Vehicle System Dynamics: International Journal of Vehicle Mechanics and Mobility

Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/nvsd20

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Published online: 09 Aug 2010.

To cite this article: Chieh Chen & Masayoshi Tomizuka (2000) Lateral Control of Commercial Heavy Vehicles, Vehicle System Dynamics: International Journal of Vehicle Mechanics and Mobility, 33:6, 391-420
To link to this article: http://dx.doi.org/10.1076/0042-3114(200006)33:6;1-M;FT391

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Lateral Control of Commercial Heavy Vehicles

CHIEH CHEN\textsuperscript{1} and MASAYOSHI TOMIZUKA\textsuperscript{2}

SUMMARY

Two nonlinear lateral control algorithms are designed for a tractor-semitrailer type commercial heavy vehicle. The baseline steering control algorithm is designed utilizing input-output linearization. To enhance the lateral stability and furthermore reduce tracking errors of the trailer, braking forces are independently controlled on the inner and outer wheels of the trailer. The coordinated steering and braking control algorithm is designed based on the multivariable backstepping technique. Simulations conducted using the complex model show that the trailer yaw errors under coordinated steering and independent braking force control are much smaller than those without independent braking force control.

1. INTRODUCTION

Traffic congestion problems and driving safety issues on highways have motivated an increased amount of research in highway automation and resulted in several programs worldwide, such as ITS in the US (see for example [1]) and ASV, SSVS and ARTS under ITS Japan [2, 3]. In the area of Advanced Vehicle Control Systems (AVCS) for an automated vehicle in the context of Automated Highway Systems (AHS), there are two basic control operations - longitudinal and lateral control. Longitudinal control involves regulating the vehicle speed to keep a proper spacing between vehicles. Lateral control, on the other hand, is concerned with automatic steering of vehicles to follow a reference along the lane center, while maintaining a good level of passenger comfort. Past research on AVCS for AHS has been emphasized on passenger vehicles [4, 5, 6, 7, 8]. The study of Commercial Heavy Vehicles (CHV) for AHS applications has gained interest only recently [9, 10, 11, 12, 13, 14, 15, 16]. The study of lateral guidance of commercial heavy vehicles is motivated by several facts. In 1993, the share of the highway miles accounted for by truck traffic was around
This is a significant percentage of the total highway miles traveled by all vehicles in U.S. According to Motor Vehicles Facts and Figures [23], the total number of registered trucks (light, commercial and truck-trailer combinations) formed approximately 10% of the national figures in 1991 and 30.9% of the highway taxes came from heavy vehicles. Also, due to several economic and policy issues, heavy vehicles have the potential of becoming the main beneficiaries of automatic guidance [14]. The main reasons are:

- On average, a truck travels six times the miles as compared to a passenger vehicle. Possible reduction in the number of drivers will reduce the operating cost substantially.
- Relative equipment cost for automating heavy vehicles is far less than that for passenger vehicles.
- Automation of heavy vehicles will have a significant impact on the overall safety of the automated guidance system. Trucking is a tedious job and automation will contribute positively to reducing stress and therefore increase safety.

In this regard, commercial heavy vehicles will gain significant benefits from Advanced Vehicle Control Systems (AVCS), and may actually become automated earlier than passenger vehicles due to economical considerations.

In this study, two kinds of control inputs will be used for lateral control of articulated heavy vehicles: the steering angle and braking forces of the wheels. We will primarily rely on the front wheel steering angle. A steering control algorithm using input/output linearization [32, 33] is designed as a baseline controller to achieve the lane following maneuver in AHS. However, as safety is always of primary concern in AHS, the braking on trailer units will also be investigated to enhance the stability of lateral motion. Specifically, braking forces can be independently distributed over the inner and outer tires of the trailer so that the relative yaw errors between the tractor and the trailer are reduced. Independent braking control has been investigated as a safety augmented system for light passenger vehicles [17, 18, 19] as well as for commercial heavy vehicles [20, 21] recently. The main control objective in these research is to assist the driver to stabilize the vehicle by applying additional differential braking force so that the directional response of the controlled vehicle is improved. In the context of Automated Highway Systems, however, differential braking control is designed to work with the steering controller in a coordinated manner. The proposed coordinated steering and braking control algorithm in the study utilizes the tractor front wheel steering and the braking force at each of the rear trailer wheels as control inputs. In designing the coordinated steering and braking control algorithm, we observe that the so called decoupling matrix for the system is singular; in other words, the vector relative degree is not well defined [32, 33]. Specifically, when differentiating the outputs, the steering input appears “earlier” than the braking torque input. Thus an input/output linearization scheme is not applicable. To overcome this difficulty, we use the braking force generated at the tire/ground interface as a virtual control input and then backstep to
determine the real braking torque applied at the wheel. The backstepping design methodology is originally introduced in the adaptive control theory to systematically and recursively construct the feedback control law, the parameter adaptation law and the associated Lyapunov function for a class of nonlinear systems satisfying some structured properties, which are known as strict feedback conditions [24, 26]. This design methodology applied to multivariable nonlinear systems whose vector relative degree are not well defined is presented in [27].

The organization of this paper is as follows. In section 2, the system model is presented. The transformation relationships between the road reference coordinate and the vehicle unsprung mass reference coordinate are explored and will be used to obtain control models. In section 3, a steering control model is formulated. Based on this model, a baseline steering controller is designed. A steering and braking control model is formulated and the coordinated steering and braking control algorithm is designed in section 4. Conclusions are given in the last section.

2. SYSTEM DESCRIPTION

Due to the popularity of the tractor-semitrailer type commercial heavy-duty vehicle in the North American region, we will use it as the benchmark vehicle in our study. Two types of dynamic models are developed for the study of lateral control of tractor-semitrailer vehicles in AHS: a complex simulation model and two simplified control models. A nonlinear complex model is developed to simulate the dynamic responses of tractor-semitrailer vehicles and will be exploited to evaluate the effectiveness of lateral control algorithms. The main distinction between this complex model and those in the literatures is that the vehicle sprung mass dynamics is derived by applying Lagrange’s equations (see [28, 29] for example). This approach has an advantage over a Newtonian mechanics formulation in that it eliminates the holonomic constraint at the fifth wheel of the tractor-semitrailer vehicle by choosing the articulation angle as a generalized coordinate. Since there is no constraint involved in the model, it is easier for both designing control algorithms and solving the differential equations numerically. The second type of dynamic models are represented by two simplified lateral control models: one for steering control and the other for coordinated steering and independent braking control.

2.1. Complex model

In the development of the complex simulation model, a coordinate system is introduced to describe the motion of the tractor-semitrailer type commercial heavy-duty vehicles. Based on the definition of the coordinate system, a set of equations describing the sprung mass dynamics are derived by using Lagrange’s mechanics. In conjunction with the equations of the unsprung mass dynamics, the expression of the generalized
force corresponding to each generalized coordinate is obtained. To complete the development of the complex model, the tire model developed by Baraket and Fancher [30] and a simplified suspension model are utilized.

Definition of coordinate system
A coordinate system is defined to precisely describe the translational and rotational motion of a tractor-semitrailer vehicle. In Figure 1, \( X_n, Y_n, Z_n \) is the inertial reference coordinate; and \( X_{u1}Y_{u1}Z_{u1} \) is the tractor unsprung mass coordinate. There is relative translational motion in the \( X-Y \) plane and relative rotational yaw motion in the direction of \( Z \) axis between \( X_nY_nZ_n \) and \( X_{u1}Y_{u1}Z_{u1} \). \( X_{s1}Y_{s1}Z_{s1} \) is the tractor sprung mass coordinate, which is body-fixed at the tractor center of mass. Coordinate \( X_{s1}Y_{s1}Z_{s1} \) has roll motions relative to coordinate \( X_{u1}Y_{u1}Z_{u1} \). These two coordinates, \( X_{u1}Y_{u1}Z_{u1} \) and \( X_{s1}Y_{s1}Z_{s1} \), can be used to completely describe the tractor translational and rotational motions relative to the inertial coordinate \( X_nY_nZ_n \). Similarly, an unsprung mass coordinate \( X_{s2}Y_{s2}Z_{s2} \) is introduced to describe the motion for the trailer. The state variables defined by the coordinate system and notations of vehicle parameters for the tractor-semitrailer defined are listed in Appendix A, Tables 1 and 2, respectively. The generalized coordinate vector is defined by \( q = [x_n, y_n, \phi, \epsilon_1, \epsilon_f]^T \).

Fig. 1. Coordinate system to describe the vehicle motion.

Equations of motion
Having completely defined the coordinate system, we can calculate the kinetic energy, \( T \), and potential energy, \( V \), of the vehicle. The equations of motion can then be derived.
by applying Lagrange’s equations,
\[
\frac{d}{dt} \frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} = F_g,
\]
where \( L = T - V \) is the Lagrangian and \( F_g \) is the generalized force. The equations of motion in the unsprung mass coordinate are listed in Appendix B.

**Generalized forces**

The external forces acting on the vehicle body are from the tire/road interface and suspensions. In applying Lagrange’s equations, we need to calculate each generalized force corresponding to each generalized coordinate explicitly: i.e. we want to transform the external forces in the Cartesian coordinate to the generalized force corresponding to each generalized coordinate. Figure 2 shows the definition of the external forces at wheel \( i, i = 1...6 \), in the Cartesian coordinate, where the longitudinal force, \( F_{ai} (i = 1, 2, 3, 4) \), is in the direction of \( X_u \) axis, the lateral force, \( F_{bi} (i = 1, 2, 3, 4) \), is in the direction of \( Y_u \) axis and the normal force, \( F_{pi} \), is in the direction of \( Z_u \) axis.

![Tire force in the Cartesian coordinate.](image)

The tire forces in the Cartesian axis directions (Xn and Yn) are

\[
F_{xni} = F_{ai} \cos \epsilon_1 - F_{bi} \sin \epsilon_1 \quad (i = 1 \sim 4), \quad F_{xni} = F_{ai} \cos(\epsilon_1 + \epsilon_f) - F_{bi} \sin(\epsilon_1 + \epsilon_f) \quad (i = 5, 6)
\]

\[
F_{ymi} = F_{ai} \sin \epsilon_1 - F_{bi} \cos \epsilon_1 \quad (i = 1 \sim 4), \quad F_{ymi} = F_{ai} \sin(\epsilon_1 + \epsilon_f) - F_{bi} \cos(\epsilon_1 + \epsilon_f) \quad (i = 5, 6)
\]

Deriving the position vector in Cartesian coordinates of each wheel in terms of \( q \) and \( z_0 \) (the height of the roll center),

\[
P_{wi} = \begin{bmatrix} f_{wx1}(x_n, y_n, \phi, \epsilon_1, \epsilon_f, z_0) \\ f_{wy1}(x_n, y_n, \phi, \epsilon_1, \epsilon_f, z_0) \\ f_{wz1}(x_n, y_n, \phi, \epsilon_1, \epsilon_f, z_0) \end{bmatrix}
\]
we can calculate the generalized forces as

\[
F_{gi} = \sum_{j=1}^{6} F_{xnj} \frac{\partial f_{wxj}}{\partial q_i} + \sum_{j=1}^{6} F_{ynj} \frac{\partial f_{wyj}}{\partial q_i} + \sum_{j=1}^{6} F_{pj} \frac{\partial f_{wzj}}{\partial q_i}
\]  

(3)

where \( q_i \in \{x_n, y_n, \phi, \epsilon_1, \epsilon_f\} \). The explicit expressions for the generalized forces are listed in Appendix C.

Tire model

Since all forces acting on the vehicle body are from tire/road interfaces and through the suspension system of a vehicle, an accurate tire force model to predict tire traction/baking force and cornering force is crucial in modeling of the vehicle. We utilize the heavy vehicle’s tire model developed by Baraket and Fancher [30], which accounts for the influence of pavement and tire conditions such as tire thread depth on frictional characteristics.

Suspension model

By far the majority of commercial vehicle suspensions employ the leaf spring [31] as the vertically compliant element. For the sake of simplicity, instead of using experimental suspension data, we will adopt an analytical approach to model the suspension as the combination of a nonlinear spring and a damper element. As shown in Figure 3, the vertical force acting on the vehicle sprung mass through the suspension system is equal to the static equilibrium force plus the perturbation force, which is denoted as \( F_s \), from the spring equilibrium point. The perturbation force can be modeled as

\[
F_{si} = \begin{cases} 
K_{f1} e_i + K_{f2} e_i^2 + D_f \dot{e}_i & \text{for } i = 1, 2 \\
K_{r1} e_i + K_{r2} e_i^2 + D_r \dot{e}_i & \text{for } i = 3, 4 \\
K_{t1} e_i + K_{t2} e_i^2 + D_t \dot{e}_i & \text{for } i = 5, 6 
\end{cases}
\]  

(4)

where \( K_{f1} \) and \( K_{f2} \) are parameters of the tractor front spring, \( K_{r1} \) and \( K_{r2} \) are parameters of the tractor rear spring, \( K_{t1} \) and \( K_{t2} \) are parameters of the trailer spring, \( D_f, D_r \), and \( D_t \) are parameters for dampers, and \( e_i \) is the deflection of the \( i \)-th spring from its equilibrium position. The expressions of the deflections in terms of the generalized coordinates are given in Appendix D.

Fig. 3. Suspension model.
2.2. Model verification

To verify the effectiveness of the complex model, simulation results of the complex vehicle model will be compared with the open loop experimental results obtained from field tests. The test vehicle is a class 8 tractor- semitrailer truck. The test truck was operated under fixed speed cruise control and a step steering command was given manually by the driver. The radius of curvature of the test track is approximately 80 meters. Measured signals for the handling tests include lateral acceleration, yaw rate, roll angle of the sprung mass, articulation angle between the tractor and the semitrailer, and the front wheel steering angle. In order to compare the simulation results of the complex vehicle model with the test vehicle, the front wheel steering angle which is recorded during experiments is used as the steering input for the simulation model. Furthermore, simulations are performed using the test vehicle parameters listed in Tables 3, 4 and 4 in Appendix E. Some of the parameters are measured values and some are estimated values. Simulation results of the complex model and the experimental results of the test vehicle are compared in Figures 4, 5, 6 and 7. In general, the predicted simulation results agree well with the field test data. We observe that the predicted response of the articulation angle between the tractor and the trailer is slower than the actual response. The discrepancies between the predicted responses and the test results may be attributed to:

1. the assumption that the roll angles of both the tractor and the trailer are the same,
2. some unknown vehicle parameters, e.g., the moment of inertia, tire cornering stiffness, the height of the roll center and the height of the vertical C.G.,
3. effects of dual tires and tandem axes, which impose nonholonomic constraints on the vehicle motion,
4. unmodeled dynamics, including roll steer and chassis compliance effect,
5. sensor calibration errors in instrumentation.

2.3. Road reference frame

The vehicle model was derived with respect to the unsprung mass reference frame in section 2.1. Since the main objective for lateral control of automated vehicles is to follow the road, the description of the relative position and the relative orientation of the controlled vehicle with respect to the road centerline need to be given explicitly. To this end, the road reference coordinate \( O_rX_rY_r \) in Figure 8 is naturally introduced to describe tracking errors of the vehicle with respect to the road centerline. The road reference frame is defined such that the \( X_r \) axis is tangent to the road centerline and the \( Y_r \) axis passes through the origin of the vehicle unsprung mass coordinate. Once the road reference frame is defined, the vehicle model with respect to the road reference frame can be obtained by state variable transformation from the vehicle model with respect to the unsprung mass reference frame.
Fig. 4. Step input response with the longitudinal vehicle speed 30 MPH, solid line: experiment, dashdot line: simulation

Fig. 5. Step input response with the longitudinal vehicle speed 35 MPH, solid line: experiment, dashdot line: simulation

Fig. 6. Step input response with the longitudinal vehicle speed 40 MPH, solid line: experiment, dashdot line: simulation
Fig. 7. Step input response with the longitudinal vehicle speed 46 MPH, solid line: experiment, dashdot line: simulation

Fig. 8. Unsprung mass and road reference coordinates.
From the definition of two reference coordinates in Figure 8, we have
\[ \dot{\epsilon}_1 = \dot{\epsilon}_r + \dot{\epsilon}_d \]  
(5)
and
\[ \ddot{\epsilon}_1 = \ddot{\epsilon}_r + \ddot{\epsilon}_d \]  
(6)
where \( \dot{\epsilon}_d \) is the vehicle’s desired yaw rate, which is equal to the yaw rate of the road centerline reference frame, and \( \dot{\epsilon}_r \) is the relative yaw rate of the vehicle unsprung mass reference frame with respect to the road centerline reference frame. Furthermore, by neglecting third and higher order terms, we obtain the transformation between unsprung mass coordinate and road reference coordinate as
\[ \dot{x}_u \simeq \dot{x}_r - y_r \dot{\epsilon}_d + y_r \epsilon_r \]  
(7)
\[ \dot{y}_u \simeq \dot{y}_r - \dot{x}_r \epsilon_r. \]  
(8)
\[ \ddot{x}_u \simeq \ddot{x}_r - y_r \ddot{\epsilon}_d - y_r \dot{\epsilon}_d + y_r \dot{\epsilon}_r + \ddot{y}_r \epsilon_r - \dot{x}_r \epsilon_r \epsilon_r. \]  
(9)
\[ \ddot{y}_u \simeq \ddot{y}_r - \dot{x}_r \epsilon_r - \ddot{x}_r \epsilon_r. \]  
(10)
Eqs. (5), (6), (7), (8), (9) and (10) will be used to formulate lateral control models in sections 3 and 4.

3. STEERING CONTROL

3.1. Steering control model (SIM1)

The steering control model will be constructed in two steps. First, a 3 d.o.f. (6 states) model is simplified from the complex model. Next, the simplified model is transformed with respect to the road reference coordinate, which is discussed in section 2.3. For the nomenclature of the simplified models, refer to Table 1 in Appendix A.

Model simplification

For the purpose of lateral guidance, the following assumptions are made to simplify the complex model to one with only lateral and yaw dynamics.

- The roll motion is negligible.
- The longitudinal acceleration \( \ddot{x}_r \) is small.
- The relative yaw angle \( \epsilon_r \) of the tractor with respect to the road centerline is small.
• The relative yaw angle $\epsilon_f$ of the tractor and the trailer is small.
• Tire slip angles of the left and the right wheels are the same.
• Tire longitudinal and lateral forces are represented by the linearized tire model.

By using the above assumptions, the complex vehicle model summarized in Appendix B can be simplified as

\[
(m_1 + m_2)\ddot{y}_u - m_2(d_1 + d_3)\ddot{e}_1 - m_2d_3\ddot{\epsilon}_f + (m_1 + m_2)\dddot{\epsilon}_u \dddot{\epsilon}_1 = F_{b1} + F_{b2} + F_{b3} + F_{b4} + F_{b5} + F_{b6},
\]

\[
-\frac{m_2(d_1 + d_3)\ddot{y}_u + (I_{z1} + I_{z2} + m_2(d_1 + d_3)^2)\dddot{\epsilon}_1}{d_2} - m_2d_3\dddot{\epsilon}_f - m_2(d_1 + d_3)\dddot{\epsilon}_u \dddot{\epsilon}_1 = (F_{b1} + F_{b2})l_1 - (F_{b3} + F_{b4})l_2 - (F_{b5} + F_{b6})(d_1 + l_3)
\]

\[
+ (F_{a2} - F_{a1})\frac{T_{\alpha_{\omega1}}}{r} + (F_{a4} - F_{a3})\frac{T_{\alpha_{\omega3}}}{r} + (F_{a6} - F_{a5})\frac{T_{\alpha_{\omega5}}}{r},
\]

and

\[
-\frac{m_2d_3\ddot{y}_u + (I_{z2} + m_2d_3^2 + m_2d_1d_3)\dddot{\epsilon}_1 + (I_{z2} + m_2d_3^2)\dddot{\epsilon}_f - m_2d_3\dddot{\epsilon}_u \dddot{\epsilon}_1}{d_2} = -(F_{b5} + F_{b6})l_3 + (F_{a6} - F_{a5})\frac{T_{\alpha_{\omega5}}}{r}.
\]

To obtain the steering control model (SIM1), we notice that longitudinal tire forces, $F_{ai}$, in (11), (12) and (13) are zero under no braking and lateral tire forces, $F_{bi}$, can be represented by the linearized tire model,

\[
F_{bi} = \begin{cases} 
C_{\alpha_{\omega}}\alpha_f & \text{for } i = 1, 2, \\
C_{\alpha_{\omega}}\alpha_r & \text{for } i = 3, 4, \\
C_{\alpha_{\omega}}\alpha_t & \text{for } i = 5, 6
\end{cases}
\]

where lateral slip angles $\alpha_f$, $\alpha_r$, and $\alpha_t$ are

\[
\alpha_f \simeq \delta - \frac{\dot{y}_u + l_1\dot{\epsilon}_1}{x_u},
\]

\[
\alpha_r \simeq -\frac{\dot{y}_u - l_2\dot{\epsilon}_1}{x_u},
\]

and

\[
\alpha_t \simeq \frac{\dot{y}_u - d_1\dot{\epsilon}_1 - l_3(\dot{\epsilon}_1 + \dot{\epsilon}_f)}{x_u} + \epsilon_f,
\]

respectively. Substituting $F_{bi}$ in (14) into the simplified vehicle model (11), (12) and (13), we obtain the control model (SIM1) as

\[
M\ddot{q} + C(\dot{q}, \dot{q}) + D\ddot{q} + Kq = F\dot{\delta},
\]

where

\[
q = [y_u, \epsilon_1, \epsilon_f]^T
\]
is the generalized coordinate vector,
\[ M = \begin{bmatrix}
  -m_1 + m_2 & -m_2(d_1 + d_3) & -m_2d_3 \\
  -m_2(d_1 + d_3) & I_{x2} + I_{x3} + m_2(d_1 + d_3)^2 & I_{x2} + m_2d_1d_3 \\
  -m_2d_3 & I_{x2} + m_2d_1d_3 & I_{x2} + m_2d_3^2
\end{bmatrix} \]

is the inertial matrix,
\[ C(q, \dot{q}) = \begin{bmatrix}
  (m_1 + m_2)\dot{x}_u \\
  -m_2(d_1 + d_3)\dot{x}_u \dot{\epsilon}_1 \\
  -m_2d_3\dot{x}_u \dot{\epsilon}_3
\end{bmatrix} \]
is the vector of the Coriolis and Centrifugal forces,
\[ D = \frac{2}{\rho} \begin{bmatrix}
  C_{af} + C_{ar} + C_{at} & l_1C_{af} - l_2C_{ar} - (l_3 + d_1)C_{at} & -l_3C_{at} \\
  l_1C_{af} - l_2C_{ar} - (l_3 + d_1)C_{at} & l_1^2C_{af} + l_2^2C_{ar} + (l_3 + d_1)^2C_{at} & l_3(l_3 + d_1)C_{at} \\
  -l_3C_{at} & l_3(l_3 + d_1)C_{at} & l_3^2C_{at}
\end{bmatrix} \]
is the damping matrix,
\[ K = \begin{bmatrix}
  0 & 0 & -2C_{at} \\
  0 & 0 & 2(l_3 + d_1)C_{at} \\
  0 & 0 & 2l_3C_{at}
\end{bmatrix} \]
is the potential matrix, and the vector \( F \in \mathbb{R}^{3 \times 1} \) is
\[ F = 2 \ C_{af} \cdot [1, \ l_1, \ 0]^T. \]

Eq. (15) represents the simplified vehicle model with respect to the unsprung mass reference coordinate.

**Control model with respect to the road reference frame**

Recall from section 2.3 that state variables with respect to the unsprung mass reference frame can be transformed into state variables with respect to the road reference frame by
\[ \dot{y}_u = \dot{y}_r - \dot{x}_r \dot{\epsilon}_r, \]  
\[ \ddot{y}_u = \ddot{y}_r - \ddot{x}_r \dot{\epsilon}_r - \dddot{x}_r \epsilon_r, \]  
\[ \dot{\epsilon}_1 = \dot{\epsilon}_r + \dot{\epsilon}_d, \]  
\[ \ddot{\epsilon}_1 = \ddot{\epsilon}_r + \dddot{\epsilon}_d. \]

By the assumptions that the longitudinal acceleration \( \dddot{x}_r \) and the relative yaw angle \( \epsilon_r \) are small, their product in (17) can be neglected. Substituting the state variable transformation equations (16), (17), (18) and (19) into the control model (15), we obtain
\[ M\ddot{q}_r + \Phi(q_r, \dot{q}_r, \dot{\epsilon}_d, \ddot{\epsilon}_d) = F\delta, \]  
where
\[ q_r = [y_r, \epsilon_r, \epsilon_1]^T \]
is the vector of state variables with respect to road centerline and is defined in Table 1, 
\( \Phi(q_r, \dot{q}_r, \epsilon_d, \dot{\epsilon}_d) \in R^{3 \times 1} \) is the vector with its components

\[
\Phi(q_r, \dot{q}_r, \epsilon_d, \dot{\epsilon}_d)(1) = \frac{2}{x_f}(C_{af} + C_{ar} + C_{at})(\dot{y}_r - \dot{x}_r 
+ (l_1 C_{af} - l_2 C_{ar} - (l_3 + d_1) C_{at})(\dot{\epsilon}_r + \dot{\epsilon}_d) - l_3 C_{at} \dot{\epsilon}_f)
- 2 C_{at} \dot{\epsilon}_f (m_1 + m_2) \dot{x}_r \dot{\epsilon}_d - m_2 (d_1 + d_3) \dot{\epsilon}_d
\]

\[
\Phi(q_r, \dot{q}_r, \epsilon_d, \dot{\epsilon}_d)(2) = \frac{2}{x_f}(-l_3 C_{at} (\dot{y}_r - \dot{x}_r \epsilon_r) + (l_1 C_{af} + l_2 C_{ar} + (l_3 + d_1)^2 C_{at})(\dot{\epsilon}_r + \dot{\epsilon}_d) + l_3 (l_3 + d_1) C_{at} \dot{\epsilon}_f)
+ 2 l_3 (l_3 + d_1) C_{at} \epsilon_f - m_2 (d_1 + d_3) \dot{x}_r \dot{\epsilon}_d + (l_2 + l_1 + l_3 + m_2 (d_1 + d_3)) \dot{\epsilon}_d.
\]

and

\[
\Phi(q_r, \dot{q}_r, \epsilon_d, \dot{\epsilon}_d)(3) = \frac{2}{x_f}(-l_3 C_{at} (\dot{y}_r - \dot{x}_r \epsilon_r) + l_3 (l_3 + d_1) C_{at} (\dot{\epsilon}_r + \dot{\epsilon}_d) + l_3^2 C_{at} \dot{\epsilon}_f)
+ 2 l_3 (l_3 + d_1) C_{at} \epsilon_f - m_2 d_3 \dot{x}_r \dot{\epsilon}_d + (l_2 + m_2 d_3 + m_2 d_1 d_3) \dot{\epsilon}_d.
\]

Eq. (20) is the simplified model which will be used to design the steering control
algorithm in section 3 for the lane following maneuver.

**Linear analysis of the control model**

The linearized model (20) can be expanded as

\[
M \ddot{q}_r + D \dot{q}_r + K q_r = F \delta + E_1 \dot{\epsilon}_d + E_2 \dot{\epsilon}_d,
\]  

(21)

where \( \dot{\epsilon}_d \) and \( \dot{\epsilon}_d \) are exogenous inputs representing the disturbance effects on curved
roads. Two interesting properties are observed from this linearized model.

1. \( M \) is a symmetric positive definite matrix which contains the inertial information
of the vehicle system.

2. The \( D \) matrix can be interpreted as a damping matrix. Each element of the \( C \)
matrix contains the tire cornering stiffness. If the cornering stiffness is small, the
vehicle system will become lightly damped and more oscillatory. For example,
if the vehicle is operated on an icy road, the vehicle stability will decrease. We
also see that the vehicle longitudinal velocity \( \dot{x}_r \) appears in the denominator of
the damping matrix. Therefore the system damping is inversely proportional
to the vehicle longitudinal velocity, which also agrees with our physical experience.

The first property that \( M \) is a positive definite matrix will be exploited in synthesizing
the input-output linearizing controller.

### 3.2 Controller design

In this section a steering control algorithm will be designed by applying the input-
output linearization scheme [32, 33]. The steering control model developed in section
3.1 is

\[
M \ddot{q}_r + \Phi(q_r, \dot{q}_r, \epsilon_d) = F \delta
\]  

(22)
where $M$ is the inertial matrix and can be partitioned into four blocks as

$$
M = \begin{bmatrix}
    m_1 + m_2 & -m_2(d_1 + d_3) & -m_2 d_3 \\
    -m_2(d_1 + d_3) & I_{x1} + I_{x2} + m_2(d_1 + d_3)^2 & I_{x2} + m_2 d_3^2 + m_2 d_1 d_3 \\
    -m_2 d_3 & I_{x2} + m_2 d_3^2 + m_2 d_1 d_3 & I_{x2} + m_2 d_3^2 \\
\end{bmatrix}
$$

Since the matrix $M$ is positive definite, both $M_{11}$ and $M_{22}$ are also positive definite. The control model in (22) can be divided into two subsystems:

$$
M_{11} \ddot{y}_r + M_{12} \begin{bmatrix} \dot{\epsilon}_r \\ \dot{\epsilon}_f \end{bmatrix} + \Phi_1 = C_{\alpha f} \delta \quad (23)
$$

and

$$
M_{21} \ddot{y}_r + M_{22} \begin{bmatrix} \dot{\epsilon}_r \\ \dot{\epsilon}_f \end{bmatrix} + \begin{bmatrix} \Phi_2 \\ \Phi_3 \end{bmatrix} = \begin{bmatrix} l_1 C_{\alpha f} \\ 0 \end{bmatrix} \delta. \quad (24)
$$

Notice that the second subsystem (24) can be rewritten as

$$
\begin{bmatrix} \dot{\epsilon}_r \\ \dot{\epsilon}_f \end{bmatrix} = M_{22}^{-1} \left\{ -M_{21} \ddot{y}_r - \begin{bmatrix} \Phi_2 \\ \Phi_3 \end{bmatrix} + \begin{bmatrix} l_1 C_{\alpha f} \\ 0 \end{bmatrix} \delta \right\}. \quad (25)
$$

Substituting Eq. (25) into Eq. (23), we obtain the input($\delta$)-output($\ddot{y}_r$) dynamics as

$$
\ddot{y}_r + \Phi = \bar{K} \delta, \quad (26)
$$

where

$$
\bar{M}_{11} = M_{11} - M_{12} M_{22}^{-1} M_{21}, \quad (27)
$$

$$
\bar{\Phi} = \Phi_1 - M_{12} M_{22}^{-1} \begin{bmatrix} \Phi_2 \\ \Phi_3 \end{bmatrix}, \quad (28)
$$

and

$$
\bar{K} = C_{\alpha f} - M_{12} M_{22}^{-1} \begin{bmatrix} l_1 C_{\alpha f} \\ 0 \end{bmatrix}. \quad (29)
$$

Note that

$$
\bar{M}_{11} = T^T M T, \quad (30)
$$

and

$$
T = \begin{bmatrix}
    I \\
    -M_{22}^{-1} M_{21} \\
\end{bmatrix}, \quad (31)
$$

which is a full rank matrix. By the facts that the matrix $M$ is positive definite and that the matrix $T$ has a full rank, we conclude that $\bar{M}_{11}$ is also positive definite. If $\bar{K} \neq 0$, we can choose the linearizing control law

$$
\bar{K} \delta = \bar{M}_{11} v + \bar{\Phi} \quad (32)
$$

With this linearizing control law, the subsystems (23) and (24) become

$$
\ddot{y}_r = v \quad (33)
$$
and
\[
M_{22} \begin{bmatrix} \dot{e}_r \\ \dot{e}_f \end{bmatrix} + \begin{bmatrix} \Phi_2 \\ \Phi_3 \end{bmatrix} = \begin{bmatrix} l_1 C_{\alpha f} \\ 0 \end{bmatrix} (M_{11} - M_{21}) v + \begin{bmatrix} l_1 C_{\alpha f} \\ 0 \end{bmatrix} \Phi
\]
(34)

Furthermore, by choosing
\[
v = k_d \dot{y}_r + k_p y_r
\]
(35)

the output \( y_r \) converges to zero asymptotically.

### 3.3. Simulation results

The simulations are conducted using the complex vehicle model and the vehicle parameters listed in Table 3. The simulation scenario we used is depicted in Figure 9. The tractor-semitrailer vehicle travels along a straight roadway with an initial lateral displacement of 15 cm and enters a curved section with a radius of curvature of 450 m at time \( t = 5 \) sec. Figures 10 and 11 show the simulation results of the input-output linearization controller at a vehicle speed of 60 MPH. We see that the lateral tracking error converges to zero asymptotically while the yaw angle of the tractor and the relative yaw angle of the trailer are small.

![Simulation scenario](image)

Fig. 9. Simulation scenario.

### 4. COORDINATED STEERING AND BRAKING CONTROL

#### 4.1. Steering and braking control model (SIM2)

In this section, the control model developed in section 3.1 is reformulated to include the left and right braking forces at the trailer as another two control inputs. Assuming that
Fig. 10. Input/output linearization control.

Fig. 11. Input/output linearization control.
the longitudinal tire forces on the tractor are zero, i.e., $F_{a1} = F_{a2} = F_{a3} = F_{a4} = 0$,
we obtain the simplified model as

$$M \ddot{q} + C(q, \dot{q}) + D\dot{q} + Kq = H \cdot U$$  \hspace{1cm} (36)

where $M$, $C(q, \dot{q})$, $D$ and $K$ are the same as in SIM1, (15), and $H$ and $U$ are

$$H = \begin{bmatrix} 2C_{\alpha f} & 0 \\ 2lC_{\alpha f} & \frac{T_w}{2} \end{bmatrix}$$

and

$$U = \begin{bmatrix} \delta \\ F_{a6} - F_{a5} \end{bmatrix} \equiv \begin{bmatrix} \delta \\ T \end{bmatrix}$$  \hspace{1cm} (37)

respectively. Eq. (36) is the model with respect to the unsprung mass reference frame.

Parallel to the development of SIM1 in section 3.1 and by using the coordinate transformations (16) (17) (18) and (19), the steering and braking control model SIM2 with respect to the road reference frame is obtained

$$M(q_r)\ddot{q}_r + \Phi(q_r, \dot{q}_r, \epsilon_d, \dot{\epsilon}_d) = H \cdot U$$  \hspace{1cm} (38)

Notice that $F_{a5}$ and $F_{a6}$ in (37) stand for the longitudinal forces at the left and right wheels of the trailer. Thus $T$ is the differential force acting on the trailer. We denote the longitudinal force $F_{ai} < 0$ when it is a braking force and $F_{ai} > 0$ when it is a traction force. In fact, the control inputs $F_{a5}$ and $F_{a6}$ at the wheels of the trailer can only be negative, i.e., we can use only braking instead of traction. However, the differential force $T$ can be both positive and negative. Furthermore, the braking forces $F_{a5}$ and $F_{a6}$ are determined by the tire force model and are functions of the tire slip ratio. Specifically, as shown in Figure 12, the wheel dynamics are

$$I_w \dot{\omega}_i = -F_{ai}r + \tau_i$$  \hspace{1cm} (39)

where $\omega_i$ is the angular velocity of the wheel, $F_{ai}$ is the braking force generated at the tire/ground interface, and $\tau_i$ is the braking torque applied at the braking disk of the wheel. The tire slip ratio is defined as

$$\lambda_i = \frac{\omega_i r - V_i}{V_i},$$  \hspace{1cm} (40)

where $V_i$ is the longitudinal velocity at $i$–th wheel. The braking force is

$$F_{ai} = C_l \lambda_i$$  \hspace{1cm} (41)

Eq. (38) as well as Eqs. (39) (40) and (41) will be used to design the coordinated steering and braking control algorithm in section 4.
4.2. Controller design

In this section, a coordinated steering and braking control algorithm will be designed. We propose to use not only the tractor front wheel steering input but also the trailer unilateral tire braking to provide the differential torque for directly controlling the trailer yaw motion. The control algorithm will be designed in two steps. In the first step, we assume that the differential force $T$ is control input. Then the desired steering command $\delta_d$ and the desired differential braking force $T_d$ are determined by input/output linearization scheme. By the nature of unilateral braking, if $T_d > 0$, we have $F_{a5d} = -T_d$ and $F_{a6d} = 0$. On the other hand, if $T_d < 0$, we have $F_{a5d} = 0$ and $F_{a6d} = T_d$. In the second step, the required braking torques $\tau_5$ and $\tau_6$ are determined to generate the desired braking forces $F_{a5d}$ and $F_{a6d}$ by utilizing backstepping design methodologies.

Step 1

First, we define the first system output $e_1$ as the lateral tracking error

$$e_1 = y_r$$

and the second output $e_2$ as the deviations from the desired articulation angle between the tractor and the trailer

$$e_2 = \epsilon_f - \epsilon_{fd}$$

where $\epsilon_{fd}$ is the steady state articulation angle and can be determined by analyzing the kinematic model as well as the dynamic model. Differentiating $e_1$ and $e_2$ twice, we obtain

$$\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \end{bmatrix} = \begin{bmatrix} M^{-1}(1) \\ M^{-1}(3) \end{bmatrix} C(q, \dot{q}, \ddot{q}) + \begin{bmatrix} M^{-1}(1) \\ M^{-1}(3) \end{bmatrix} HU - \begin{bmatrix} 0 \\ \dot{\epsilon}_{fd} \end{bmatrix}$$

where the number $i$ in the parenthesis $M^{-1}(i)$ denotes the $i$-th row of the $M^{-1}$ matrix. For notational simplicity, we define

$$J = \begin{bmatrix} M^{-1}(1) \\ M^{-1}(3) \end{bmatrix} H$$
If the matrix $J$ is nonsingular, we can choose the control input $U$ as

$$U = J^{-1} \left\{ - \left[ \begin{array}{c} M^{-1}(1) \
M^{-1}(3) \end{array} \right] C(\dot{q}, \dot{\epsilon}_d, \epsilon_d) + \left[ \begin{array}{c} 0 \\ \dot{\epsilon}_f \end{array} \right] - KD \left[ \begin{array}{c} e_1 \\ e_2 \end{array} \right] - KP \left[ \begin{array}{c} e_1 \\ e_2 \end{array} \right] \right\} \quad (46)$$

This control law cancels the system nonlinearities and inserts the desired error dynamics. Thus the closed loop system becomes

$$\left[ \begin{array}{c} \ddot{e}_1 \\ \ddot{e}_2 \end{array} \right] + KD \left[ \begin{array}{c} \dot{e}_1 \\ \dot{e}_2 \end{array} \right] + KP \left[ \begin{array}{c} e_1 \\ e_2 \end{array} \right] = 0. \quad (47)$$

**Step 2**

In Step 1 we regard $T$ as a real control input; then the desired steering command and the desired differential braking forces $T_d$ are set in (46). In this step we will ‘back-step’ to determine the braking torques $\tau_5$ and $\tau_6$ on the trailer’s left and right wheels.

Combining equations (39), (40) and (41) of wheel dynamics, we obtain

$$\dot{F}_{ai} = C_{li} \left( \frac{\dot{\lambda}_i}{\dot{V}_i} + \frac{2\lambda_i}{\dot{V}_i} \dot{\omega}_i \right) = C_{li} \left( -\frac{\dot{\lambda}_i}{\dot{V}_i} + \frac{r}{\dot{V}_i} \dot{V}_i - C_{li} \lambda_i r + \tau_i \right) \quad (48)$$

Thus the equations governing the vehicle dynamics and wheel dynamics are

$$\left[ \begin{array}{c} \dot{e}_1 \\ \dot{e}_2 \end{array} \right] = \left[ \begin{array}{c} M^{-1}(1) \\ M^{-1}(3) \end{array} \right] C(\dot{q}, \dot{\epsilon}_d, \epsilon_d) + J \cdot U \quad (49)$$

and

$$\dot{F}_{ai} = C_{li} \left( -\frac{\dot{\lambda}_i}{\dot{V}_i} + \frac{r}{\dot{V}_i} \dot{V}_i - C_{li} \lambda_i r + \tau_i \right) \quad (50)$$

Recall that $T = F_{a6} - F_{a5}$ and both $F_{a6}$ and $F_{a5}$ are negative. In this unilateral braking scheme, if $T_d$ determined in (46) is positive, we have $F_{a5d} = -T_d$ and $F_{a6d} = 0$. Thus the braking controller will apply the brake torque on the trailer left wheel. On the other hand, if $T_d$ is negative, we have $F_{a5d} = 0$ and $F_{a6d} = T_d$, so the braking controller will apply the brake torque on the trailer right wheel. From Eq. (46) in step 1, the control inputs are chosen as

$$\delta = \delta_d(q_r, \dot{q}_r, \dot{\epsilon}_d) \quad (51)$$

and

$$T = T_d(q_r, \dot{q}_r, \dot{\epsilon}_d) \quad (52)$$

such that the error dynamics becomes

$$\dot{e}_1 + k_{d1} \dot{e}_1 + k_{p1} e_1 = 0 \quad (53)$$

and

$$\ddot{e}_2 + k_{d2} \dot{e}_2 + k_{p2} e_2 = 0. \quad (54)$$
Note that $T$ is determined by the braking force $F_{ai}$, and that braking force $F_{ai}$ can be adjusted only through equation (50), i.e., the braking torque $\tau_{i}$ is the actual control input. Therefore, $T$ cannot be simply set to $T_d$ all the time, and $\tau_{i}$ must be adjusted so that the difference between $T_d$ and $T$ is brought to zero asymptotically. We define two new variables $\eta_1$ and $\eta_2$ as

$$\eta_1 = F_{a5} - F_{a5d}$$

and

$$\eta_2 = F_{a6} - F_{a6d},$$

where $\eta_1$ and $\eta_2$ represent the differences of the actual braking force and the desired braking force on the left and right trailer wheels, respectively. The differential torque $T$ can then be re-parameterized in terms of $T_d$, $\eta_1$ and $\eta_2$ as

$$T = T_d + \eta_2 - \eta_1.$$  

where the term $(\eta_2 - \eta_1)$ quantifies the deviation errors between the desired differential torque and actual differential torque generated by differential braking on the trailer. The braking controller needs to ensure that the deviation errors converge to zero asymptotically. Noting

$$\dot{\eta}_1 = \dot{F}_{a5} - \dot{F}_{a5d} = C_{lt} (\frac{\omega_{a5}}{\bar{v}_5} \dot{V}_5 + \frac{1}{C_{la5}} (C_{lt} \lambda_5 r + \tau_5)) - \dot{F}_{a5d}$$

and

$$\dot{\eta}_2 = \dot{F}_{a6} - \dot{F}_{a6d} = C_{lt} (\frac{\omega_{a6}}{\bar{v}_6} \dot{V}_6 + \frac{1}{C_{la6}} (C_{lt} \lambda_6 r + \tau_6)) - \dot{F}_{a6d},$$

we choose the braking control laws for the left and right wheels as

$$\tau_5 = C_{lt} \lambda_5 r + \frac{\lambda_{12}}{\bar{v}_5} (\frac{1}{C_{la5}} \dot{V}_5 + \frac{1}{C_{la5}} (F_{a5d} - k_1 \eta_1))$$

and

$$\tau_6 = C_{lt} \lambda_6 r + \frac{\lambda_{12}}{\bar{v}_6} (\frac{1}{C_{la6}} \dot{V}_6 + \frac{1}{C_{la6}} (F_{a6d} - k_2 \eta_2)).$$

It can be easily verified that the steering control law Eq. (51), which is developed in step 1, and the braking control laws Eqs. (60) and (61), which are designed in step 2, render the the closed-loop system dynamics as

$$\dot{e}_1 + k_{pd} e_1 + k_{p1} e_1 + J_{12} (\eta_2 - \eta_1) = 0,$$

$$\dot{e}_2 + k_{pd} e_2 + k_{p2} e_2 + J_{22} (\eta_2 - \eta_1) = 0,$$

$$\dot{\eta}_1 + k_1 \eta_1 = 0,$$

and

$$\dot{\eta}_2 + k_2 \eta_2 = 0.$$
where $J_{12}$ and $J_{22}$ are the (1, 2) and (2, 2) elements of the matrix $J$.

To show that the closed-loop system dynamics described by Eqs. (62), (63), (64), and (65) are stable, we define the state vector $(x_1, x_2, x_3, x_4)^T$ as $(e_1, \dot{e}_1, e_2, \dot{e}_2)^T$ and transform equations (62) and (63) to state space form. We obtain

$$
\frac{d}{dt} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ \eta_1 \\ \eta_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ k_{p1} & k_{d1} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & k_{p2} & k_{d2} & 0 & 0 \\ 0 & 0 & 0 & 0 & -k_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -k_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ \eta_1 \\ \eta_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ J_{12} \\ -J_{12} \end{bmatrix} \begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix}
$$

(66)

Then the overall closed-loop system can be rewritten as

$$
\frac{d}{dt} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ \eta_1 \\ \eta_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -k_{p1} & -k_{d1} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -k_{p2} & -k_{d2} & 0 & 0 \\ 0 & 0 & 0 & 0 & -k_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -k_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ \eta_1 \\ \eta_2 \end{bmatrix}
$$

(67)

We see that the overall system matrix can be divided by four blocks and the lower off-diagonal block is identically zero. Thus the eigenvalues of the overall system are the union of those of the block diagonal matrices. Since each block diagonal matrix is asymptotically stable, the overall system is asymptotically stable.

4.3. Simulation results

We use the same scenario shown in Figure 9. Vehicle longitudinal speed is 26.4 m/s (60 MPH). Figures 13, 14 and 15 show the simulation results of the coordinated steering and independent braking control. To better show the differential braking capacity on the trailer, we intentionally set the desired articulation angle $\epsilon_{fd}$ to zero in the simulation study. Notice also that in implementing this control algorithm, we impose upper and lower bounds on the braking torque input to avoid tire force saturation. Comparison of the steering control designed in section 3. and the coordinated steering and independent braking control is shown in Figure 16, from which we see that the peak trailer yaw errors are reduced from 2.64° to 0.97°. We also observe that the longitudinal velocity decreases when the independent braking control algorithm is activated. As we stated in section 3.1. that the system damping is inversely proportional to the longitudinal velocity, so a decrease of the longitudinal velocity will contribute to decreases of both tractor and trailer yaw errors. To see the effect caused only by the differential forces distribution over the inner and outer tires of the trailer, we assume that the longitudinal controller will give traction force commands on the tractor to counteract the braking forces on the trailer. Simulation results for this scenario is given in Figure 17, which shows that the trailer yaw errors is reduced from 2.64° to 1.13°. Recall from Table 3 that the length of the trailer is 9.65 m. Thus a decrease of 1.51° in yaw errors corresponds to a decrease of 25.4 cm in lateral tracking errors of the trailer.
Fig. 13. Input/output linearization control with trailer independent braking.

Fig. 14. Input/output linearization control with trailer independent braking.

Fig. 15. Input/output linearization control with trailer independent braking.
Fig. 16. Comparison of I/O linearization control with (solid line) and without (dashdot line) trailer independent braking.

Fig. 17. Comparison of I/O linearization control with (solid line) and without (dashdot line) trailer independent braking for constant longitudinal speed.
5. CONCLUSIONS

Two control algorithms for lateral guidance of tractor-semitrailer vehicles were designed. The first was a baseline steering control algorithm and the second was a coordinated steering and independent braking control algorithm. In the design of the second control algorithm, we utilized tractor front wheel steering angles and trailer independent braking forces to control the tractor and the trailer motion. The multivariable backstepping design methodology was utilized to determine the coordinated steering angle and braking torques on the trailer wheels. Simulations showed that both the tractor and the trailer yaw errors under coordinated steering and independent braking force control were smaller than those without independent braking force control.

REFERENCES

Appendix A: Nomenclature

Table 1. State variables.

<table>
<thead>
<tr>
<th>variable</th>
<th>description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x_n)</td>
<td>position of the tractor in (X_n) direction of the inertial coordinate (X_nY_nZ_n)</td>
</tr>
<tr>
<td>(\dot{x}_n)</td>
<td>velocity of the tractor in (X_n) direction of the inertial coordinate (X_nY_nZ_n)</td>
</tr>
<tr>
<td>(y_n)</td>
<td>position of the tractor in (Y_n) direction of the inertial coordinate (X_nY_nZ_n)</td>
</tr>
<tr>
<td>(\dot{y}_n)</td>
<td>velocity of the tractor in (Y_n) direction of the inertial coordinate (X_nY_nZ_n)</td>
</tr>
<tr>
<td>(y_r)</td>
<td>lateral displacement at the origin of the tractor</td>
</tr>
<tr>
<td>(\dot{y}_r)</td>
<td>lateral velocity at the origin of the tractor</td>
</tr>
<tr>
<td>(\epsilon_1)</td>
<td>tractor yaw angle with respect to inertial coordinate (X_nY_nZ_n)</td>
</tr>
<tr>
<td>(\dot{\epsilon}_1)</td>
<td>tractor yaw rate with respect to inertial coordinate (X_nY_nZ_n)</td>
</tr>
<tr>
<td>(\epsilon_r)</td>
<td>tractor relative yaw angle with respect to road centerline coordinate (X_rY_rZ_r)</td>
</tr>
<tr>
<td>(\dot{\epsilon}_r)</td>
<td>tractor relative yaw rate with respect to road centerline coordinate (X_rY_rZ_r)</td>
</tr>
<tr>
<td>(\phi)</td>
<td>tractor roll angle</td>
</tr>
<tr>
<td>(\dot{\phi})</td>
<td>rate of change of tractor roll angle</td>
</tr>
<tr>
<td>(\epsilon_f)</td>
<td>articulation angle between the tractor and the trailer</td>
</tr>
<tr>
<td>(\dot{\epsilon}_f)</td>
<td>rate of change of the articulation angle between the tractor and the trailer</td>
</tr>
<tr>
<td>(\rho)</td>
<td>radius of curvature of the road</td>
</tr>
<tr>
<td>(\dot{\epsilon}_d)</td>
<td>desired yaw rate set by the road and is equal to (\frac{1}{\rho})</td>
</tr>
<tr>
<td>(\delta)</td>
<td>tractor front wheel steering angle</td>
</tr>
<tr>
<td>(F_{a5})</td>
<td>braking force on the trailer left wheel</td>
</tr>
<tr>
<td>(F_{a6})</td>
<td>braking force on the trailer right wheel</td>
</tr>
<tr>
<td>(\tau_5)</td>
<td>braking torque on the trailer left wheel</td>
</tr>
<tr>
<td>(\tau_6)</td>
<td>braking torque on the trailer right wheel</td>
</tr>
<tr>
<td>(V_5)</td>
<td>longitudinal velocity at the trailer left wheel</td>
</tr>
<tr>
<td>(V_6)</td>
<td>longitudinal velocity at the trailer right wheel</td>
</tr>
<tr>
<td>(\lambda_i)</td>
<td>longitudinal slip ratio</td>
</tr>
<tr>
<td>(\alpha_i)</td>
<td>lateral slip angle</td>
</tr>
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</table>
Fig. 18. Schematic diagram of complex vehicle model.

Table 2. Parameters of complex vehicle model.

<table>
<thead>
<tr>
<th>parameter</th>
<th>description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_1$</td>
<td>tractor’s mass</td>
</tr>
<tr>
<td>$I_{x1}, I_{y1}, I_{z1}$</td>
<td>tractor’s moment of inertia</td>
</tr>
<tr>
<td>$m_2$</td>
<td>semitrailer’s mass</td>
</tr>
<tr>
<td>$I_{x2}, I_{y2}, I_{z2}$</td>
<td>semitrailer’s moment of inertia</td>
</tr>
<tr>
<td>$l_1$</td>
<td>distance between the tractor C.G. and front wheel axle</td>
</tr>
<tr>
<td>$l_2$</td>
<td>distance between tractor C.G. and real wheel axle</td>
</tr>
<tr>
<td>$l_3$</td>
<td>distance between joint (fifth wheel) and trailer real wheel axle</td>
</tr>
<tr>
<td>$d_1, d_2$</td>
<td>relative position between tractor’s C.G. to fifth wheel</td>
</tr>
<tr>
<td>$d_3, d_4$</td>
<td>relative position between semitrailer’s C.G. to fifth wheel</td>
</tr>
<tr>
<td>$T_{w1}$</td>
<td>tractor front axle track width</td>
</tr>
<tr>
<td>$T_{w2}$</td>
<td>tractor rear axle track width</td>
</tr>
<tr>
<td>$T_{w3}$</td>
<td>semitrailer rear axle track width</td>
</tr>
<tr>
<td>$h_2$</td>
<td>distance from tractor roll center to C.G.</td>
</tr>
<tr>
<td>$C_{\alpha f}$</td>
<td>cornering stiffness of tractor front wheel</td>
</tr>
<tr>
<td>$C_{\alpha r}$</td>
<td>cornering stiffness of tractor rear wheel</td>
</tr>
<tr>
<td>$C_{\alpha t}$</td>
<td>cornering stiffness of semitrailer rear wheel</td>
</tr>
<tr>
<td>$S_{x f}$</td>
<td>longitudinal stiffness of tractor front wheel</td>
</tr>
<tr>
<td>$S_{x r}$</td>
<td>longitudinal stiffness of tractor rear wheel</td>
</tr>
<tr>
<td>$S_{x t}$</td>
<td>longitudinal stiffness of semitrailer rear wheel</td>
</tr>
</tbody>
</table>
APPENDIX B: EQUATIONS OF MOTION

\[\begin{align*}
(m_1 + m_2)\ddot{x}_u + (m_1 h_2 \phi + m_2 (h_2 - d_2 + d_4) \phi + m_2 d_3 \sin \epsilon f) \dot{\epsilon} + m_2 d_4 \sin \epsilon f &= F_{y_u} \times \cos \epsilon f + F_{\dot{y}_u} \times \sin \epsilon f \\
&= (m_1 + m_2) (1 + \phi^2) \dot{y}_u - (m_1 h_2 + m_2 (h_2 - d_2 + d_4) - m_2 d_4 \sin \epsilon f) \phi + m_2 d_1 \cos \epsilon f (1 + \phi^2) \epsilon f + m_2 d_2 \cos \epsilon f \dot{\epsilon} f + m_2 d_3 \cos \epsilon f \phi f \\
&= -F_{y_u} \times \sin \epsilon f + F_{\dot{y}_u} \times \cos \epsilon f
\end{align*}\]

\[\begin{align*}
(m_1 + m_2)\ddot{x}_u + (m_1 h_2 \phi + m_2 (h_2 - d_2 + d_4) - m_2 d_3 \sin \epsilon f) \dot{\epsilon} + m_2 d_4 \sin \epsilon f &= F_{y_u} \times \cos \epsilon f + F_{\dot{y}_u} \times \sin \epsilon f \\
&= (m_1 + m_2) (1 + \phi^2) \dot{y}_u - (m_1 h_2 + m_2 (h_2 - d_2 + d_4) - m_2 d_3 \sin \epsilon f) \phi + m_2 d_1 \cos \epsilon f (1 + \phi^2) \epsilon f + m_2 d_2 \cos \epsilon f \dot{\epsilon} f + m_2 d_3 \cos \epsilon f \phi f \\
&= -F_{y_u} \times \sin \epsilon f + F_{\dot{y}_u} \times \cos \epsilon f
\end{align*}\]
APPENDIX C: GENERALIZED FORCES

\[ F_{g,x} = F_{x,1} + F_{x,2} + F_{x,3} + F_{x,4} + F_{x,5} + F_{x,6}. \]  
\[ F_{g,y} = F_{y,1} + F_{y,2} + F_{y,3} + F_{y,4} + F_{y,5} + F_{y,6}. \]  
\[ F_{g,z} = F_{z,1} - (z_0 \cos \phi - \frac{T_0}{2} \sin \phi) + F_{z,2} - (z_0 \cos \phi + \frac{T_0}{2} \sin \phi) + F_{z,3} - (z_0 \cos \phi - \frac{T_0}{2} \sin \phi) \]
\[ + (F_{z,5} \cos \phi + F_{z,5} \sin \phi) \cdot (z_0 \cos \phi + \frac{T_0}{2} \cos \phi + l_3 \sin \phi \sin \phi) \]
\[ + (F_{z,6} \cos \phi + F_{z,6} \sin \phi) \cdot (z_0 \cos \phi + \frac{T_0}{2} \cos \phi + l_3 \sin \phi \sin \phi) \]
\[ + F_{z,7} \cdot (\frac{T_0}{2} \cos \phi + z_0 \sin \phi) + F_{z,8} \cdot (\frac{T_0}{2} \cos \phi - z_0 \sin \phi) + F_{z,9} \cdot (\frac{T_0}{2} \cos \phi + z_0 \sin \phi) \]
\[ + F_{z,10} \cdot (\frac{T_0}{2} \cos \phi - z_0 \sin \phi). \]
\[ F_{g,k} = \begin{align*}
(\frac{F_{k,1} + F_{k,2}}{l_1} - (F_{k,3} + F_{k,4})l_2 & - (F_{k,5} \cos \phi + F_{k,5} \sin \phi) \cdot (\frac{T_0}{2} \sin \phi + l_3 \cos \phi + d_1) \\
- (F_{k,6} \cos \phi + F_{k,6} \sin \phi) \cdot (\frac{T_0}{2} \sin \phi + l_3 \cos \phi + d_1) & - F_{k,1} \cdot (\frac{T_0}{2} \cos \phi + z_0 \sin \phi) + F_{k,2} \cdot (\frac{T_0}{2} \cos \phi - z_0 \sin \phi) \\
- F_{k,3} \cdot (\frac{T_0}{2} \cos \phi + z_0 \sin \phi) + F_{k,4} \cdot (\frac{T_0}{2} \cos \phi - z_0 \sin \phi) & + (F_{k,5} \cos \phi - F_{k,5} \sin \phi) \cdot (\frac{T_0}{2} \cos \phi + l_3 \sin \phi \sin \phi) \\
+ (F_{k,6} \cos \phi - F_{k,6} \sin \phi) \cdot (\frac{T_0}{2} \cos \phi + l_3 \sin \phi \sin \phi) & - (F_{k,7} \cos \phi + F_{k,7} \sin \phi) \cdot (\frac{T_0}{2} \cos \phi - l_3 \sin \phi \sin \phi) \\
- (F_{k,8} \cos \phi + F_{k,8} \sin \phi) \cdot (\frac{T_0}{2} \cos \phi - l_3 \sin \phi \sin \phi). &
\end{align*} \]  

APPENDIX D: DEFLECTIONS OF SUSPENSION

\[ e_1 = -\frac{T_0}{2} \sin \phi \]  
\[ e_2 = \frac{T_0}{2} \sin \phi \]  
\[ e_3 = -\frac{T_0}{2} \sin \phi \]  
\[ e_4 = \frac{T_0}{2} \sin \phi \]  
\[ e_5 = -\frac{T_0}{2} \pi \cos \phi + (l_3 \phi) \sin \phi \]  
\[ e_6 = \frac{T_0}{2} \pi \cos \phi + (l_3 \phi) \sin \phi \]
APPENDIX E: VEHICLE PARAMETERS

Table 3. Parameters for a tractor-semitrailer vehicle. (Parameters marked with an asterisk are estimated values).

<table>
<thead>
<tr>
<th>parameter</th>
<th>unit</th>
<th>value</th>
<th>parameter</th>
<th>unit</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_1$</td>
<td>Kg</td>
<td>8444.0</td>
<td>$m_2$</td>
<td>Kg</td>
<td>23472.0</td>
</tr>
<tr>
<td>$I_{x1}$</td>
<td>Kg m^2</td>
<td>12446.5*</td>
<td>$I_{x2}$</td>
<td>Kg m^2</td>
<td>35523.7*</td>
</tr>
<tr>
<td>$I_y1$</td>
<td>Kg m^2</td>
<td>65734.6*</td>
<td>$I_y2$</td>
<td>Kg m^2</td>
<td>181565.5*</td>
</tr>
<tr>
<td>$I_z1$</td>
<td>Kg m^2</td>
<td>65734.6*</td>
<td>$I_y2$</td>
<td>Kg m^2</td>
<td>181565.5*</td>
</tr>
<tr>
<td>$l_1$</td>
<td>m</td>
<td>2.59</td>
<td>$T_{w1}$</td>
<td>m</td>
<td>2.02</td>
</tr>
<tr>
<td>$l_2$</td>
<td>m</td>
<td>3.29</td>
<td>$T_{w2}$</td>
<td>m</td>
<td>1.82</td>
</tr>
<tr>
<td>$z_0$</td>
<td>m</td>
<td>9.65</td>
<td>$T_{w3}$</td>
<td>m</td>
<td>1.82</td>
</tr>
<tr>
<td>$d_1$</td>
<td>m</td>
<td>1.20*</td>
<td>$h_2$</td>
<td>m</td>
<td>0.20*</td>
</tr>
<tr>
<td>$d_2$</td>
<td>m</td>
<td>0.60*</td>
<td>$d_3$</td>
<td>m</td>
<td>4.20</td>
</tr>
</tbody>
</table>

Table 4. Suspension parameters.

<table>
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<tr>
<th>parameter</th>
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<th>parameter</th>
<th>unit</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_{f1}$</td>
<td>N/m</td>
<td>2.72e5*</td>
<td>$K_{f2}$</td>
<td>N/m</td>
<td>3.36e10*</td>
</tr>
<tr>
<td>$K_{r1}$</td>
<td>N/m</td>
<td>8.53e5*</td>
<td>$K_{r2}$</td>
<td>N/m^5</td>
<td>1.05e11*</td>
</tr>
<tr>
<td>$K_{t1}$</td>
<td>N/m</td>
<td>1.55e6*</td>
<td>$K_{t2}$</td>
<td>N/m^5</td>
<td>1.92e12*</td>
</tr>
<tr>
<td>$D_f$</td>
<td>N·sec/m</td>
<td>9080*</td>
<td>$D_r$</td>
<td>N·sec/m</td>
<td>9080*</td>
</tr>
</tbody>
</table>

Table 5. Tire and wheel parameters.

<table>
<thead>
<tr>
<th>parameter</th>
<th>unit</th>
<th>value</th>
<th>parameter</th>
<th>unit</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_w$</td>
<td>Kg m^2</td>
<td>13.15*</td>
<td>$R$</td>
<td>m</td>
<td>0.3*</td>
</tr>
<tr>
<td>$C_{ax}$</td>
<td>N/rad</td>
<td>143330.0</td>
<td>$C_{lf}$</td>
<td>N</td>
<td>127120.0</td>
</tr>
<tr>
<td>$C_{ar}$</td>
<td>N/rad</td>
<td>143330.0 × 4</td>
<td></td>
<td>$C_{lr}$</td>
<td>N</td>
</tr>
<tr>
<td>$C_{aw}$</td>
<td>N/rad</td>
<td>80312.0 × 4</td>
<td></td>
<td>$C_{lt}$</td>
<td>N</td>
</tr>
</tbody>
</table>