New methods for students' evaluation using fuzzy sets

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Abstract

In an earlier work, Biswas (1995) presented two methods for the application of fuzzy sets in students' answerscripts evaluation. In this paper, we extend his work to propose two new methods for evaluating students' answerscripts using fuzzy sets. The proposed methods can overcome the drawbacks in Biswas (1995) due to the fact that they do not need to perform the complicated matching operations and they can evaluate students' answerscripts in a more fair manner.

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1. Introduction

In 1965, Zadeh proposed the theory of fuzzy sets [15]. In recent years, some research on the application of fuzzy set theory in education has begun [2, 4, 10]. In [10], Chiang et al. presented a method for the application of fuzzy set theory to teaching assessment. In [4], Chang et al. presented a method for fuzzy assessment of learning performance of junior school students. In [2], Biswas pointed out that the chief aim of education institutions should be to provide the students with the evaluation reports regarding their test/examination as sufficient as possible and with unavoidable error as small as possible. He also presented a fuzzy evaluation method (fem) for the application of fuzzy sets in students' answerscripts evaluation. The fem presented in [2] is a computer based fuzzy approach, where a vector valued marking is used. Furthermore, in [2], Biswas also generalized the fem to propose a generalized fuzzy evaluation method (gfem) in which a matrix-valued marking is adopted. However, the methods presented in [2] have the following drawbacks:

1. Because they use a matching function $S$ to measure the degrees of similarity between the standard fuzzy sets and the fuzzy marks of the questions, they will take a large amount of time to perform the matching operations.

2. In Biswas's methods, two different fuzzy marks may be translated into the same awarded grade and this is unfair in students' evaluation.
Because Biswas's methods have the above two drawbacks in the task of students' answerscripts evaluation, it is necessary to develop new methods to overcome the above drawbacks.

In this paper, we present two new methods for the application of fuzzy sets in students' answerscripts evaluation. They can overcome the drawbacks of the ones presented in [2]. The proposed methods have the advantages of much faster execution and are more fair in the task of students' evaluation than the ones presented in [2].

The rest of this paper is organized as follows. In Section 2, we briefly review the theory of fuzzy sets from [5, 6, 12, 15, 16]. In Section 3, we briefly review Biswas's methods for students' answerscripts evaluation. In Section 4, we present a new method for students' answerscripts evaluation using fuzzy sets. In Section 5, we present a generalized fuzzy evaluation method for students' evaluation. The conclusions are discussed in Section 6.

2. Fuzzy set theory

The theory of fuzzy sets was proposed by Zadeh in 1965 [15]. Roughly speaking, a fuzzy set is a class with fuzzy boundaries. Let \( X \) be the universe of discourse, \( X = \{x_1, x_2, \ldots, x_n\} \), and let \( A \) be a fuzzy set of \( X \), then the fuzzy set \( A \) can be represented as

\[
A = \{(x_1, f_A(x_1)), (x_2, f_A(x_2)), \ldots, (x_n, f_A(x_n))\}, \tag{1}
\]

where \( f_a \) is the membership function of the fuzzy set \( A \), \( f_a : X \rightarrow [0, 1] \), \( f_A(x_i) \) indicates the degree of membership of \( x_i \) in \( A \). If the universe of discourse \( X \) is an infinite set, then the fuzzy set \( A \) can be expressed as

\[
A = \int x f_a(x_i)/x_i, \quad x_i \in X. \tag{2}
\]

Example 2.1. Let \( X \) be the universe of discourse, \( X = \{\text{red, black, yellow, blue, white, brown, green}\} \), and let "dark" be a fuzzy set of the universe of discourse \( X \) subjectively defined as follows:

\[
dark = \{(\text{red, 0.5}), (\text{black, 1.0}), (\text{yellow, 0.1}), (\text{blue, 0.6}), (\text{white, 0.0}), (\text{brown, 0.8}), (\text{green, 0.3})\}. \tag{3}
\]

where "black" has the largest membership value (i.e., 1.0) in the fuzzy set "dark", and "white" has the smallest membership value (i.e., 0.0) in the fuzzy set "dark". Thus, "black" is most pertinent to the fuzzy set "dark", and "white" is impertinent to the fuzzy set "dark".

Example 2.2. Let \( X \) be the universe of discourse, \( X = [0, 100] \). Then, the fuzzy sets "young" and "old" may subjectively be defined as follows:

\[
f_{\text{young}}(x) = \begin{cases} 
1, & 0 < x \leq 20, \\
(1 + ((x - 20)/15)^2)^{-1}, & 20 < x \leq 100,
\end{cases} \tag{5}
\]

\[
f_{\text{old}}(x) = \begin{cases} 
0, & 0 < x \leq 40, \\
(1 + ((x - 40)/10)^2)^{-1}, & 40 < x \leq 100,
\end{cases} \tag{6}
\]

where \( f_{\text{young}} \) and \( f_{\text{old}} \) are the membership functions of the fuzzy sets "young" and "old", respectively.

3. Biswas's methods for students' answerscripts evaluation

In [2], Biswas used a matching function \( S \) to measure the degree of similarity between two fuzzy sets. Let \( A \) and \( B \) be two fuzzy sets of the universe of discourse \( X \), where

\[
A = \{(x_1, f_A(x_1)), (x_2, f_A(x_2)), \ldots, (x_n, f_A(x_n))\},
\]

\[
B = \{(x_1, f_B(x_1)), (x_2, f_B(x_2)), \ldots, (x_n, f_B(x_n))\},
\]

\[
X = \{x_1, x_2, \ldots, x_n\}.
\]
By using the vector representation method, the fuzzy sets \( A \) and \( B \) can be represented by the vectors \( \vec{A} \) and \( \vec{B} \), respectively, where

\[
\vec{A} = \langle f_A(x_1), f_A(x_2), \ldots, f_A(x_n) \rangle,
\vec{B} = \langle f_B(x_1), f_B(x_2), \ldots, f_B(x_n) \rangle.
\]

Then, the degree of similarity \( S(\vec{A}, \vec{B}) \) between the fuzzy sets \( A \) and \( B \) can be defined by

\[
S(\vec{A}, \vec{B}) = \frac{\vec{A} \cdot \vec{B}}{\max(\vec{A} \cdot \vec{A}, \vec{B} \cdot \vec{B})},
\]

where \( S(\vec{A}, \vec{B}) \in [0, 1] \). The larger the value of \( S(\vec{A}, \vec{B}) \), the more the similarity between the fuzzy sets \( A \) and \( B \).

Based on the matching function \( S \), Biswas introduced a "fuzzy evaluation method" (fem) for evaluating students' answerscripts. In the following, we briefly review Biswas's methods for students' answerscripts evaluation. In [2], Biswas used five fuzzy linguistic hedges (called Standard Fuzzy Sets (SFS)) for students' answerscripts evaluation, i.e., \( E \) (excellent), \( V \) (very good), \( G \) (good), \( S \) (satisfactory), and \( U \) (unsatisfactory), where

\[
X = \{0\%, 20\%, 40\%, 60\%, 80\%, 100\%\},
E = \{(0\%, 0), (20\%, 0.1), (40\%, 0.2), (60\%, 0.4), (80\%, 0.4), (100\%, 1)\},
V = \{(0\%, 0), (20\%, 0.8), (40\%, 0.9), (60\%, 0.9), (80\%, 0.9), (100\%, 0.8)\},
G = \{(0\%, 0.1), (20\%, 0.8), (40\%, 0.8), (60\%, 0.9), (80\%, 0.4), (100\%, 0.2)\},
S = \{(0\%, 0.4), (20\%, 0.4), (40\%, 0.9), (60\%, 0.6), (80\%, 0.2), (100\%, 0)\},
U = \{(0\%, 1), (20\%, 1), (40\%, 0.4), (60\%, 0.2), (80\%, 0), (100\%, 0)\}.
\]

Biswas's algorithm [2] for students' answerscript evaluation is summarized as follows.

**Step 1:** For each attempted question in the answerscript repeatedly perform the following steps:

<table>
<thead>
<tr>
<th>Question No.</th>
<th>Fuzzy mark</th>
<th>Grade</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q.1</td>
<td>0 0.1 0.2 0.4 0.6</td>
<td></td>
</tr>
<tr>
<td>Q.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Q.3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Total mark =
(1) The evaluator awards a fuzzy mark $F_i$ to the question $Q_i$ by his best possible judgement and fills up the cells of the $i$th row for the first seven columns. Let $F_i$ be the vector representation of $F_i$.

(2) Calculate the following degrees of similarities:

\[ S(E, F_i), S(V, F_i), S(G, F_i), S(S, F_i), \text{ and } S(U, F_i), \]

where $E$, $V$, $G$, $S$, and $U$ are the vector representations of the standard fuzzy sets $E$ (excellent), $V$ (very good), $G$ (good), $S$ (satisfactory), and $U$ (unsatisfactory), respectively.

(3) Find the maximum among the five values $S(E, F_i)$, $S(V, F_i)$, $S(G, F_i)$, $S(S, F_i)$, and $S(U, F_i)$. Assume that $S(V, F_i)$ is the maximum value among the values of $S(E, F_i)$, $S(V, F_i)$, $S(G, F_i)$, $S(S, F_i)$, and $S(U, F_i)$, then award grade “B” to the question $Q_i$ due to the fact that grade “B” corresponds to $V$ (very good) of the standard fuzzy set.

**Step 2:** Calculate the total score using the following formula:

\[
\text{Total score} = \frac{1}{100} \sum \left[ T(Q_i) \times P(g_i) \right],
\]

(8)

where $T(Q_i)$ is the mark allotted to $Q_i$ in the question paper, and $g_i$ is the grade awarded to $Q_i$ by Step 1 of the algorithm. Put this total score in the appropriate box at the bottom of the fuzzy grade sheet.

Furthermore, in [2], Biswas also presented a generalized fuzzy evaluation method (gfem), where a generalized fuzzy grade sheet shown in Table 2 is used to evaluate the students’ answerscripts. In the grade sheet of Table 2, for all $j = 1, 2, 3, 4$, and for all $i$, $g_{ij}$ is the calculated grade by fem for the awarded fum $F_{ij}$, and $m_i$ is the calculated mark to be awarded to the attempted question $Q_i$ using the formula:

\[
m_i = \frac{1}{400} \cdot T(Q_i) \cdot \sum_{j=1}^{4} P(g_{ij})
\]

(9)

and Total mark = $\sum m_i$.

However, the methods presented in [2] have the following drawbacks:

(1) Because they use a matching function $S$ to measure the degrees of similarity between the standard fuzzy sets and the fuzzy marks of the questions, it will take a large amount of time to perform the matching operations. Especially, when the number of questions in the test/examination is very big.

(2) In Biswas’s method, two different fuzzy marks may be translated into the same awarded grade and this is unfair in students’ evaluation. For example, let $F_i$ and $F_j$ be two different fuzzy marks represented by fuzzy sets of the universe of discourse $X$, respectively, and let $E$ (excellent), $V$ (very good), $G$ (good), $S$ (satisfactory), and $U$ (unsatisfactory) be standard fuzzy sets of the universe of discourse $X$, where $X = \{0\%, 20\%, 40\%, 60\%, 80\%, 100\%\}$ and the corresponding awarded grade of the standard fuzzy sets “E”, “V”, “G”, “S”, and “U” are “A”, “B”, “C”, “D”, and “E”, respectively. Then, based on [2], we can calculate the following degrees of similarities:

**Case 1:** If $S(V, F_i)$ is the maximum value among the values of $S(E, F_i)$, $S(V, F_i)$, $S(G, F_i)$, $S(S, F_i)$, $S(U, F_i)$, then the fuzzy mark $F_i$ is translated to the awarded grade “B” due to the fact that the grade “B” corresponds to $V$ (very good).

**Case 2:** If $S(V, F_j)$ is the maximum value among the values of $S(E, F_j)$, $S(V, F_j)$, $S(G, F_j)$, $S(S, F_j)$, $S(U, F_j)$, then the fuzzy mark $F_j$ is translated to the
awarded grade “B” due to the fact that the grade “B” corresponds to V (very good).

From Cases 1 and 2, we can see that two different fuzzy marks \( F_i \) and \( F_j \) are translated to the same awarded grade “B”, and this is unfair in the task of students' answer scripts evaluation.

Because Biswas's methods have the above two drawbacks in the task of students' answer scripts evaluation, a new method for students' answer scripts evaluation is required to overcome the above drawbacks.

4. A new method for student's evaluation using fuzzy sets

In this section, we present a new method for students' answer scripts evaluation. Assume that there are eleven satisfication levels to evaluate the students' answer scripts regarding a question of a test/examination, i.e., extremely good (EG), very very good (VVG), very good (VG), good (G), more or less good (MG), fair (F), more or less bad (MB), bad (B), very bad (VB), very very bad (VVB), and extremely bad (EB), where the degrees of satisfication of the eleven satisfication levels are shown in Table 3.

Let \( X \) be a set of satisfication levels, \( X = \{ \text{extremely good (EG)}, \text{very very good (VVG)}, \text{very good (VG)}, \text{good (G)}, \text{more or less good (MG)}, \text{fair (F)}, \text{more or less bad (MB)}, \text{bad (B)}, \text{very bad (VB)}, \text{very very bad (VVB)}, \text{extremely bad (EB)} \} \), and let \( T \) be a mapping function which maps a satisfication level to the maximum degree of satisfication of the corresponding satisfication level, where \( T: X \rightarrow [0, 1] \). From Table 3, we can see that

\[
T(\text{extremely good}) = 1.00 \quad (\text{i.e., } T(\text{EG}) = 1.00),
\]

\[
T(\text{very good}) = 0.99 \quad (\text{i.e., } T(\text{VVG}) = 0.99),
\]

\[
T(\text{good}) = 0.90 \quad (\text{i.e., } T(\text{VG}) = 0.90),
\]

\[
T(\text{more or less good}) = 0.80 \quad (\text{i.e., } T(\text{G}) = 0.80),
\]

\[
T(\text{fair}) = 0.70 \quad (\text{i.e., } T(\text{MG}) = 0.70),
\]

\[
T(\text{bad}) = 0.60 \quad (\text{i.e., } T(\text{F}) = 0.60),
\]

\[
T(\text{more or less bad}) = 0.50 \quad (\text{i.e., } T(\text{MB}) = 0.50),
\]

\[
T(\text{bad}) = 0.40 \quad (\text{i.e., } T(\text{B}) = 0.40),
\]

\[
T(\text{very bad}) = 0.24 \quad (\text{i.e., } T(\text{VB}) = 0.24),
\]

\[
T(\text{very very bad}) = 0.09 \quad (\text{i.e., } T(\text{VVB}) = 0.09),
\]

\[
T(\text{extremely bad}) = 0 \quad (\text{i.e., } T(\text{EB}) = 0). \quad (10)
\]

Assume that an evaluator can evaluate the students' answer scripts using extended fuzzy grade sheets. The definition of the extended fuzzy grade sheets is presented as follows.

**Definition 4.1.** Extended fuzzy grade sheet: An extended fuzzy grade sheet is a matrix type structure containing thirteen columns and \( n \) rows, where \( n \) is the total number of questions in a test/examination. An example of an extended fuzzy grade sheet is shown in Table 4. At the bottom of the sheet there is a box which tells the total score. The first column reveals the serial numbers of the questions; in any row, the columns from the second to the twelfth shows the fuzzy mark awarded to the answer to the corresponding question in the first column, where the fuzzy mark is represented as a fuzzy set in the universe of discourse \( X \), \( X = \{ \text{extremely good (EG)}, \text{very very good (VVG)}, \text{very good (VG)}, \text{good (G)}, \text{more or less good (MG)}, \text{fair (F)}, \text{more or less bad (MB)}, \text{bad (B)}, \text{very bad (VB)}, \text{very very bad (VVB)}, \text{extremely bad (EB)} \} \), The last (i.e., the
Table 4
An extended fuzzy grade sheet

<table>
<thead>
<tr>
<th>Question No.</th>
<th>Satisfication levels</th>
<th>Degree of satisfaction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>EG</td>
<td>VVG</td>
</tr>
<tr>
<td>Q.1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Q.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>...</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Q.n</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Total mark =

Table 5
An example of an extended fuzzy grade sheet

<table>
<thead>
<tr>
<th>Question No.</th>
<th>Satisfication levels</th>
<th>Degree of satisfaction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>EG</td>
<td>VVG</td>
</tr>
<tr>
<td>Q.1</td>
<td>0</td>
<td>0.9</td>
</tr>
<tr>
<td>...</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Q.n</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Total mark =

The thirteenth column shows the degree of satisfication evaluated by the proposed method awarded to each question. The box at the bottom shows the total mark awarded to the student.

For example, assume that an evaluator is using an extended fuzzy grade sheet to evaluate the fuzzy mark of the first question (i.e., Q. 1) of a test/examination of a student as shown in Table 5. From Table 5, we can see that the satisfication level regarding the first question of the student’s answerscript is represented by a fuzzy set \( F(Q.1) \) of the universe of discourse \( X \), where \( X = \{ \text{EG, VVG, VG, G, MG, F, MB, B, VB, VVB, EB} \} \), and

\[
F(Q.1) = \{(\text{EG, 0}), (\text{VVG, 0.9}), (\text{VG, 0.8}), (\text{G, 0.5}), (\text{MG, 0}), (\text{F, 0}), (\text{MB, 0}), (\text{B, 0}), (\text{VB, 0}), (\text{VVB, 0}), (\text{EB, 0})\}. \quad (11)
\]

For convenience, the fuzzy set \( F(Q.1) \) can also be abbreviated into

\[
F(Q.1) = \{(\text{VVG, 0.9}), (\text{VG, 0.8}), (\text{G, 0.5})\}. \quad (12)
\]

It indicates that the satisfication level of the student’s answerscript with respect to the first question is described as 90% very very good, 80% very good, and 50% good.

The method for students’ answerscripts evaluation is now presented as follows:

1. **Step 1**: Assume that the fuzzy mark of the question \( Q.i \) of a student’s answerscript evaluated by an evaluator is shown in Table 6, where \( y_i \in [0, 1] \) and \( 1 \leq i \leq 11 \). From formula (10), we can see that \( T(\text{EG}) = 1 \), \( T(\text{VVG}) = 0.99 \), \( T(\text{VG}) = 0.90 \), \( T(\text{G}) = 0.80 \), \( T(\text{MG}) = 0.70 \), \( T(\text{F}) = 0.60 \), \( T(\text{MB}) = 0.50 \), \( T(\text{B}) = 0.40 \), \( T(\text{VB}) = 0.24 \), \( T(\text{VVB}) = 0.09 \), and \( T(\text{EB}) = 0 \). In this case, the degree of satisfication \( D(Q.i) \) of the question \( Q.i \) of the student’s answerscript can be evaluated by the function \( D \),

\[
D(Q.i) = \frac{y_1 * T(\text{EG}) + y_2 * T(\text{VVG}) + \cdots + y_{11} * T(\text{EB})}{y_1 + y_2 + \cdots + y_{11}} \quad (13)
\]
Table 6
Fuzzy mark of question Q.1 in an extended fuzzy grade sheet

<table>
<thead>
<tr>
<th>Question No.</th>
<th>Satisfaction levels</th>
<th>Degree of satisfication</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>EG</td>
<td>VVG</td>
</tr>
<tr>
<td></td>
<td>y_1</td>
<td>y_2</td>
</tr>
<tr>
<td>Q.1</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Total mark =

where $D(Q.i) \in [0, 1]$. The larger the value of $D(Q.i)$, the more the degree of satisfication that the question Q.1 of the student’s answer script satisfies the evaluator’s opinion.

For example, let us consider the example shown in Table 5. From formula (10), we can see that $T(VVG) = 0.99$, $T(VG) = 0.90$, and $T(G) = 0.80$. By applying formula (13), the degree of satisfication $D(Q.1)$ of the student’s answer script regarding question Q.1 can be evaluated as follows:

$$D(Q.1) = \frac{0.9 \cdot 0.99 + 0.8 \cdot 0.90 + 0.5 \cdot 0.80}{0.9 + 0.8 + 0.5}$$

$$D(Q.1) = 0.9141.$$  (14)

It indicates that the degree of satisfication of the question Q.1 of the student’s answer script evaluated by the evaluator is 0.9141 (i.e., 91.41%).

Step 2: Consider a candidate’s answer script to a paper of 100 marks. Assume that in total there were n questions to be answered:

**TOTAL MARKS = 100**

Q.1 carries $s_1$ marks
Q.2 carries $s_2$ marks
: 
Q. $n$ carries $s_n$ marks,

where $\sum_{i=1}^{n} s_i = 100$, $0 \leq s_i \leq 100$, and $1 \leq i \leq n$. Assume that the evaluated degree of satisfication of the question Q.1, Q.2, ..., and Q. $n$ are $D(Q.1)$, $D(Q.2)$, ..., and $D(Q.n)$, respectively, then the total score of the student can be evaluated as follows:

$$s_1 \cdot D(Q.1) + s_2 \cdot D(Q.2) + \cdots + s_n \cdot D(Q.n).$$  (15)

Put this total score in the appropriate box at the bottom of the extended fuzzy grade sheet.

In the following, we use an example to illustrate the students’ answer script evaluation process.

**Example 4.1.** Consider a candidate’s answer script to a paper of 100 marks. Assume that in total there were four questions to be answered:

**TOTAL MARKS = 100**

Q.1 carries 20 marks
Q.2 carries 30 marks
Q.3 carries 25 marks
Q.4 carries 25 marks.

Assume that an evaluator awards the students’ answer script by an extended fuzzy grade sheet as shown in Table 7.

[Step 1] Based on formula (10) and by applying formula (13), we can see that

$$D(Q.1) = \frac{0.8 \cdot T(VVG) + 0.9 \cdot T(VG)}{0.8 + 0.9} = \frac{0.8 \cdot 0.99 + 0.9 \cdot 0.90}{0.8 + 0.9} = 0.9424.$$  (16)
Table 7
Extended fuzzy grade sheet of Example 4.1

<table>
<thead>
<tr>
<th>Question No.</th>
<th>Satisfication levels</th>
<th>Degree of satisfication</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>EG</td>
<td>VVG</td>
</tr>
<tr>
<td>Q.1</td>
<td>0</td>
<td>0.8</td>
</tr>
<tr>
<td>Q.2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Q.3</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Q.4</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Total mark = 67

D(Q.2) = \( \frac{0.6 \cdot T(G) + 0.9 \cdot T(MG) + 0.5 \cdot T(F)}{0.6 + 0.9 + 0.5} \)

\[ = \frac{0.6 \cdot 0.80 + 0.9 \cdot 0.70 + 0.5 \cdot 0.60}{0.6 + 0.9 + 0.5} \]

\[ = 0.7050. \quad (17) \]

D(Q.3) = \( \frac{0.8 \cdot T(VG) + 0.7 \cdot T(G) + 0.5 \cdot T(MG)}{0.8 + 0.7 + 0.5} \)

\[ = \frac{0.8 \cdot 0.90 + 0.7 \cdot 0.80 + 0.5 \cdot 0.70}{0.8 + 0.7 + 0.5} \]

\[ = 0.8150. \quad (18) \]

D(Q.4) = \( \frac{0.5 \cdot T(B) + 0.9 \cdot T(VB) + 0.2 \cdot T(VVB)}{0.5 + 0.9 + 0.2} \)

\[ = \frac{0.5 \cdot 0.40 + 0.9 \cdot 0.24 + 0.2 \cdot 0.09}{0.5 + 0.9 + 0.2} \]

\[ = 0.2713. \quad (19) \]

[Step 2] By applying formula (15), the total mark of the student can be evaluated as follows:

\[ 20 \cdot D(Q.1) + 30 \cdot D(Q.2) + 25 \cdot D(Q.3) + 25 \cdot D(Q.4) \]

\[ = 20 \cdot 0.9424 + 30 \cdot 0.7050 + 25 \cdot 0.8150 + 25 \cdot 0.2713 \]

\[ = 18.848 + 21.15 + 20.375 + 6.7825 \]

\[ = 67.155 \]

\[ = 67 \] (assuming that no half mark is given in the total score). \( \quad (20) \)

5. A generalized fuzzy evaluation method

In this section, we generalize the method presented in Section 4 to propose a weighted method for students' answerscripts evaluation using fuzzy sets. Consider a candidate's answerscript to a paper of 100 marks.

Step 1: Assume that in total there are \( n \) questions to be answered:

TOTAL MARKS = 100

Q.1 carries \( s_1 \) marks
Q.2 carries \( s_2 \) marks
Q.3 carries \( s_3 \) marks
Q.4 carries \( s_4 \) marks

Assume that an evaluator evaluates the questions of students' answerscripts using the following four criteria [2]:

C1: Accuracy of information,
C2: Adequate coverage,
C3: Conciseness,
C4: Clear expression,
Table 8
A generalized extended fuzzy grade sheet

<table>
<thead>
<tr>
<th>Question No.</th>
<th>Criteria</th>
<th>Satisfaction levels</th>
<th>Degree of satisfication for criteria</th>
<th>Degree of satisfication for questions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>EG VVG VG G MG F MB B VB VVB EB</td>
<td>D(C11)</td>
<td>P(Q.1)</td>
</tr>
<tr>
<td>Q.1</td>
<td>C1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>C2</td>
<td></td>
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<tr>
<td></td>
<td>C3</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>C4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Q.2</td>
<td>C1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>C2</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>C3</td>
<td></td>
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</tr>
<tr>
<td></td>
<td>C4</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Q.n</td>
<td>C1</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>C2</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>C3</td>
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<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>C4</td>
<td></td>
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</tr>
</tbody>
</table>

Total mark = s1 * P(Q.1) + s2 * P(Q.2) + ... + sn * P(Q.n)

and assume that the weights of the criteria C1, C2, C3, and C4 are w1, w2, w3, and w4, respectively, where w_i ∈ [0, 1] and 1 ≤ i ≤ 4. Furthermore, assume that the evaluator can evaluate each question of the students’ answerscripts using the above four criteria based on the method described in Section 4. In this case, an evaluator can evaluate the student’s answerscripts using a generalized extended fuzzy grade sheet as shown in Table 8, where the degrees of satisfaction of the question Q.i of a student’s answerscript regarding to the criteria C1, C2, C3, and C4 evaluated by the method presented in section 4 are D(Ci1), D(Ci2), D(Ci3), and D(Ci4), respectively, where 0 ≤ D(Ci1) ≤ 1, 0 ≤ D(Ci2) ≤ 1, 0 ≤ D(Ci3) ≤ 1, 0 ≤ D(Ci4) ≤ 1, and 1 ≤ i ≤ n.

Step 2: The degree of satisfaction P(Q.i) of the question Q.i of the student’s answerscript can be evaluated as follows:

\[
P(Q.i) = \frac{w_1 \cdot D(Ci1) + w_2 \cdot D(Ci2) + w_3 \cdot D(Ci3) + w_4 \cdot D(Ci4)}{w_1 + w_2 + w_3 + w_4},
\]

where P(Q.i) ∈ [0, 1] and 1 ≤ i ≤ n. The total score of the student can be evaluated and is equal to

\[
s_1 \cdot P(Q.1) + s_2 \cdot P(Q.2) + \ldots + s_n \cdot P(Q.n).
\]

Put this total score in the appropriate box at the bottom of the extended fuzzy grade sheet.
6. Conclusions

In [2], Biswas has presented two fuzzy evaluation methods for students’ answercripts evaluation. In this paper, we extend the work of [2] to present two new methods for students’ answercripts evaluation. The proposed methods can be executed much faster than the ones presented in [2] due to the fact that they do not need to perform the complicated matching operations. Furthermore, they can make a more fair evaluation of students’ answercripts. The proposed methods can overcome the drawbacks of the ones presented in [2].

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References