Neutrino-photon scattering and its crossed processes in a background magnetic field

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Abstract

We study the neutrino-photon processes such as $\gamma\gamma \rightarrow \nu\bar{\nu}$, $\nu\gamma \rightarrow \nu\gamma$, and $\nu\bar{\nu} \rightarrow \gamma\gamma$ in a background magnetic field smaller than the critical magnetic field $B_c = m_e^2/e$. Using Schwinger’s formalism, we extract leading magnetic-field contributions to the above processes. Our result is valid throughout the kinematic regime where both neutrino and photon energies are significantly smaller than $m_e$. We briefly discuss the astrophysical implications of our result. © 1999 Published by Elsevier Science B.V. All rights reserved.

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The relevance of neutrino-photon interactions in astrophysics and cosmology has been studied extensively [1]. For example, the plasmon decay $\gamma^+ \rightarrow \nu\bar{\nu}$ in horizontal branch stars and red giants leads to a strong constraint on the neutrino magnetic-moment [2]. Similarly, the decay process $\nu^+ \rightarrow \nu\gamma$ was also calculated [3], and its partial width has been constrained by various astrophysical observations [1]. It is natural to ask whether the two-photon processes such as the scatterings $\gamma\gamma \rightarrow \nu\bar{\nu}$, $\nu\gamma \rightarrow \nu\gamma$ or the decay $\nu^+ \rightarrow \nu\gamma\gamma$ are also relevant in astrophysics and cosmology. It turns out that, due to the left-handedness of the weak interaction, the $O(G_F)$ contributions to the amplitudes of the above processes are proportional to the mass of the neutrino [4]. Hence the resulting cross sections or decay rates are very suppressed. On the other hand, similar processes involving three photons such as $\gamma\gamma \rightarrow \nu\bar{\nu}\gamma$ or $\nu\gamma \rightarrow \nu\gamma\gamma$ are not suppressed by the same mechanism [5]. Consequently, one expects that the cross sections for $\gamma\gamma \rightarrow \nu\bar{\nu}$ and its crossed processes should be enhanced under a strong background magnetic field. In fact, under a background magnetic field $B$, the cross section $\sigma(\gamma\gamma \rightarrow \nu\bar{\nu})$ with photon energy $E_\gamma \ll m_e$ is enhanced by a factor $(m_w/m_e)^4(B/B_c)^2$ [6] as compared to its counterpart in the vacuum, where $m_w$ and $m_e$ are the $W$ boson and the electron masses respectively; $B_c = m_e^2/e$ is the critical magnetic field.

The previous calculation on $\gamma\gamma \rightarrow \nu\bar{\nu}$ [6] applies an effective Lagrangian for $\gamma\gamma \rightarrow \nu\nu\gamma$ [5] and replaces one of the external photon with the classical magnetic field. It is clear that such an approach is valid only in the limit that $E_\gamma, E_\nu \ll m_e$. In this work, we shall extend the previous analysis by study-
ing the processes $\gamma\gamma \rightarrow \nu\bar{\nu}$, $\gamma\nu \rightarrow \nu\nu$ and $\nu\bar{\nu} \rightarrow \gamma\gamma$
with $E_\gamma$ and $E_\nu$ larger than $m_\nu$, but still considerably
smaller than $m_\mu$. This generalization is motivated by the
fact that the above processes may take place in stars
with temperatures higher than $m_\nu$. In this case,
the effective-Lagrangian approach is no longer ap-
propriate.

Let us begin with the process $\gamma\gamma \rightarrow \nu\bar{\nu}$ in a
background magnetic field. For convenience, the
cross section of this process is denoted as $\sigma_{\nu}(\gamma\gamma \rightarrow
\nu\bar{\nu})$. The relevant Feynman diagram is depicted in
Fig. 1. The effective four-fermion interactions be-
tween leptons and neutrinos can be written as

$$\mathcal{L} = -\frac{G_F}{\sqrt{2}} \left( \bar{\nu}_1 \gamma_\mu (1 - \gamma_5) \nu_1 \right) \left( \bar{\nu}_2 \gamma^\alpha (g_\nu - g_A \gamma_5) \nu_2 \right),$$

(1)

where $g_\nu = 1/2 + 2\sin^2\theta_\nu$ and $g_A = 1/2$ for $l = e$;
$g_\nu = -1/2 + 2\sin\theta_\nu$ and $g_A = -1/2$ for $l = \mu, \tau$. We
should remark that the contribution due to $g_\nu$
is proportional to the neutrino mass in the limit of
vanishing magnetic field. At $O(B)$ in the limit $B \ll B_\nu$, it
gives no contribution to the amplitude by the
charge conjugation invariance. Therefore we shall
neglect the contribution by $g_\nu$. Likewise, we shall
also neglect contributions by $g_A$ for $l = \mu, \tau$, since
$-1/2 + 2\sin\theta_\nu = 0.04 \ll 1$. The amplitude for
$\gamma(k_1) \gamma(k_2) \rightarrow \nu(p_1) \nu(p_2)$ in a background
magnetic field reads

$$M = \frac{G_F g_\nu}{\sqrt{2}} 4\pi i \epsilon \left( p_2 \right) \gamma^\mu (1 - \gamma_5) \epsilon \left( p_1 \right) \int d^4V \times d^4W \text{tr} \left[ \gamma_\mu \mathcal{G}(W) \gamma_\mu \mathcal{G}(-V - W) \gamma_5 \mathcal{G}(V) \right] \times e^{-i(k_1 \cdot V - k_2 \cdot W)} \times \exp \left( -\frac{ie}{2} V^A F_A^\mu W^\mu \right) \times \exp \left( i (k_1 \cdot V - k_2 \cdot W) + (k_1, \mu \leftrightarrow k_2, \nu) \right).$$

(2)

where $V = z - x$, and $W = x - y$; $\epsilon(k_1)$ and $\epsilon(k_2)$
are polarization vectors of the photons; $\mathcal{G}(W) \equiv \mathcal{G}(x - y)$ is a part of the full electron propagator
$G(x, y)$ which has the following form under a con-
stant magnetic field [7]:

$$G(x, y) = \Phi(x, y) \mathcal{G}(x - y),$$

with

$$\Phi(x, y) = \exp \left( -\frac{ie}{2} V^\mu F_{\mu\nu} W^\nu \right).$$

(4)

For a constant magnetic field along the $+z$ direction, we have $F_{12} = -F_{21} = B$ while other components of $F_{\mu\nu}$ vanish.
At this stage, the calculation of $M$ remains non-trivial since the function $\mathcal{G}$ as given by Eq. (3) is complicated. To find a simplification for $\mathcal{G}$, we go to the momentum space, which amounts to writing

$$
\mathcal{G}(x, y) = \int \frac{d^4 p}{(2\pi)^4} e^{-i p (x - y)} \mathcal{G}(p),
$$

with

$$
\mathcal{G}(p) = \int_0^\infty ds \cos(eBs)
\times \exp \left[ -is \left( m_c^2 - p_T^2 - \tan(eBs) \frac{p_T^2}{eBs} \right) \right]
\times \left[ e^{i\epsilon B s} (m_c + \gamma \cdot p_T) + \gamma \cdot p_T \right].
$$

(5)

It is useful to write $\mathcal{G}$ in terms of Landau levels [8]

$$
\mathcal{G}(p) = \sum_{n=0}^{\infty} \frac{-id_n(\alpha) \gamma p_T + m_c + \gamma \cdot p_T + d_n(\alpha) \gamma (m_c + \gamma \cdot p_T)}{p_T^2 + 2mB}
+ \frac{\gamma \cdot p_T}{p_T^2},
$$

(6)

where

$$
p_T^2 = m_c^2 - p_T^2, \quad \alpha = -p_T^2 / eB,
$$

$$
d_n(\alpha) = (-1)^n e^{-a} (L_n(2\alpha) - L_{n-1}(2\alpha)),
$$

with $L_n$ the Laguerre polynomials. As indicated by Eq. (6), the $B$ dependence of $\mathcal{G}(p)$ resides in $d_n(\alpha)$, $d_n(\alpha)$, and the propagator $1/(p_T^2 + 2mB)$. For $B \ll B_c$, the propagator $\mathcal{G}$ and the phase factor $\exp\left[ (-ie/2)V^A F_\alpha W^* \right]$ can be expanded in powers of $eB$. To the linear order in $eB$, we have [9]

$$
\mathcal{G}(p) = i \frac{\gamma \cdot p + m_c}{p^2 - m_c^2 + ie} - \frac{\gamma \cdot p}{p^2 - m_c^2 + ie} e^{2B^2},
$$

+ $O(e^2 B^2),
$$

(7)

and

$$
\exp \left( \frac{-ie}{2} V^A F_\alpha W^* \right) = 1 - \frac{ie}{2} (V^A F_\alpha W^*)
+ O(e^2 B^2).
$$

The above expansions can be used to compute the amplitude $M$ in powers of $eB$. Indeed, by dimensional analysis, any given power of $eB$ in the expansion of $M$ is accompanied by an equal power of $1/m_c^2$ (for $m_c > p$) or $1/p^2$ (for $p > m_c$) with $p$ the typical energy scale of external particles. Clearly, for $B \ll B_c = m_c^2/e$, both $(eB/m_c^2)^n$ and $(eB/p^2)^n$ (applicable when $p > m_c$) are much smaller than unity.

From Eqs. (2) and (7) and the expansion of the phase factor, the amplitude $M$ to the linear order in $eB$ is

$$
M = G_{eV} e^{\alpha \epsilon} \frac{e\alpha}{\sqrt{2}} \pi (p_2) \gamma_1 (1 - \gamma_2) (p_1) J^p,
$$

(8)

with

$$
J^p = C_1 \left[ (e_1 F_{e2}) (k^p - k^{e2}) \right]
+ C_2 \left[ (e_1 F_{k1}) (k_1 \cdot e_2) k^p \right]
+ (e_1 F_{k1}) (k_1 \cdot e_2) k^p
+ C_3 \left[ (e_1 F_{k2}) (k_2 \cdot e_2) k^p \right]
+ (e_2 F_{k1}) (k_2 \cdot e_1) k^p
+ C_5 \left[ (e_1 F_{k2}) (k_1 \cdot e_2) k^p \right]
+ (e_2 F_{k1}) (k_1 \cdot e_1) k^p
+ C_6 \left[ (e_1 F_{k2}) (e_2 F_{k2}) (k^p - k^{e2}) \right]
+ C_7 \left[ (k_2 \cdot e_1) (k_1 \cdot e_2) \left( (F_{k1})^p + (F_{k2})^p \right) \right]
+ C_8 \left[ (e_1 \cdot e_2) \left( (F_{k1})^p + (F_{k2})^p \right) \right]
+ C_9 \left[ (k_1 F_{k2}) (e_1, e_2) (k^p - k^{e2}) \right]
+ C_{10} \left[ (k_1 F_{k2}) (k_2 e_1) (k_1 e_2) (k^p - k^{e2}) \right]
+ C_{11} \left[ (k_1 F_{k2}) (k_2 e_1, e_2) + (k_1 e_1 e_2) \right],
$$

(9)

where $C_1, C_2, \cdots, C_{11}$ are linear combinations of the integrals

$$
I[a, b, c] = \int_0^1 dx \int_0^x dy \frac{x^b y^{a-b}}{(1 - sy + ie)},
$$

with $t = 2k_1 \cdot k_2/m_c^2$. The detailed structures of these coefficients will be presented elsewhere [9]. We have checked our result by taking the limit $2k_1 \cdot k_2/m_c^2 \ll 1$. It agrees with the result of Ref. [6], which is obtained via the effective-Lagrangian approach.

From Eqs. (8) and (9), we can calculate the cross section for $\gamma \gamma \rightarrow \nu \bar{\nu}$ in a background magnetic field.
respectively, with equal magnitudes. The result for $\sigma_{gg}(\gamma\gamma \to \nu\bar{\nu})$ with $B = 0.1B_c$ and $B$ perpendicular to the collision axis is plotted in Fig. 2. For other relative alignments between $B$ and the collision axis, the cross section $\sigma_{gg}$ varies by no more than an order of magnitude. To explore the validity of the effective-Lagrangian approach, we also plot the cross section $\sigma_{gg}(\gamma\gamma \to \nu\bar{\nu})$ obtained in this method [6]. It is found that $\sigma_{gg}$ and $\sigma_{gg}^e$ agree reasonably well at a small incoming photon energy ($\omega$), i.e., $\omega/m_e < 0.5$. For $\omega$ slightly greater than $m_e$, the internal electron could become on shell, and $\sigma_{gg}$ would dominate over $\sigma_{gg}^e$ due to the rescattering effect by $e^+e^- \to \nu\bar{\nu}$. Such a dominance lasts till $\omega/m_e = 2.2$ where $\sigma_{gg}^e$ begins to overtake $\sigma_{gg}$. Finally, for comparisons, we also display the $2 \to 3$ scattering cross section $\sigma(\gamma\gamma \to \nu\bar{\nu}\gamma)$ obtained in Refs. [10,11]. For $\omega/m_e < 5$, this cross section is seen to be suppressed compared to $\sigma_{gg}(\gamma\gamma \to \nu\bar{\nu})$. At higher energies, it becomes equally important as the latter.

The stellar energy-loss rate $Q$ due to $\gamma\gamma \to \nu\bar{\nu}$ in a background magnetic field has been calculated [6]. We repeat the calculation using our updated result of $\sigma_{gg}(\gamma\gamma \to \nu\bar{\nu})$. The temperature dependencies of $Q$ are listed in Table 1. For comparisons, we also list corresponding results obtained from the effective-Lagrangian approach [1]. For temperatures below 0.01 MeV, the effective-Lagrangian approach works very well. On the other hand, this approach becomes rather inaccurate for temperatures greater than 1 MeV. At $T = 0.1$ MeV, our exact calculation gives an energy-loss rate almost two orders of magnitude greater than the result from the effective Lagrangian. Such a behavior can be understood from the energy dependence of the scattering cross section, as shown in Fig. 2. It is clear that, for $T = 0.1$ MeV, $Q$ must have received significant contributions from scatterings with $\omega \approx m_e$. At this energy, the full calculation gives a much larger scattering cross section than the effective Lagrangian does.

By comparing the predictions of the full calculation and the effective-Lagrangian approach [6], we conclude that the applicability of the latter to the energy-loss rate is quite restricted. While the effective Lagrangian works reasonably well with $\omega < 0.1m_e$, it would give a poor approximation on $Q$ unless $T < 0.01m_e$.

Besides $\gamma\gamma \to \nu\bar{\nu}$, the crossed processes $\nu(\bar{\nu})\gamma \to \nu(\bar{\nu})\gamma$ and $\nu\bar{\nu} \to \gamma\gamma$ in a background magnetic field also play some roles in astrophysics. For example, one expects that these two processes might be relevant for the mean free paths of supernova neutrinos. In fact, it was recently suggested that [12], for supernova neutrinos, the $2 \to 3$ scatterings $\nu\nu \to \nu\gamma\gamma$ and $\bar{\nu}\bar{\nu} \to \gamma\gamma\gamma$ give neutrino mean free paths less than the supernova core radius. Thus they could

### Table 1

<table>
<thead>
<tr>
<th>$Q/T$ [MeV]</th>
<th>exact</th>
<th>effective</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.001</td>
<td>$1.2 \times 10^{-14}$</td>
<td>$5.6 \times 10^{-3}$</td>
</tr>
<tr>
<td>0.01</td>
<td>$1.2 \times 10^{-14}$</td>
<td>$1.7 \times 10^{-3}$</td>
</tr>
<tr>
<td>0.1</td>
<td>$3.1 \times 10^{-14}$</td>
<td>$1.2 \times 10^{-2}$</td>
</tr>
<tr>
<td>1</td>
<td>$1.2 \times 10^{-14}$</td>
<td>$1.2 \times 10^{-1}$</td>
</tr>
<tr>
<td>10</td>
<td>$1.2 \times 10^{-14}$</td>
<td>$1.2 \times 10^{-1}$</td>
</tr>
</tbody>
</table>

Fig. 2. $\sigma_{gg}(\gamma\gamma \to \nu\bar{\nu})$ is the cross section obtained from the exact calculation, while $\sigma_{gg}^e(\gamma\gamma \to \nu\bar{\nu})$ is obtained from the effective Lagrangian approach. The magnetic field direction is taken to be parallel to the collision axis. For comparison, the $2 \to 3$ cross section $\sigma(\gamma\gamma \to \nu\bar{\nu}\gamma)$ is also displayed.

---

1. In the low temperature limit, i.e., $T < m_e$, our $Q$ differs from that of Ref. [6], but the discrepancy is within one order of magnitude.
The solid line and the dotted line depict cross sections \( \sigma_b(\gamma\nu \to \gamma\nu) \) and \( \sigma_b(\gamma\nu \to \gamma\nu) \) respectively. The dashed line depicts the cross section \( \sigma(\gamma\nu \to \gamma\gamma) \). Figure 3.

Table 2

<table>
<thead>
<tr>
<th>( E_\nu [\text{MeV}] )</th>
<th>0.01</th>
<th>0.1</th>
<th>1</th>
<th>5</th>
<th>20</th>
<th>50</th>
</tr>
</thead>
</table>
| \( \lambda_{1,2} \) relevant to the scatterings \( \nu\gamma \to \nu\gamma \) and \( \nu\tilde{\nu} \to \gamma\gamma \), respectively, in a background magnetic field \( B = B_1/10 \). We fix \( T = 20 \text{ MeV} \), \( \mu_e = 0 \) and vary the neutrino energy \( E_\nu \) from 0.01 MeV to 50 MeV.

\[
\begin{array}{cccccccc}
0.01 & 0.1 & 1 & 5 & 20 & 50 \\
\hline
\lambda_{1,2} & 1 \times 10^{22} & 2 \times 10^{19} & 1 \times 10^{19} & 5 \times 10^{19} & 3 \times 10^{18} & 4 \times 10^{18} & 1 \times 10^{13} & 5 \times 10^{16} & 3 \times 10^{15} \\
\end{array}
\]
concerned. Hence this process is not expected to affect the supernova dynamics. Furthermore, since the cross section \( \sigma(\nu\gamma \rightarrow \nu\gamma\gamma) \) is at most comparable to \( \sigma_B(\nu\gamma \rightarrow \nu\gamma) \)\(|_{\parallel}\), the neutrino mean free path implied by the former process should also be much greater than the supernova core radius. This is in a sharp contrast to the small neutrino mean free path \((\lambda_{\nu\gamma \rightarrow \nu\gamma\gamma} \approx 10^{-3} \text{ cm})\) for \( T = 20 \text{ MeV}, E_\nu = 20 \text{ MeV}, \text{ and } \mu_\nu = 0 \) obtained in Ref. [12]. This discrepancy confirms that the extrapolation performed in Ref. [12] indeed underestimates the mean free paths of high energy neutrinos with \( E_\nu > m_\nu \).

Now let us turn to the last process, \( \nu\nu \rightarrow \gamma\gamma \) in a background magnetic field. This process behaves rather similarly as the reversed process \( \gamma\gamma \rightarrow \nu\nu \) discussed before. The scattering cross section \( \sigma_B(\nu\nu \rightarrow \gamma\gamma) \)\(|_{\parallel}\) is depicted in Fig. 4. For comparisons, the corresponding \( 2 \rightarrow 3 \) cross section \( \sigma(\nu\nu \rightarrow \gamma\gamma\gamma) \)\(|_{\parallel}\) is also shown. One can see that \( \sigma_B(\nu\nu \rightarrow \gamma\gamma) \)\(|_{\parallel}\) peaks locally in the vicinity of \( \omega = m_\nu \), where the threshold effect of electron pair-production emerges. Furthermore, from \( \omega = 0.1 m_\nu \) to \( \omega = m_\nu \), the \( 2 \rightarrow 2 \) cross section dominates the \( 2 \rightarrow 3 \) cross section by a few orders of magnitude. The two curves cross at \( \omega = 5 m_\nu \), at which point the \( 2 \rightarrow 3 \) process begins to dominate. At \( \omega = m_\nu \), \( B = 0.1 B_\nu \), we have, for example, \( \sigma_B(\nu\nu \rightarrow \gamma\gamma) \)\(|_{\parallel}\) \( = 10^{-43} \text{ cm}^2 \), and \( \sigma(\nu\nu \rightarrow \gamma\gamma\gamma) = 1.5 \times 10^{-53} \text{ cm}^2 \)\(|_{\parallel}\) [10,11]. The former cross section becomes \( 3 \times 10^{-50} \text{ cm}^2 \) at \( \omega = 50 m_\nu \) while the latter cross section is roughly an order of magnitude larger. The neutrino mean free path due to \( \nu\nu \rightarrow \gamma\gamma \), which we denote as \( \lambda_{\nu\nu \rightarrow \gamma\gamma} \), can be calculated using \( \lambda_{\nu\nu \rightarrow \gamma\gamma} = 1/n_\nu \sigma_B \), where \( n_\nu \) is the number density of the antineutrino, and \( \sigma_B \) is the average cross section of \( \nu\nu \rightarrow \gamma\gamma \). The results on \( \lambda_{\nu\nu \rightarrow \gamma\gamma} \) for different neutrino energies are listed in Table 2. For \( T = 20 \text{ MeV}, E_\nu = 20 \text{ MeV}, \mu_\nu = 0 \) and \( B = 0.1 B_\nu \), we find \( \lambda_{\nu\nu \rightarrow \gamma\gamma} = 5 \times 10^{16} \text{ cm} \). The neutrino mean free path decreases to \( 3 \times 10^{15} \text{ cm} \) for \( E_\nu = 50 \text{ MeV} \), and increases to \( 3 \times 10^{18} \text{ cm} \) for \( E_\nu = 1 \text{ MeV} \). Once again, the above neutrino mean free paths are all much greater than the supernova core radius. Furthermore, by comparing \( \sigma(\nu\nu \rightarrow \gamma\gamma\gamma) \) with \( \sigma_B(\nu\nu \rightarrow \gamma\gamma) \)\(|_{\parallel}\), we conclude that the neutrino mean free path relevant to the former process should also be much greater than the supernova core radius. This is again in a sharp contrast to a small neutrino mean free path \((\lambda_{\nu\nu \rightarrow \gamma\gamma\gamma} \approx 10^{-3} \text{ cm})\) for \( T = 20 \text{ MeV}, E_\nu = 20 \text{ MeV}, \text{ and } \mu_\nu = 0 \) obtained in Ref. [12]. Once more, this discrepancy is due to the cross section extrapolation performed in Ref. [12].

In conclusion, we have illustrated the weak-field expansion technique for processes occurring in a background magnetic field. Specifically, we apply this technique to calculate the cross sections of \( \gamma\gamma \rightarrow \nu\nu, \gamma\nu \rightarrow \gamma\nu, \) and \( \nu\nu \rightarrow \gamma\gamma \) under a background magnetic field. We found that the effective-Lagrangian approach is inappropriate for computing the stellar energy-loss rate due to \( \gamma\gamma \rightarrow \nu\nu \), unless the star temperature is less than \( 0.01 m_\nu \). We also found that the neutrino mean free paths relevant to \( \gamma\nu \rightarrow \gamma\nu \) and \( \nu\nu \rightarrow \gamma\gamma \) in a background magnetic field are much greater than the supernova core radius. The same conclusions are reached for the neutrino mean free paths relevant to \( \gamma\nu \rightarrow \gamma\nu \) and \( \nu\nu \rightarrow \gamma\gamma \). Therefore both neutrino-photon scatterings and neutrino-antineutrino annihilations into photons are not expected to affect the supernova dynamics.

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