Study of linear and nonlinear optical properties of distorted Ti–O₆ perovskite structure in BaₓSr₁₋ₓTiO₃

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Abstract

We report the linear and nonlinear optical properties of sol–gel prepared BaₓSr₁₋ₓTiO₃ (BST) crystalline powders. The results from powder X-ray diffraction, birefringence, and second-order optical nonlinearity measurements indicate that the paraelectric–ferroelectric phase transition of Ba₀.₇₅Sr₀.₂₅TiO₃ occurs near room temperature. The second-order optical nonlinearity of the ferroelectric BST is about 7.9 pm/V. The index of refraction, \( n \), and birefringence, \( \Delta n_B \), are also appropriate for practical nonlinear optical applications.

Keywords: A. Optical materials; B. Sol–gel growth; D. Ferroelectricity; D. Optical properties; D. Phase transitions

1. Introduction

Ferroelectric materials possess useful properties, which relate to their field-reversible spontaneous polarization below the Curie temperature \( T_C \). Two remanent polarizations with opposite polarity are available. The lower frequency transverse optical phonons of some ferroelectrics soften near the ferro–paraelectric phase transition [1] and give rise to highly nonlinear dielectric constant. Ferroelectric materials have been widely used in many applications ranging from sensors and actuators [2], new nonvolatile random access memory [3], to various microwave devices such as frequency-agile filters, phase shifters and tunable high-Q resonators [3–5]. An ideal material for these applications must possess high dielectric constant, low dielectric loss, and tunability in material properties.

Ternary and more complex oxides with perovskite structure such as BaₓSr₁₋ₓTiO₃ represent an important class of ferroelectric materials, whose electrical properties are tunable with their compositions and structures. Barium titanate (BaTiO₃) belongs to the displacement type of ferroelectric material, for which the origin of ferroelectricity derives from the displacement of ions relative to each other with a Curie temperature of 393 K [6], while SrTiO₃ is paraelectric and never becomes ferroelectric even at 0 K. It is known that the dielectric constant of BaₓSr₁₋ₓTiO₃ (BST) varies with \( x = \frac{Ba}{Ba + Sr} \) and becomes anomalously large at the structural phase transition [7]. The mechanism giving rise to the giant dielectric constant has remained unclear. The large dielectric constant of BST thin films can also be tuned with an external electric field, which makes the material attractive for the development of various microwave devices [3].

The electrical properties of BST films had been subjected to detailed investigation, however little has been done to reveal their optical properties at various compositions [8]. Optical applications with BST are interesting as many advanced nonlinear optical devices with BST films can be realized by using the periodic poling technique [9,10].

This paper is organized as follows: in Section 2, we first describe the experimental procedure used to prepare BST samples. The measurement methods to be used for characterizing the linear and nonlinear optical properties are then given. The experimental results are presented in Section 3. Finally, in Section 4, the discussion is given and the conclusions drawn.

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2. Experimental procedures

2.1. Preparation of BST powders with a sol–gel method

The sol–gel technique [11] was employed for preparing Ba$_x$Sr$_{1-x}$TiO$_3$ crystalline powders in order to yield samples with high composition accuracy. We first boiled acetic acid to 120°C for producing dehydrated solvent. A proper amount of barium acetate and strontium acetate (99% purity from Gransman Inc.) was dissolved in dehydrated acetic acid at 90°C and was sufficiently stirred for 20 min. Titanium isopropoxide and some di-ethanol were then added to the solution and were stirred for another 20 min. We dried and solidified the solution by illuminating with a 400-W infrared lamp for two days. The resulting white solid was heated to 165°C for 1 h and then ground into powders. We then sintered the powders at 1000°C for 150 min.

The powders were pressed into pellets with a pressure of 10,000 psi and the pellets were sintered again at 1350°C. The surfaces of the resulting pellets were polished for measuring the indices of refraction and powder X-ray diffraction patterns. For powder SHG measurements, the pellets were ground and then sieved to obtain powder samples with particle sizes of 45, 60, 100, 140, 270 and 400 meshes.

2.2. Linear optical measurement

To rapidly test and screen a library of new nonlinear optical materials, it is highly desired to develop some simple methods for evaluating the linear and nonlinear optical properties of the powder samples.

Stagg and Charampopoulos [12] devised a technique for measuring the index of refraction, $n = n - i\kappa$, of a powder sample. They found that the power reflectance from a rough surface at an incident angle, $\theta$, can be described with

$$R(n, \theta, \sigma, \lambda) = \rho(\theta, \sigma, \lambda) R_0(n, \theta),$$

(1)

Here $R_0$ represents the reflectance from an ideal smooth surface, and $\rho$ is the scattering factor from surface roughness. Assuming that the rough surface can be modeled with a surface profile that follows Gaussian distribution with a root-mean-squared (rms) roughness of $\sigma\lambda$, the scattering factor can then be derived with Kirchhoff’s scalar diffraction theory. For a surface with roughness $\sigma\lambda < 1$ and correlation length $L_c/\lambda < 1$, the scattering factor can be simplified to a polarization-independent term [12]

$$\rho(\theta, \sigma, \lambda) = \exp\left[-\frac{4\pi \cos \theta}{\lambda} \left(\frac{\sigma}{\lambda}\right)^2\right].$$

(2)

Therefore, the ratio of the two power reflectances with $p$- and $s$-polarized light becomes

$$r = \frac{R_p(n, \theta, \sigma, \lambda)}{R_s(n, \theta, \sigma, \lambda)} = \frac{R_{0p}(n, \theta)}{R_{0s}(n, \theta)}.$$  

(3)

This ratio can be related to the complex index of refraction $\tilde{n} = n - i\kappa$ of material by

$$r = \frac{a^2 + b^2 - 2a \sin \theta \tan \theta + \sin^2 \theta \tan^2 \theta}{a^2 + b^2 + 2a \sin \theta \tan \theta + \sin^2 \theta \tan^2 \theta},$$

(4)

where

$$2a^2 = (n^2 - \kappa^2 - \sin^2 \theta)^2 + 4n^2\kappa^2 + (n^2 - \kappa^2 - \sin^2 \theta)$$

and

$$2b^2 = (n^2 - \kappa^2 - \sin^2 \theta)^2 + 4n^2\kappa^2 - (n^2 - \kappa^2 - \sin^2 \theta).$$

2.3. Characterization of second-order nonlinear optical properties

Kurtz and Perry [13] were the first to investigate the second-order NLO responses of crystalline powders. Considering a fundamental beam with wavelength $\lambda$ normally incident on a crystal plate with thickness $L$, the total second-harmonic intensity can be expressed as [14]

$$I_{2w} = \frac{128\pi^2 d_{eff}^2 v_s^2 L^2}{n^2_{2w} n_{2w}^2 \lambda_{2w}^2} \frac{\sin^2(\Delta kL/2)}{(\Delta kL/2)^2},$$

(5)

where $\Delta k = k(2\omega) - 2k(\omega)$, $I_w$ is the intensity of the incident fundamental beam, $n_{2w}$, $n_{2w}$, and $d_{eff}$ are the indices of refraction and the effective nonlinearity of the crystal plate. When the plate is made with crystalline powders, the second-harmonic intensity becomes [15]

$$I_{2w} = \frac{512\pi^2 d_{eff}^2 v_s^2 L^2}{n^2_{2w} n_{2w}^2 \lambda_{2w}^2 \Delta v^2} \frac{\sin^2 \left(\frac{\pi}{\Delta v} \left(\frac{r}{L_c}\right)\right)}{\left(\frac{r}{L_c}\right)},$$

(6)

Here $\bar{r}$ denotes the averaged particle size, $L_c = 4(n_{2w} - n_{2w})$ is the coherent length, and $(d_{eff})$ is the square of the effective nonlinearity averaged over the orientation distribution of crystalline powders. When the second-harmonic generation is not phase matchable, Eq. (6) leads to the following asymptotic forms [15]:

$$I_{2w} \sim \begin{cases} \frac{128\pi^2 d_{eff}^2 v_s^2 L^2}{n^2_{2w} n_{2w}^2 \lambda_{2w}^2 \Delta v^2}, & \bar{r} \ll L_c, \\ \frac{256\pi^2 d_{eff}^2 v_s^2 L^2}{n^2_{2w} n_{2w}^2 \lambda_{2w}^2 \Delta v^2}, & \bar{r} \gg L_c. \end{cases}$$

(7)

If the second-harmonic generation satisfies the type-I phase matching condition, we can rewrite Eq. (5) as [16]

$$I_{2w}(\theta) = \frac{128\pi^2 (d_{eff}^2) v_s^2 L^2}{n^2_{2w} n_{2w}^2 \lambda_{2w}^2 \Delta v^2} \frac{\sin^2 \left[\frac{\pi}{2\Delta v} (\theta - \theta_{pm})\right]}{\left[\frac{\pi}{2\Delta v} (\theta - \theta_{pm})\right]^2},$$

(8)

where $\Delta v = \lambda/\Delta n_{2w} |\sin 2\theta_{pm}|$, and $\theta_{pm}$ is the phase matching angle. Here $\Delta n_{2w} = n_{2w} - n_{2w}$ denotes the birefringence of material at the second-harmonic wavelength. In the event that $\bar{r} \gg \bar{r}_{pm}$ or $\bar{r} \ll \bar{r}_{pm}$, Eq. (8) can
We derived a useful empirical formula, which possesses the correct asymptotic forms in Eq. (9), to depict the overall variation in second harmonic intensity with particle size.

$$I_{2\omega} = I_0\sqrt{1 - \exp\left[-\left(\hat{r}/A\right)^2\right]}$$ (10)

with

$$I_0 = \frac{256\pi^4I^2_{\omega}}{n^2_{\omega}n_{2\omega}\lambda^2_{2\omega}} L\hat{L}_{pm}, \quad \hat{r} \gg \hat{L}_{pm},$$

and

$$A \approx q_{pm}.\,$$

An experimental arrangement for measuring the second-harmonic scattering pattern from crystalline powders is described in Fig. 1. In this setup, the fundamental beam normally incidents on the sample cell. A liquid light guide with its input end attached on a rotation stage is employed to collect the second harmonic intensity at various scattering angles. We can integrate the second harmonic pattern over scattering angle to yield the total second harmonic intensity, $I_{2\omega}$.

3. Results

3.1. Structural determination of Ba$_x$Sr$_{1-x}$TiO$_3$ with powder X-ray diffraction

It had been revealed that the mechanical, electrical and optical properties of solid solutions of Ba$_x$Sr$_{1-x}$TiO$_3$ strongly depend on the mole fraction of Ba. We are therefore interested in studying the structures of BST perovskites with accurately controlled Ba compositions.

We first employ X-ray diffraction (XRD) for probing the unit cell dimensions of Ba$_x$Sr$_{1-x}$TiO$_3$ with various $x$-values. The resulting XRD patterns were then analyzed with the Rietveld refinement procedure [17]. In Fig. 2, the XRD patterns from four powder samples ($x$ = 1, 0.8, 0.5 and 0, from top to bottom) are presented. The averaged cell dimension as a function of $x$ is summarized in Fig. 3.

The unit cell of our synthesized BaTiO$_3$ crystalline powder has an averaged dimension of $a = 3.9864 \pm 0.003$ Å and $c = 4.0187 \pm 0.002$ Å (for BaTiO$_3$ (P4MM) see Refs. [18,19]). For SrTiO$_3$, our measured result ($a = b = c = 3.897$ Å) is slightly smaller than the published data ($a$ varies from 3.900 to 3.905 Å) (for SrTiO$_3$ (PM-3M) see Refs. [20,21]), but the difference is within our experimental error. It can also be seen that Vegard’s linear scaling law [8] is satisfied only within $0.2 < x < 0.7$. Outside the range, the cell dimension does not change significantly with $x$.  

![Fig. 1. Experimental setup used for measuring the second harmonic scattering pattern from a crystalline powder sample.](image)
3.2. Linear optical properties of $\text{Ba}_x\text{Sr}_{1-x}\text{TiO}_3$

To estimate the measuring accuracy of the index of refraction from Eq. (4), we first apply this linear optical technique on some well-known NLO materials. The results of $\text{KH}_2\text{PO}_4$ (KDP) crystalline powder at $\lambda = 633\, \text{nm}$ are presented in Fig. 4. By using Eq. (4), the index of refraction of KDP was determined to be 1.49, which is fairly close to the averaged value of $n_E$ and $n_O$ of KDP single crystal (see Table 1) [22].

We then apply this method to probe linear optical properties of BST powders with various $\text{Ba}/(\text{Ba} + \text{Sr})$ ratios. The
results are presented in Fig. 5, which show that the index of refraction of BST is fairly constant (~2.20). The value is about 8% lower than that taken from single crystal BaTiO$_3$ ($n_{av}$ = 2.39) and SrTiO$_3$ ($n$ = 2.39) [23]. The deviation is most likely to have originated from the larger light scattering loss from our powder samples.

3.3. Nonlinear optical properties of Ba$_x$Sr$_{1-x}$TiO$_3$

For calibration, the nonlinear optical (NLO) characterization apparatus described in Fig. 1 was first applied to investigate NLO responses of KDP and β-BaB$_2$O$_4$ (BBO) powders. The results show that second harmonic generation from these two standard materials is phase matchable at 1.06 μm. The NLO response from BBO is about four times that of KDP, which agrees very well with the published values [22].

We then apply this experimental setup to measure NLO properties of BST powders. The results are shown in Fig. 6. The second harmonic generation from BST is also phase matchable at 1.06 μm. The effective second-order nonlinearities of Ba$_x$Sr$_{1-x}$TiO$_3$ with $x$ = 0.8 and 0.7 were found to be about 7.2 and 3.5 pm/V, respectively. The effective nonlinearities, $\sqrt{(d_{eff}^2)}$, and birefringence, $\Delta n_B = n_{E,2\omega} - n_{O,2\omega}$, of BST with various $x$ are summarized in Fig. 7. It is interesting to note that the $\sqrt{(d_{eff}^2)}$ and $\Delta n_B$ exhibit a discontinuous change with $x$. This supports that BST with Ba/(Ba + Sr) ~ 0.75 undergoes a structural phase transition near room temperature. Table 1 summarizes our results of the optical properties of KDP, BBO and BST.

4. Discussion and conclusions

From the XRD patterns shown in Fig. 2, we found that the peaks near 45 and 51° are split into two neighboring subpeaks when the ratio of Ba/(Sr + Ba) is higher than 0.7. To further reveal the origin of the splittings, we perform XRD simulations on BaTiO$_3$ with a crystal structure of $P4MM$ ($C_{av}$) and $PM-3M$ ($O_b$).

The simulated results, which are shown in Fig. 8, indicate
that the structural change of unit cell can cause the atomic plane spacing in {200} (corresponding to $2\theta \sim 45^\circ$) to be different. The similar behavior also happens in {210} ($2\theta \sim 50^\circ$). Thus the observed splittings in our XRD patterns provide a direct evidence of a structural change from $O_h$ to $C_{4v}$ in BST when Ba/(Ba + Sr) is higher than 0.75.

As pointed out previously barium titanate (BaTiO$_3$) belongs to the displacement type of ferroelectric material for which the origin of ferroelectricity derives from the displacement of ions relative to each other. It is believed that BST undergoes the same sequence of structural phase transitions as pure BaTiO$_3$ does, but at progressively lower phase transition temperatures as the concentration of Ba is reduced. The Ti–O$_6$ octahedron in the ferroelectric BaTiO$_3$ is distorted with $C_{4v}$-symmetry. The resulting spontaneous polarization, $P_s$, can be expressed as [24]

$$P_s = P_0' \Delta z,$$

(11)

where $\Delta z$ denotes the displacement of ions from the symmetric positions which are occupied in the paraelectric phase. $P_s$ serves as an order parameter of the phase transition and therefore above the transition the parameter varies with temperature by [25]

$$P_s = a_0 \sqrt{T_c - T}.$$

(12)

Recently, Wang [26] had proposed a simple two-band model suited for describing polarization in a ferroelectric crystal. With the model, the second-order NLO coefficient
of a ferroelectric crystal had been derived to be

\[ d_{\text{eff}} = \frac{\pi C(n_{\omega}^2 + 2)^2 E_0^6}{18(E_0^2 - \hbar^2\omega^2)(E_0^2 - 4\hbar^2\omega^2)} P_s. \]  

(13)

Here \( E_0 \) is the energy gap of the ferroelectric material and \( C \) denotes a simple constant. By combining Eqs. (11) and (13), we then have

\[ d_{\text{eff}} = d'\Delta \varepsilon. \]  

(14)

Note that the displacement of ions may lead to a change in unit cell dimension. Based on the model, birefringence can also be related to spontaneous polarization by \( \Delta n_B = B P_s' \) [26]. Therefore, we are expecting to observe that both the second-order optical nonlinearity and birefringence vary with the mole fraction of Ba. Indeed, this is what we have observed, as shown in Fig. 7.

We should point out that the linear optical dispersion from 1.064 to 0.532 \( \mu \)m in a single crystal BaTiO\(_3\) is about 0.13 [23], which is larger than the birefringence \( (\Delta n_B = -0.06) \). Therefore the type-I phase matching condition should not be satisfied in BaTiO\(_3\) at 1.064 \( \mu \)m.

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**Fig. 7.** The measured \( \sqrt{d_{\text{eff}}^2} \) and \( \Delta n_B \) of Ba\(_x\)Sr\(_{1-x}\)TiO\(_3\) are plotted as a function of \( x \).

**Fig. 8.** Calculated powder X-ray patterns BaTiO\(_3\) crystalline powder with a point group symmetry of \( O_h \) (dashed curve) and (b) \( C_4v \) (solid curve).
Table 1
Linear and second-order nonlinear optical properties of some crystalline powders

<table>
<thead>
<tr>
<th></th>
<th>Refractive indices of crystal at 0.633 μm</th>
<th>Second-order nonlinearity of crystal at 1.064 μm</th>
<th>n (0.633 μm)</th>
<th>l_{pm} (μm)</th>
<th>Δn_B (0.532 μm) =</th>
<th>Δn_B/Δn (KDP) @ 1.064 μm</th>
<th>d_{eff} (pm/V) @ 1.064 μm</th>
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<td>KDP</td>
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<td>n₀ = 1.512</td>
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<td>1.49</td>
<td>11.1</td>
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<td>n₉ = 1.417</td>
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<td>n₆ = 1.492</td>
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<td>Δn_B = −0.041</td>
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<td>n₀ = 1.675</td>
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<td>1.58</td>
<td>3.7</td>
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<td>n₆ = 1.615</td>
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<td>Δn_B = −0.120</td>
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<td>Ba₀.₇Sr₀.₃TiO₃</td>
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<td>2.12</td>
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<td>9.7</td>
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</table>
However, note that the single crystal BaTiO$_3$ usually contains various transition metal impurities, which often results in red shift of the absorption edge. Our BST crystalline powders prepared with the sol–gel method do not contain such unintended dopants. Therefore our samples more accurately reflect the intrinsic properties of the materials and possess smaller dispersion in the visible light spectrum region. This can lead to the observed type-I phase matched second-harmonic generation in BST, as shown in Fig. 6.

Note that the index of refraction at the zero frequency limit ($\hbar \omega \ll E_0$) can be expressed as [27]

$$n^2 = 1 + \frac{8\pi^2\hbar^2}{m^2V} \sum_k \sum_{vc} p_{vc}(k)p_{vc}(k) \frac{E_{cv}(k)}{E_{cv}(k)}.$$ (15)

It is dominated by those interband transitions which lie near band gap (smaller $E_{cv}$) and possess large momentum matrix elements, $p_{vc}$. In BST, oxygen’s 2p and titanium’s 3d orbitals dominate these transitions, where Ba and Sr do not play an important role. This indicates that the index of refraction of BST should be irrelevant to the concentration of Ba. Indeed this is exactly what we discover in Fig. 5. It should be pointed out that Ba/Sr play the major role in causing unit cell distortion, which more sensitively reflects in second-order optical nonlinearity and birefringence.

The birefringence of BST shown in Fig. 7 varies from 0.03 to 0.06 as the mole fraction of Ba is increased from 0.03 to 0.05, are also appropriate. The measured linear and nonlinear optical properties of BST warrant a high efficacy in second-order nonlinear optical applications.

In summary, we investigate linear and nonlinear optical properties of Ba$_x$Sr$_{1-x}$TiO$_3$ with various mole fractions of Ba. Our results indicate ferroelectric BST possess an effective second-order nonlinearity of about 7.9 pm/V, which is comparable to LiNbO$_3$. In addition, the index of refraction, $n = 2.2$, and birefringence, $\Delta n_B \sim 0.05$, are also appropriate for practical NLO applications. By combining with their excellent electrical and mechanical characteristics, BST could serve as an ideal multifunctional, smart material in micro optical electro-mechanical systems (MOEMS) [28,29].

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References