Complex refractive-index measurement based on Fresnel’s equations and the uses of heterodyne interferometry

Ming-Horng Chiu, Ju-Yi Lee, and Der-Chin Su

The phase difference between s and p polarization of the light reflected from a material is used for measuring the material’s complex refractive index. First, two phase differences that correspond to two different incidence angles are measured by heterodyne interferometry. Then these two phase differences are substituted into Fresnel’s equations, and a set of simultaneous equations is obtained. Finally, the equations are solved by use of a personal computer by a numerical analysis technique, and the complex refractive index of the material can be estimated. © 1999 Optical Society of America

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1. Introduction

A complex refractive index $N(n, k)$ is an important characteristic constant of thin-film materials, where $n$ is the refractive index and $k$ is the extinction coefficient. There are several methods for measuring the complex refractive index of a material, e.g., $R$-versus-$\theta$ (reflectance versus incidence angle) methods$^{1-6}$ and ellipsometry.$^7,8$ In those methods, the reflectances of $s$ and $p$ polarization at several different incidence angles or polarization conditions must be measured. Consequently, most of them are related to the measurement of light-intensity variations. However, the stability of the light source, the scattering light, the internal reflection, and other factors influence the accuracy of the measurement results. Recently, Feke et al.$^9$ proposed a novel method to measure the complex refractive index. The phases that correspond to two orthogonal polarizations at different incidence angles are measured with a phase-shifting interferometric technique. These data are substituted into the special equations derived from Fresnel’s equations, and the complex refractive index is obtained.

In this paper a simple method for measuring a complex refractive index is proposed. A light beam of either $s$ or $p$ polarization reflected from a material with a complex refractive index has a phase shift. The phase difference between these two kinds of polarization is a function of $n$, $k$, and incidence angle $\theta$. The phase differences that correspond to two different incidence angles are measured with the heterodyne interferometry proposed by Chiu et al.$^{10}$ These data are substituted into Fresnel’s equations,$^{11}$ and a set of simultaneous equations is obtained. Then these equations are solved by use of a personal computer by a numerical analysis technique, and the complex refractive index of the material can be estimated. The technique has several merits, including a simple optical setup, easy operation, high stability, high measurement accuracy, and rapid measurement. We demonstrate its feasibility.

2. Principles

A. Phase Difference Resulting from Reflection from an Absorbing Material

A ray of light in air is incident at $\theta$ onto an absorbing material with a complex refractive index $N(n, k)$, as shown in Fig. 1. According to Fresnel’s equa-
polarization can be expressed as

\[ r_s = \frac{\cos \theta - (u + iv)}{\cos \theta + (u + iv)} = |r_s| \exp(i\delta_s), \quad (1) \]

\[ r_p = \frac{N^2 \cos \theta - (u + iv)}{N^2 \cos \theta + (u + iv)} = |r_p| \exp(i\delta_p), \quad (2) \]

respectively, where

\[ u^2 = \frac{1}{2}((n^2 - k^2 - \sin^2 \theta)
+ ((n^2 - k^2 - \sin^2 \theta)^2 + 4n^2k^2)^{1/2}), \quad (3) \]

\[ v^2 = \frac{1}{2}(-(n^2 - k^2 - \sin^2 \theta)
+ ((n^2 - k^2 - \sin^2 \theta)^2 + 4n^2k^2)^{1/2}), \quad (4) \]

and \( \delta_s \) and \( \delta_p \) are the phase shifts of \( s \) and \( p \) polarizations and can be expressed as

\[ \delta_s = \tan^{-1}\left(\frac{2v \cos \theta}{u^2 + v^2 - \cos^2 \theta}\right), \quad (5) \]

\[ \delta_p = \tan^{-1}\left(\frac{2v \cos \theta(n^2 - k^2 - 2u^2)}{u^2 + v^2 - (n^2 + k^2)^2 \cos^2 \theta}\right), \quad (6) \]

respectively. Hence the phase difference of \( s \) polarization relative to \( p \) polarization is

\[ \phi = \delta_s - \delta_p = \tan^{-1}\left(\frac{ad - bc}{ac + bd}\right), \quad (7) \]

where

\[ a = 2v \cos \theta, \]
\[ b = u^2 + v^2 - \cos^2 \theta, \]
\[ c = 2v \cos \theta(n^2 - k^2 - 2u^2), \]
\[ d = u^2 + v^2 - (n^2 + k^2)^2 \cos^2 \theta. \quad (8) \]

From Eqs. (7) and (8) it is obvious that phase difference \( \phi \) is a function of \( n \), \( k \), and \( \theta \), and \( \phi \) can be experimentally measured for each given \( \theta \). To evaluate the values of \( n \) and \( k \) we require two phase differences \( \phi_1 \) and \( \phi_2 \) that correspond to two incidence angles, \( \theta_1 \) and \( \theta_2 \). Hence a set of simultaneous equations

\[ \phi_1 = \phi_1(n, k, \theta_1), \quad (9) \]
\[ \phi_2 = \phi_2(n, k, \theta_2) \quad (10) \]

is obtained. If these simultaneous equations are solved, the complex refractive index of the material can be estimated.

B. Phase-Difference Measurements with Heterodyne Interferometry

Chiu et al.\textsuperscript{10} proposed a method for measuring the refractive index of a transparent material by using total-internal-reflection heterodyne interferometry. A schematic diagram of the optical arrangement of our method, which is based on similar considerations, was designed and is shown in Fig. 2(a). Linearly polarized light passing through an electro-optic modulator (EO) is incident upon a beam splitter (BS) and divided into two parts, a reference beam and a test beam. The reference beam passes through analyzer ANr, and enters photodetector Dr. If the amplitude of light detected by Dr is \( E_r \), then the intensity mea-

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**Fig. 1.** Reflection at the surface of an absorbing material.

**Fig. 2.** Schematic diagram of measurement of the phase differences owing to reflection at (a) an absorbing material and (b) a beam splitter. Other abbreviations are defined in text.
Table 1. Experimental Conditions and Measurement Results

<table>
<thead>
<tr>
<th>Material</th>
<th>θ₁ (°)</th>
<th>θ₂ (°)</th>
<th>φ₁ (°)</th>
<th>φ₂ (°)</th>
<th>n</th>
<th>k</th>
<th>Δn</th>
<th>Δk</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ni</td>
<td>60</td>
<td>80</td>
<td>144.96</td>
<td>74.15</td>
<td>2.007</td>
<td>3.781</td>
<td>0.01</td>
<td>0.006</td>
</tr>
<tr>
<td>Cu</td>
<td>70</td>
<td>80</td>
<td>121.32</td>
<td>74.15</td>
<td>2.006</td>
<td>3.782</td>
<td>0.01</td>
<td>0.006</td>
</tr>
</tbody>
</table>

Reference values from Ref. 14: N(Ni) = 1.98 + i3.74 and N(Cu) = 0.272 + i3.42.

4. Discussion

From Eqs. (9) and (10) we get

\[
\begin{align*}
\Delta n & \equiv \left| \frac{\partial \Phi_2}{\partial k} \Delta \Phi_1 + \frac{\partial \Phi_1}{\partial n} \Delta \Phi_2 \right|, \\
\Delta k & \equiv \left| \frac{\partial \Phi_1}{\partial n} \Delta \Phi_1 + \frac{\partial \Phi_2}{\partial k} \Delta \Phi_2 \right|,
\end{align*}
\]

where \(\Delta n\) and \(\Delta k\) are the errors in \(n\) and \(k\) and \(\Delta \Phi_1\) and \(\Delta \Phi_2\) are the errors in the phase differences at two different incidence angles \(\theta_1\) and \(\theta_2\), respectively. The errors in the phase differences in this technique may be influenced by the following factors:

1. Angular resolution of the phasemeter
   The angular resolution \(\Delta \Phi_p\) of the phasemeter can be expressed as
   \[
   \Delta \Phi_p = \frac{\pi}{f_c} \times 360^\circ,
   \]
   where \(f\) and \(f_c\) are the frequencies of the input waves and the reference clock in the phasemeter, respectively. In our experiments we used \(f = 800\) Hz and \(f_c = 32\) MHz, so the angular resolution of the phasemeter is better than 0.01°.

2. Second-harmonic error
   The second-harmonic error comes from the deviation angle \(\theta_0\), between the polarization directions of \(p\) polarization of the incident beam and the incidence plane. It introduces an error in the phase difference:
   \[
   \Delta \Phi_p = \frac{\tan \phi (\sec 2\theta_0 - 1)}{1 + \sec 2\theta_0 \tan^2 \phi}
   \]

where \(\Delta \Phi_p\) is the error in the phase difference, \(\phi\) is the phase difference, and \(\theta_0\) is the deviation angle.
into \( \phi \). The relation curves of \( \Delta \phi_r \) versus \( \phi \) for \( \theta_r = 0.5^\circ, 1.0^\circ, 1.5^\circ, 2.0^\circ \) are depicted in Fig. 3, and their maximum errors are 0.004\(^\circ\), 0.02\(^\circ\), 0.04\(^\circ\), and 0.07\(^\circ\), respectively. It is obvious that \( \Delta \phi_r \) equals zero as \( \phi = -180^\circ, -90^\circ, 90^\circ, 180^\circ \). This error can be made nearly zero by accurate modification of the azimuthal angle of every polarization component, as was done by Chiu et al.\(^{13}\). Owing to the extinction ratio effect of a polarizer, mixing of light polarization occurs. After a ray light passes through a polarizer, its Jones vector should be affected by the reflection from the beam splitter, and they are influenced by the extinction ratio of the polarizer. Consequently, we have

\[
\begin{align*}
\phi_{ar} &= \phi_{br}, \\
\phi_{br} &= \phi_{br}, \\
|A_r|/|B_r| &= |A_r|/|B_r|.
\end{align*}
\]

Consequently, Eq. (19) can be rewritten as

\[
I_r = (|A_r| + |\alpha_1|^2)^2 + (|B_r| + |\beta_1|^2)^2 + 2(|A_r| |B_r| + |A_r| |\alpha_1| + |B_r| |\beta_1|) \cos(2\pi f t + \phi_{BS}).
\]

Similarly, the intensity of the test signal is given as

\[
I_t = |A_t|^2 + |B_t|^2 + |\alpha_t|^2 + |\beta_t|^2 + 2(|A_t| |\alpha_t| + |B_t| |\beta_t|) \cos \phi + 2(|A_t| |\beta_t|)^2 \sin^2 \phi \frac{1}{2} \cos(2\pi f t - \phi'),
\]

where \( \alpha_t \) and \( \beta_t \) are the complex amplitudes of components with vibration directions along the \( x \) and \( y \) axes, and they are mixed into \( E_x \) and \( E_y \), respectively; \( f \) is the frequency difference produced by the EO. For clarity, the coefficients related to the reference signal and the test signal are expressed by the subscripts \( r \) and \( t \), respectively. Based on the derivations of Jones calculus, the intensity of the reference signal can be written as\(^{17,18}\)

\[
I_r = |A_r|^2 + |B_r|^2 + |\alpha_r|^2 + |\beta_r|^2 + 2|A_r| |\alpha_r| \cos(\phi_{ar} - \phi_{ar}) + 2|B_r| |\beta_r| \cos(\phi_{br} - \phi_{br}) + 2|A_r| |B_r| \cos(2\pi f t + \phi_{ar} - \phi_{br}) + 2|\alpha_r| |B_r| \cos(2\pi f t + \phi_{ar} - \phi_{br}) + 2|\alpha_r| |\beta_r| \cos(2\pi f t + \phi_{ar} - \phi_{br}).
\]

Because both interfering beams (\( s \) and \( p \) polarization) propagate along the same optical path and they are influenced by the reflection from the beam splitter, we have \( |A_r| = |B_r|, \phi_{ar} - \phi_{br} = \phi_{ar} - \phi_{br} = \phi_{BS}, \)
For $D_n$ and $D_k$ of nickel and copper at $\theta_2 = 80^\circ$ by substituting $|\Delta \phi_1| = |\Delta \phi_2| = 0.03^\circ$ into relations (14) and (15). The results are shown in Figs. 5(a) and 5(b), respectively. Obviously, the best resolution can be obtained when $\theta_1$ is close to $60^\circ$. $\Delta n$ and $\Delta k$ corresponding to our experimental conditions were calculated and are included in Table 1.

To investigate the effects of experimental conditions on the measurements we depict in Fig. 6 the curves of constant $n$ and $k$ as functions of phase difference $f$ for $\theta_1 = 60^\circ$ and $\theta_2 = 80^\circ$. In the figure the values of $n$ and $k$ are shown from 0.2 to 4 in steps of 0.2. According to Humphreys-Owen, the sensitivity to the experimental conditions is indicated by the spacing between contours: If the spacing is large, the sensitivity is good and vice versa. It is obvious that our experimental conditions are useful if $n$ and $k$ are small. For good sensitivity, it is better to choose optimum incidence angles with the method proposed by Logofatu et al. Moreover, curves of constant $n$ and $k$ as functions of $D_n$ and $D_k$ are shown in Fig. 7. We obtained them by substituting the experimental conditions $\theta_1 = 60^\circ$, $\theta_2 = 80^\circ$, and $|\Delta \phi_1| = |\Delta \phi_2| = 0.03^\circ$ into relations (14) and (15). In Fig. 7 the values of $n$ and $k$ are from 0.5 to 4 in steps of 0.5. It can be seen that both $\Delta n$ and $\Delta k$ are smaller than $1 \times 10^{-2}$ for the test materials with $n > 1$ and $k < 2$. Furthermore, this method is highly stable against air turbulence because of its common path configuration.

**5. Conclusion**

Based on Fresnel’s equations and the use of heterodyne interferometry, we have developed a new method for measuring a complex refractive index. The phase difference between $s$ and $p$ polarization of the reflected light from an absorbing material is measured with a heterodyne interferometer. Two phase differences, corresponding to two different incidence angles, are measured. These two phase differences are substituted into Fresnel’s equations to yield a set of simultaneous equations. Then the equations are solved by use of a personal computer by a numerical analysis technique, and the complex refractive index
of the material can be estimated. The method has several merits, including a simple optical setup, easy operation, high stability, high measurement accuracy, and rapid measurement. Its feasibility has been demonstrated.

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References