, we give simulation results that illustrate the performance of our proposed method. Finally, concluding remarks are given in Section 6.

2. Statement of the problem

Consider the VOD system shown in Fig. 2. There is one VOD server and four switching nodes (e.g., ATM switches) in this system. The VOD server is responsible for providing the video
programs, and the switching node has the ability to duplicate the video stream that comes from the server. The customer's terminal device is connected to a switching node and the customer can receive the video program from the down link of the connected switching node. The switching nodes and the server are connected to each other in some fashion by directed links. The available capacity of a link is defined as the maximum number of video streams that can be served. In our model, each video program requires a unit of the link capacity for transmission. In order to guarantee the quality of service, we also assume that when a multicast session is established, the bandwidth allocated for this session is reserved until the session finishes.

Suppose there are \( m \) video programs \( P_1, P_2, \ldots, P_m \) available on the server. Because the switching node can duplicate the video stream, if a video stream flows into a switching node, all subscribers connected to that switching node can obtain the program in that stream. Therefore, the bid \( b^j_k \) of a video program \( P_k \) at a switching node \( j \) is defined as the number of customers who subscribe to video program \( P_k \). The bid vector \( (b^1_j, b^2_j, \ldots, b^n_j) \) for switching node \( j \) is the collection of bid \( b^j_k \) at switching node \( j \). We assume that any video program that flows in each link requires a unit of link capacity. Hence, we associate a nonnegative integer (link capacity) \( r_{ij} \) for each link \( (i, j) \) to represent the maximum number of video programs which can flow on that link \( (i, j) \).

Therefore, a VOD system shown in Fig. 2 is modeled as a network \( G = (V, E) \) shown in Fig. 3a where \( V \) is a set of nodes containing a server node (node 0) and all switching nodes (nodes 1, 2, \ldots, \( n-1 \)) and \( E \) is a set of communication links between these nodes. Let \( v^k_j = 1 \) denote the video stream \( k \) flowing into node \( j \) and \( v^k_j = 0 \), otherwise. Thus, the total number of subscribers being served (called source gain) is \( \sum_{j=1}^{n-1} \sum_{k=1}^{m} v^k_j \cdot b^j_k \) where \( m \) is number of streams and \( n-1 \) is number of switching nodes. For example, the source gain for the multicast scheme in Fig. 3b is 23. Fig. 3c shows that the video distribution scheme in Fig. 3b is composed using a set of multicast trees \( \{T_1, T_2, T_3\} \) where \( T_1, T_2 \) and \( T_3 \) can be used for multicast sessions of video programs \( P_1, P_2 \) and \( P_3 \), respectively.

In this research, the problem was formulated to find a multicast scheme such that the source gain is maximized and the bandwidth constraint is maintained. We call this problem an optimum source gain multicast problem (OSGMP).

Notation:

- \( n \): number of nodes.
- \( m \): number of video streams.
- \( r_{ij} \): capacity of link \((i, j)\).
- \( b^j_k \): switch \( j \)'s bid for stream \( k \).
- \( v^k_j \): the decision variable, \( v^k_j = \begin{cases} 1, & \text{stream } k \text{ flows into node } j, \\ 0, & \text{otherwise}. \end{cases} \)
- \( x^k_{ij} \): the decision variable, \( x^k_{ij} = \begin{cases} 1, & \text{stream } k \text{ flows in link } (i, j), \\ 0, & \text{otherwise}. \end{cases} \)
- \( M \): big \( M \); a very large number.

Assumptions:

1. Each stream requires one unit of link capacity.
2. Each link has integral units of capacity.
3. Node 0 is the source node and nodes 1, 2, \ldots, \( n-1 \) are switching nodes.
4. The video server has bid information.
The problem can be mathematically stated as follows.

Maximize \( \sum_{j=1}^{m} \sum_{k=1}^{n-1} v_j^k b_j^k \)

subject to

\[
\begin{align*}
\sum_{k=1}^{m} x_{ij} & \leq r_{ij}, & 0 \leq i, j \leq n - 1, & (1) \\
\sum_{i=0}^{n-1} x_{io}^k & = 0, & 1 \leq k \leq m, & (2) \\
\sum_{i=0}^{n-1} x_{ij}^k & = v_j^k, & 1 \leq j \leq n - 1, 1 \leq k \leq m. & (3)
\end{align*}
\]
\[ n \sum_{s=0}^{n-1} x_{js}^k \leq M v_j^k, \quad 1 \leq j \leq n - 1, 1 \leq k \leq m, \quad (4) \]

\[ x_{ij}^k \in \{0, 1\} \quad 0 \leq i, j \leq n - 1, 1 \leq k \leq m; \quad v_j^k \in \{0, 1\} \quad 0 \leq j \leq n - 1, 1 \leq k \leq m. \quad (5) \]

Constraint (1) ensures that the multicast streams in each link do not exceed the bandwidth boundaries. Constraint (2) ensures that no stream can flow back to the server. The remaining constraints indicate that a video stream can flow out from a switch node only if it has received the video stream from its upstream neighbors. Furthermore, for each node, all video sets which were sent by its upstream neighbors are mutually exclusive.

3. A branch-and-bound algorithm for directed acyclic graphs

In order to solve the OSGMP, we use a tree, called a state-space tree to represent all of the feasible solutions, and apply the branch-and-bound algorithm on the state-space tree to search for the optimal solution.

Note that for a DAG there exists a topological order for its nodes. That is, we can label the nodes such that \( i < j \) for every directed link \((i, j)\). For example, Fig. 4 shows a topological order for a DAG. Based on the topological order, we can find all feasible solutions for each node. For example, in Fig. 5, links \((1, 3), (2, 3)\) are the only incoming links of node 3. Let node 1 can receive video programs \( P_1, P_2 \) and \( P_3 \). That is, node 1 has video program set \( \{P_1, P_2, P_3\} \). Similarly, let node 2 have video program set \( \{P_2, P_3\} \). The capacity of link \((1, 3)\) is 2, it means two video programs can be chosen from set \( \{P_1, P_2, P_3\} \) and send them to node 3. Similarly, we can choose one video program from set \( \{P_2, P_3\} \) at node 2 and then send it to node 3. Therefore, all possible video program sets for node 3 are \( \{P_1, P_2\}, \{P_1, P_2, P_3\}, \{P_1, P_3\}, \{P_2, P_3\} \). Note that we only keep dominating video program sets for node 3. For example, \( \{P_1, P_2\}, \{P_1, P_3\}, \{P_2, P_3\} \) are dominated by \( \{P_1, P_2, P_3\} \) at node 3 because \( \{P_1, P_2, P_3\} \) for node 3 has a better gain than others. Thus, we keep \( \{P_1, P_2, P_3\} \) as a dominating video program set at node 3. In the following, we will present how to generate dominating video set for each node. Let

\[ v_j^k = \begin{cases} 1, & \text{video program } P_k \text{ can be received at switching node } j, \\ 0, & \text{otherwise.} \end{cases} \]

Fig. 4. Topological order of a DAG.
Let vector \( v_j = (v_j^1, v_j^2, \ldots, v_j^n) \) denote the video vector received at the switching node \( j \). The or operation for vector \( v = (v_1, v_2, \ldots, v_m) \) and \( u = (u_1, u_2, \ldots, u_m) \) is defined as

\[
v \lor u = (v_1 \lor u_1, v_2 \lor u_2, \ldots, v_m \lor u_m)
\]

where \( \lor \) is logic or operation. Let \( e_j \) be a unit vector with all entries at zero except entry \( j \), which is a one. Thus, \( v = \sum_{i=1}^{m} v_i e_i \). Let \( S_x \) be the received video set for \( v_x \) at node \( x \), such that if \( v_i^j = 1 \) then \( e_j \) is an element in \( S_x \). For example, in Fig. 5, node 2 has video vector \( v_2 = (0, 1, 1) \), then \( v_2 = e_2 + e_3 \) and \( S_2 = \{e_2, e_3\} \). Suppose that link capacities of \((x, z)\) and \((y, z)\) are 1, then the set that includes all possible video vectors received at node \( z \) is defined as

\[
S_x \lor S_y = \{e_i \lor e_j | e_i \in S_x, e_j \in S_y\}.
\]

For example, \( S_x = \{e_1, e_2, e_3\} \) and \( S_y = \{e_2, e_3\} \), then the set that includes all possible video vectors received at node \( z \) are \( S_x \lor S_y = \{(1, 1, 0), (1, 0, 1), (0, 1, 1), (0, 1, 1), (0, 0, 1)\} \). Let \( S^{(2)} \) denote \( S \lor S \) and \( S^{(0)} = S^{(1-1)} \lor S \). For any node \( j \), assume that links \((i_1, j), (i_2, j), \ldots, (i_a, j)\) are the incoming links for node \( j \) and node \( i_s \) \((s = 1, 2, \ldots, a)\) has video vector received \((v_{i_s}^1, v_{i_s}^2, \ldots, v_{i_s}^m)\). Let \( F_j \) be the set that includes all of the possible received video vectors for node \( j \). Thus,

\[
F_j = S^{(r_{i_1})}_1 \lor S^{(r_{i_2})}_1 \lor \cdots \lor S^{(r_{i_a})}_a,
\]

where \( r_{i_s} \) is the link capacity of \((i_s, j)\), \( s = 1, 2, \ldots, a \).

We say a vector \( v \) is dominated by vector \( u \) if and only if \( v \lor u = u \). For example, \((0, 1, 1)\) is dominated by \((1, 1, 1)\) because \((0, 1, 1) \lor (1, 1, 1) = (1, 1, 1) \). Let \( [F] \) denote the set of vectors in which all the dominated vectors in \( F \) are deleted. That is, any two vectors in \([F]\) are not dominated by one another. Fig. 5 shows an example for how to compute all possible video vectors received for a node. The video vectors received at nodes 1 and 2 are \((1, 1, 1)\) and \((0, 1, 1)\), respectively. Then, set \( F_3 \) can be generated by computing the result of \([[[S^{(2)}_1 \lor S_2] \lor S_2] \lor S_2] \). The possible video sets received, \( S_1 \) and \( S_2 \) with respect to \( F_1 \) and \( F_2 \) are \( \{e_1, e_2, e_3\} \) and \( \{e_2, e_3\} \), respectively. Therefore, \( S^{(2)}_1 = \{e_1, e_2, e_3\} \lor \{e_1, e_2, e_3\} = \{(1, 0, 0), (1, 1, 0), (1, 0, 1), (1, 1, 0), (0, 1, 0), (0, 1, 1), (0, 1, 1), (0, 0, 1)\} \), and \( S^{(2)}_2 = \{(1, 1, 0), (1, 0, 1), (0, 1, 1)\} \) by deleting the dominated vectors from set \( S^{(2)}_1 \). In the final computation, \([[[S^{(2)}_1 \lor S_2] \lor S_2] \lor S_2] \lor ([\{(1, 1, 0), (1, 0, 1), (0, 1, 1)\} \lor \{(0, 1, 0), (0, 0, 1)\}]) = \{(1, 1, 1)\} \). This means that node 3 can receive video programs \( P_1, P_2 \) and \( P_3 \) from its incoming links.
The state-space tree is generated from the network $G$. The node for the state-space tree is labeled by a received video vector $(v_1^j, v_2^j, \ldots, v_m^j)$, which specifies that switching node $j$ can receive video programs $P_k$ whenever $v_k^j = 1$, $k = 1, 2, \ldots, m$. The path from the root to a leaf node in the state-space tree will be defined to represent a sequence of possible video vectors, i.e., a sequence of received video vectors $(v_0^1, v_0^2, \ldots, v_0^m), (v_1^1, v_1^2, \ldots, v_1^m), \ldots, (v_{n-1}^1, v_{n-1}^2, \ldots, v_{n-1}^m)$. Note that the feasible solution $x_{ij}^k$, $\forall i, j, k$ can be found easily from the sequence of received video vectors.

For example, Fig. 7 is a partial state-space tree generated from Fig. 6. As shown in Fig. 6, node 0 is the server which provides three video programs $P_1$, $P_2$, and $P_3$, hence the received video vector for the root $x_0$ in the state-space tree is $(1, 1, 1)$ (i.e., $F_0 = \{(1, 1, 1)\}$). For switching node 1, because link from node 0 to node 1 is the only incoming link and the boundary capacity for this link is 2, hence, the possible video vectors that can be received by node 1 can be derived using $F_1 = [S_0^{12}] = \{(1, 1, 0), (1, 0, 1), (0, 1, 1)\}$. Thus, the second level for the state-space tree is generated and labeled using $(1, 1, 0), (1, 0, 1)$ and $(0, 1, 1)$. Similarly, for switching node 2, because link from node 0 to node 2 and link from node 1 to node 2 are the only incoming links and $r_{02} = 1$ and $r_{12} = 1$, the children of node 1 in state-space tree can be determined by $[\{e_1, e_2\} \lor \{e_1, e_2, e_3\}] = \{(0, 1, 1), (1, 0, 1), (1, 1, 0)\}$. Therefore, we labeled these nodes using $(0, 1, 1), (1, 0, 1)$ and $(1, 1, 0)$. By continuing the same process, the entire state-space tree for this OSGMP can be obtained.

In fact, to find an optimal solution, we will not consider all of the feasible sequences since it is very time consuming. We will apply a best-first branch-and-bound algorithm to find the optimal

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**Fig. 6. Example of a OSGMP.**

**Fig. 7. A portion of state-space tree.**
solution by traversing only a portion of the state-space tree. The branch-and-bound method is basically characterized by two decision rules. One provides the method to estimate the upper bound of an objective function at every node of the state-space tree. The other specifies a choice criterion for the selection of a node for the next branch.

3.1. The estimation of the upper bound for the objective function at node $x$

Assume that the current by visited node is $x_i$ in the state-space tree. Then the partial sequence for the received video vectors can be defined by the path $A_{x_i}$ from the root $x_0$ to node $x_i$ is $(v_0^1, v_0^2, \ldots, v_0^m), (v_1^1, v_1^2, \ldots, v_1^m), \ldots, (v_i^1, v_i^2, \ldots, v_i^m).$ That is, there are $i$ switching nodes being assigned to received video vectors. The current value of the objective function at node $x_i$ is:

$$f(A_{x_i}) = \sum_{j=1}^{i} \sum_{k=1}^{m} b_j^i v_j^i.$$ 

Note that there are remaining $n - (i + 1)$ switching nodes unvisited. Let switching node $j$ be a node in $G$ with bid vector $(b_j^1, b_j^2, \ldots, b_j^m)$ and incoming links $(i_1, j), (i_2, j), \ldots, (i_n, j).$ We sort the bids at node $j$ such that $b_j^{i_1} \geq b_j^{i_2} \geq \cdots \geq b_j^{i_n}.$ An upper bound $ub_j$ for gain contributed from node $j$ can be estimated by $ub_j = \sum_{k=1}^{h} b_j^k$ where $h = \sum_{i=1}^{n} r_{i,j}.$ The value $UB_i = f(A_{x_i}) + \sum_{j=1}^{i} ub_j$ is an upper bound for the objective function value in the complete assignment generated from partial assignment $(v_1^1, v_1^2, \ldots, v_1^m), (v_2^1, v_2^2, \ldots, v_2^m), \ldots, (v_i^1, v_i^2, \ldots, v_i^m).$

3.2. The selection process in a branching node.

To facilitate the generation of the state-space tree, a data structure heap called live-node heap is used to record all live nodes that are waiting to be branched. The search strategy of the proposed branch-and-bound algorithm is a best-first search. That is, the node, say node $x$, selected for the next branch is the live node whose $UB_x$ value is the largest among all of the nodes in the live-node heap. Note that the maximal value is at the top of the heap. Traversing the state-space tree begins from the root and stops when the live-node heap is empty. In addition, a current maximal source gain ($G_{\text{max}}$) is associated with the branch-and-bound algorithm. Initially, $G_{\text{max}}$ is set at 0 and updated to be $G_{\text{max}} = \max(G_{\text{max}}, f(A_y))$ whenever a leaf node $y$ is reached. If a node $x$ satisfies $UB_x \leq G_{\text{max}}$, then node $x$ is bounded since further branching from $x$ will not lead to a better solution. When the live-node heap becomes empty, we obtain the optimal solution $(v_1^1, v_1^2, \ldots, v_1^m), (v_2^1, v_2^2, \ldots, v_2^m), \ldots, (v_{n-1}^1, v_{n-1}^2, \ldots, v_{n-1}^m)$ with optimal value $\sum_{j=1}^{n-1} \sum_{k=1}^{m} b_j^k v_j^k = G_{\text{max}}.$ The detailed algorithm is presented in Fig. 8, and the method is introduced using a recursive version.

A simple numerical example is given in Fig. 9. In Fig. 9a, each switching node is associated with a bid vector and each link is associated with a capacity. Fig. 9b shows the generation of the state-space tree. Initially, set the current maximal source gain ($G_{\text{max}}$) to be 0 and the root node $x_0$ of the state-space tree is $(1, 1, 1)$ (i.e., $S_0 = \{e_1, e_2, e_3\}$). The upper bound for gain contributed from switching node 1 is $ub_1 = 7.$ This is because $b_1^2 > b_1^1 > b_1^1$ and $r_{01} = 2.$ Then, $ub_1 = b_1^2 + b_1^1 = 7.$ Similarly, we have $ub_2 = 7$ and $ub_3 = 3.$ Therefore, the possible received video vectors set for switching node 1 can be computed and obtained using $[S_0^{(2)}] = \{(0, 1, 1), (1, 0, 1), (1, 1, 0)\}.$ This generates nodes 1, 2 and 3 of the state-space tree.
Fig. 8. Algorithm of optimal-stream distribution.

Now, we estimate the upper bound for the objection function (UB) at each node. We observed that at node 1, $UB_1 = (0, 1, 1)(1, 4, 3) + ub_2 + ub_3 = 17$ was the biggest value among $UB_1$, $UB_2$ and $UB_3$. Therefore, we selected node 1 to be our next branching node. Continuing the same process, we can reach the first leaf node 7 and update the current maximal source gain ($G_{max}$) to be 17. Similarly, nodes 5, 2 and 3 are bounded by $G_{max}$. Hence, no more nodes are waiting for branching and the algorithm terminates.

4. A DAG’s-based heuristic algorithm for general graphs

In order to solve the OSGMP for a general graph case, we propose a two phased algorithm to find an approximate solution. At the first phase, we properly choose a directed acyclic subgraph from the given general graph and then apply the branch-and-bound algorithm to find an initial solution. In the second phase, the residual capacities for links are considered so that the source gain of the initial solution will be increased. To achieve this goal, for each residual link $(i, j)$, we can select a set of video programs which appear on node $i$ but not on node $j$, and send them along the link until the capacities are exhausted. The details are given as follows.

4.1. Phase I: choose a directed acyclic subgraph from a general graph

We can choose a directed acyclic subgraph from a general graph by labeling the node order. Based on the node order, we can obtained a DAG by removing all links $(i, j)$ if $i > j$. Different node orders produce different subgraphs. A good directed acyclic subgraph is a subgraph with greater source gain at the first phase. A subroutine Find-node-order is proposed to determine a node order for a general graph such that a good directed acyclic subgraph can be produced. Subroutine Find-node-order gives each node with a value called gain-lost to estimate the maximum source gain that may be lost in the first phase when a node order is given.

Initially, we set the gain-lost for each node to be a big number, say 999 except for the source node. The gain-lost for the source node is set to be 0. For each iteration, Find-node-order will choose a node whose number of incoming links (in-degree) is zero and then label it. If no zero in-degree
Fig. 9. A numerical example for branch-and-bound algorithm.
nodes are found, then the node with the smallest gain-lost value among the nodes in graph $G$ will be selected and labeled. The chosen node is then deleted from graph $G$. Next, we update the bid vectors of its outgoing neighbors. Let $(j, i_1), (j, i_2), \ldots (j, i_a)$ be nodes $j$’s outgoing links and $(b_{i_1}^1, b_{i_1}^2, \ldots, b_{i_1}^m), (b_{i_2}^1, b_{i_2}^2, \ldots, b_{i_2}^m), \ldots, (b_{i_a}^1, b_{i_a}^2, \ldots, b_{i_a}^m)$ be the bid vectors of node $j$’s outgoing neighbors $i_1, i_2, \ldots i_a$, respectively. Note that, after link $(j, i_k)$ is deleted, the largest gain that can be achieved is the sum of first $r_{jik}$ biggest bids among bid vector $(b_{i_k}^1, b_{i_k}^2, \ldots, b_{i_k}^m)$. We set the bid values of the first $r_{jik}$ biggest bids to 0. Also, we re-estimate the gain-lost values for nodes $i_k$ using the updated bid vector. A complete description of this subroutine is shown in Fig. 10.

For node $j$ with bid vector $(b_j^1, b_j^2, \ldots, b_j^m)$ and incoming links $(i_1, j), (i_2, j), \ldots, (i_a, j)$, if we label node $j$ as $l(j) = \min\{l(i_1), l(i_2), \ldots, l(i_a)\}$, then all incoming links for node $j$ will not appear on the directed acyclic subgraph and the total capacities $r = r_{i_1,j} + r_{i_2,j} + \ldots + r_{i_a,j}$ will not be used for the first phase. Therefore, we let the gain-lost for node $j$ be the gain contributed from the total lost capacities $r$. That is, we sort the bids at node $j$ such that $b_j^1 \geq b_j^2 \geq \ldots \geq b_j^m$. A gain-lost, denoted as $glost(j)$ for node $j$ can be estimated by $glost(j) = \sum_{i=1}^{r} b_j^i$. Obviously, when in-degree for node $j$ is 0 then the value of $glost(j)$ should be set to 0.

Fig. 11 shows a numerical example for how to determine the node order for a given graph. At first, the algorithm initializes the gain-lost value for each node. At the first iteration, node $i$ was depicted and labeled by 0, for in-degree($i$) is equal to zero. Now, execute the update process to modify $b_j$ and $b_k$. For example, updates bid vectors $b_j$. Because the capacity of link $(i, j)$ is 2, therefore, we choose the first 2 biggest bids among $b_j$ and reset them to 0. This secures modified bid vector $b_j = (0, 0, 4)$. Similarly, we can obtain the modified bid vector $b_k = (1, 3, 0)$. Next, delete node $i$ from graph $G$ and then update $glost(k)$ and $glost(j)$. For updating the value for $glost(k)$, $(j, k)$ is the only incoming link of node $k$, hence, the total capacities $r$ which will not be used for phase I is equal to 1. Now, we recompute $glost(k)$ using the summation of the first $r (= 1)$ biggest bids among $b_k$, and obtain $glost(k) = 3$. Similarly, we can secure the value $glost(j) = 4$. Repeat the same iteration until graph $G$ becomes empty. Hence, a node order is produced by this algorithm.
Based on the node order, we can obtain a DAG by removing all links \((i, j)\) if \(i > j\). At the same time, we collect the removed links to form a subgraph called a residual subgraph. Note that, if the given graph is a DAG, obviously the residual subgraph will be empty. Now, we apply the branch-and-bound algorithm on the directed acyclic subgraph, and obtain an optimal received video vector for each node of the resulting DAG.

4.2. Phase II: video streams distribution on residual graph

In order to increase the source gain, for every link \((i, j)\) in the residual subgraph, node \(i\) can send video streams to node \(j\). Let link \((i, j)\) have capacity \(r_{ij}\) in the residual subgraph, and \((b_{1j}^i, b_{2j}^i, \ldots, b_{mj}^i)\) be the bid vector of node \(j\). Let \((v_1^i, v_2^i, \ldots, v_{ni}^i)\) and \((v_1^j, v_2^j, \ldots, v_{nj}^j)\) be the received video vectors for node \(i\) and node \(j\), respectively. A video can contribute to the source gain when it is already received.
at node $i$ but not received at node $j$. That is, we want to choose a video $q$ such that $v^q_i = 1$ and $v^q_j = 0$. However, link $(i, j)$ only has capacity $r_{ij}$. Let $Q = \{q|v^q_i = 1 \land v^q_j = 0\}$. Thus, we select a subset $Q' \subseteq Q$, where $|Q'| = r_{ij}$, such that $\sum_{q \in Q'} b^q_j$ is maximized. This can be done by selecting the first $r_{ij}$ biggest $b^q_j$, $q \in Q$.

Fig. 12a shows the graph partition by giving the node order which is obtained from Fig. 11. Fig. 12b shows the results from applying the two-phased heuristic algorithm. The two-phased heuristic algorithm for solving the OSGMP over a general graph is stated in Fig. 13.

5. Simulation results

In this section, the performance of the branch-and-bound algorithm for solving OSGMP over DAG and the two-phased heuristic algorithm for a general graph case are studied. The criterion we
algorithm. Two-phased heuristic;
begin
apply subroutine Find-nodes-order;
partition graph $G$ into a subgraph pairs $(D, R)$ where $D$ is the directed acyclic subgraph, and $R$ is the residual subgraph;
apply DAG’s Optimal-stream-distribution $(D)$;
apply video streams distribution on residual graph $R$;
end;

Fig. 13. Algorithm of two-phase heuristic.

adopted to evaluate the branch-and-bound algorithm was the traversing ratio of the state-space tree, which is defined as $n_b/N$, where $n_b$ is the number of nodes that are traversed by the branch-and-bound algorithm, and $N$ is the number of nodes in the complete state-space tree. On the other hand, the criterion we adopted to evaluate the two-phased algorithm was the gap ratio, which is defined as $1 - X_{heu}/X_{opt}$, where $X_{heu}$ is the source gain of the two-phased heuristic algorithm, and $X_{opt}$ is the optimal source gain.

The performance of these algorithms are influenced by the following five factors:

1. number of nodes,
2. the capacity constraint for each link,
3. the bid vector with respect to each node,
4. number of video programs available for customer subscription, and
5. the shapes of the given graph.

Hence, the following assumptions were made about the experiment to address these factors.

1. The capacity constraint for each link was randomly assigned to either 5 or 6.
2. The bid of each video program at any switching node was randomly generated between intervals $[0, 10]$.
3. Number of video programs $m$, available for customers subscription $m = 8, 10, 12$ were considered.
4. Twenty instances of bid vectors were run for each given graph, and the traversing ratio (gap ratio) was also computed when the branch-and-bound algorithm (two-phased heuristic algorithm) was applied.
5. Five DAGs were depicted from randomly generated graphs, then run for the branch-and-bound algorithm. The number of nodes $n = 10, 12$ were considered. Fig. 14a shows these graph topologies.
6. Five general graphs were depicted from randomly generated graphs, then run for the two-phased algorithm. The number of nodes $n = 10$ was considered. Fig. 14b shows these graph topologies.

The results of the average traversing ratios are listed in Table 1, with the number of nodes $n = 10, 12$ and number of video programs available for customers subscription $m = 8, 10$ and 12, respectively. On average, the traversing ratios were no more than 12% (3%) for 10-node (12-node) networks.
The other experiment was done by applying the two-phased algorithm to five general graphs with \( n = 10, m = 10 \), and for each given graph, 20 instances of bid vectors were considered. The experimental results are listed in Table 2. It shows the source gains which were found using the two-phased heuristic algorithm are very close to the optimal source gains and most of the gap ratios were less than 0.07. Therefore, the proposed two-phase algorithm is very efficient.

Fig. 14. Network topologies used in the performance evaluation. (a) DAGs run for branch-and-bound algorithm (b) General graphs run for two-phased algorithm.
Table 1
Variation in average traversing ratios for branch-and-bound algorithm

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Table 2
Comparision of two-phase algorithm and optimal algorithm on source gain and computational time

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*These samples were run on IBM PC with Pentium-PRO-S CPU of 200 MHz.

6. Concluding remarks

In this paper, we formally modelled the optimum source gain multicast problem as an integer programming problem. A branch-and-bound method was proposed to solve this problem for a DAG case. We presented a two-phased algorithm to find an approximate solution for a general graph case. From the computation results, it is shown that the objective function value of the approximation solution is very close to the optimal value. On the other hand, in our design issue of the two-phased heuristic algorithm, a *good* directed acyclic subgraph was determined by choosing a node order for the given graph to achieve higher gain in the first phase. We used *gain-lost* values as the criterion for labeling the node order. In the update process for each iteration, we updated the bid vector of the chosen node’s outgoing neighbors by setting first $r$ biggest bids to be 0, where $r$ denoted the capacity for the outgoing link. This was based on assuming that the chosen node would receive every kind of video from the server node. However, this assumption is not accurate enough and it will cause an under-estimate of the *gain-lost* values. Therefore, a more accurate estimation of the *gain-lost* values to enhance the performance of the two-phase algorithm will be our future research.

The postoptimality analysis is also a possible future research. For example, it may happen that a customer decide to discontinue viewing a particular video and switches to some other video stream. One way to accomplish this is to solve the problem anew, but this may be computationally
inefficient. If one makes use of the properties of the multicast tree solution, it is possible to reduce additional computations.

References