An Analysis for Measurement of Thermal Diffusivity Components of Anisotropic Platelike Samples by AC Calorimetric Method

This content has been downloaded from IOPscience. Please scroll down to see the full text.

2000 Jpn. J. Appl. Phys. 39 L690
(http://iopscience.iop.org/1347-4065/39/7A/L690)

View the table of contents for this issue, or go to the journal homepage for more

Download details:

IP Address: 140.113.38.11
This content was downloaded on 28/04/2014 at 07:47

Please note that terms and conditions apply.
An Analysis for Measurement of Thermal Diffusivity Components of Anisotropic Platelike Samples by AC Calorimetric Method

Long-Jye SHEU and Jenn-Der LIN
Department of Mechanical Engineering, National Chiao Tung University, 1001 Tai Hsueh Rd., Hsinchu, Taiwan, R.O.C.

(Received January 17, 2000; accepted for publication May 9, 2000)

This work examines the effects of anisotropy and transparency on measurements of thermal diffusivity components with an ac calorimetric method associated with laser heating. Analytical results indicate that the region where the two-dimensional effect occurs increases with the decrease of the ratio of cross-plane to in-plane thermal diffusivity. The region also increases with the optical thickness of the sample. The linear relations, as indicated by a decay constant from which the cross-plane thermal diffusivity is deduced, are not obtained for media of both optically moderate and thin thickness, while at sufficiently large optical thickness, anisotropy and two-dimensional effects are found insignificant.

KEYWORDS: thermal diffusivity, ac calorimetric, two-dimensional, anisotropy, semi-transparent

1. Introduction

Hatta et al.,(1,2) Gu and Hatta,(3) Kato et al.(4,5) and Gu et al.(5) developed an ac calorimetric method for measuring the in-plane thermal diffusivity of a thin film. That research assumed a one-dimensional (1D) temperature distribution be-

\begin{equation}
q(x, y, t) = Qa(1 - R) \left\{ e^{-\alpha y} + \sum_{n=1}^{\infty} \left[ R^{2n-1} e^{-\alpha(2nd-y)} + R^{2n} e^{-\alpha(2nd+y)} \right] \right\} [H(x+a) - H(x)] e^{\alpha f t}, \quad (2)
\end{equation}

with the multiple reflections at boundaries included. \( \alpha \) is the absorption coefficient and \( R \) denotes the boundary reflectivity. \( H \) represents the heaviside function. \( Q \) is the heat flux per unit area of the incident beam. \( i \) is equal to \( \sqrt{-1} \). \( f \) denotes the frequency of the incident beam and \( a \) represents the spatial width of the incident beam.

The initial and boundary conditions for surfaces with convection heat loss are

\begin{align}
T(x, y, t) &= 0 \quad \text{at} \quad t = 0, \quad (3) \\
T(-\infty, y, t) &= T(\infty, y, t) = 0, \quad (4) \\
\frac{\partial T(x, 0, t)}{\partial y} &= -\frac{h}{\kappa_x} T(x, 0, t) = 0, \quad (5) \\
\frac{\partial T(x, d, t)}{\partial y} &= -\frac{h}{\kappa_y} T(x, d, t) = 0, \quad (6)
\end{align}

where \( h \) is the convective heat transfer coefficient. By using the Fourier transform technique and introducing the following

\begin{align*}
\kappa_x \frac{\partial^2 T}{\partial x^2} + \kappa_y \frac{\partial^2 T}{\partial y^2} + q(x, y, t) &= \rho c \frac{\partial T}{\partial t}, \quad (1)
\end{align*}

where \( \kappa_x \) and \( \kappa_y \) are, respectively, the in-plane and cross-plane thermal conductivity components. \( \rho \) is the density and \( c \) the specific heat. The heat source term \( q(x, y, t) \) can be described as

\begin{center}
Fig. 1. Physical model and coordinate systems.
\end{center}
nondimensional parameters,

\[ x' = \frac{x}{d/\pi}, \quad y' = \frac{y}{d/\pi}, \quad f' = \frac{f}{\pi^2 D_x/d^2}, \]

\[ \frac{T_{ac}(x', y', f')}{Qd/2\pi^3 \kappa_y} = \int_{-\infty}^{\infty} \frac{\tau(1-R)(e^{-iw\tau} - 1)}{q^2 + \pi^2 \tau^2 - \omega^2 + 2\pi f' i/\kappa} \]

where the D’s are the thermal diffusivity components. \( \tau \) denotes the optical depth of the sample, and B represents the Biot number.

\[ q = \sqrt{(w^2 + 2\pi f i)/\kappa}, \quad (9) \]

and

\[ P(y') = e^{-\gamma y'/\pi} + \sum_{n=1}^{\infty} [R^{2n} e^{-\pi(2n+y'/\pi)} + R^{2n-1} e^{-\pi(2n-y'/\pi)}] \]

with

\[ EE = \left( \frac{\tau}{\pi} + \frac{B}{\kappa} \right) \left[ 1 + \sum_{n=1}^{\infty} R^{2n} e^{-2\pi} \right] + \left( \frac{B}{\kappa} - \frac{\tau}{\pi} \right) \sum_{n=1}^{\infty} R^{2n-1} e^{-2\pi} \]

\[ FF = \left( \frac{\tau}{\pi} - \frac{B}{\kappa} \right) e^{-\tau} + \sum_{n=1}^{\infty} R^{2n-1} e^{-(2n-1)\tau} \]

\[ - \left( \frac{B}{\kappa} + \frac{\tau}{\pi} \right) \sum_{n=1}^{\infty} R^{2n-1} e^{-(2n-1)\tau}. \]

The solution \( T_{ac}(x', y', f') \) of eq. (8) is a complex value. The amplitude \( |T_{ac}(x, y, f)| \) and phase \( \phi(x, y, f) \) of the ac temperature response are then obtained from

\[ |T_{ac}(x', y', f')| = \sqrt{T_{RE}^2(x', y', f') + T_{IM}^2(x', y', f')}, \]

\[ \phi(x', y', f') = \arctan(T_{IM}(x', y', f')/T_{RE}(x', y', f')). \]

The subscripts RE and IM respectively denote the real and imaginary parts of \( T_{ac} \).

When heat loss can be neglected, the apparent thermal diffusivity of either \( D^*_a \) or \( D^*_p \), respectively derived from the decay of the amplitude and the shift of the phase, can be used to accurately derive the thermal diffusivity \( D_x \). To compare \( D^*_a \) and \( D^*_p \) derived from 2D temperature responses with the 1D results, both \( D^*_a \) and \( D^*_p \) are obtained as follows. The plots of \( \ln |T_{ac}(x', y', f')|/\sqrt{\tau} \) and \( \phi(x', y', f')/\sqrt{\tau} \) versus \( x' \) decay with a constant slope of \( \sqrt{\tau} \) under 1D analysis, and either plot would deduce the thermal diffusivity \( D_x \). In 2D analysis, the plots of \( \ln |T_{ac}(x', y', f')|/\sqrt{\tau} \) and \( \phi(x', y', f')/\sqrt{\tau} \) versus \( x' \) decay with slopes of \( a_a \) and \( a_p \).

\[ \kappa = \frac{D_y}{D_x}, \quad \tau = \alpha d, \quad B = \frac{hd}{\kappa_x \pi}, \]

the ac temperature is then derived as

\[ \left[ (q + B/k)e^{2\pi y} FF - (q - B/k)e^{2\pi(1-y')} EE \right] \]

\[ = \left[ (q + B/k)e^{2\pi y} FF - (q - B/k)e^{2\pi(1-y')} EE \right] + P(y'). \]

3. Results and Discussion

The trapezoid method is used to calculate the integral in eq. (8). In numerical computations, the limit, \( \infty \), of the integration is replaced by 10, which has been found sufficiently large. With respect to the amplitude, a quantity of \( \ln(Qd/2\pi \kappa^2) \) is subtracted since only the relative values of \( \ln |T_{ac}(x', y', f')|/\sqrt{\tau} \) are necessary.

Figure 2(a) shows the effect of \( \kappa \) on the measured \( \sqrt{D^*_a/D_x}(D^*_p/D_x) \) along \( x' \) at the back surface \( y' = \pi \) for \( f' = 0.00001, B = 0.0001 \) and \( R = 0.0 \). The optical thickness of the sample is taken to be \( \tau = 1000 \), for which the sample can be regarded as opaque, making the absorption of heating energy a surface phenomenon. For isotropic materials (\( \kappa = 1 \)), Figure 2(a) shows that in the region \( x' > 6 \), the values of \( \sqrt{(D^*_a/D_x)(D^*_p/D_x)} \) are unity. This phenomenon implies that in this region the thermal system can be regarded as 1D. In the study of Yamane et al., 3D analysis for isotropic materials (\( \kappa = 1 \)) is valid in the region where \( x' > 100 \). In another study of Yamane et al., a two-layer system was considered, on the basis of which the special case of a one-layer system was also discussed. However, their study concludes that 1D analysis is valid for \( x' > 10 \). Our results for isotropic materials, \( x' > 6 \), are consistent with those of Hatta et al. Figure 2(a) also reveals that when \( \kappa \) is less than 1, the value of \( x' \) when \( \sqrt{(D^*_a/D_x)(D^*_p/D_x)} = 1 \) increases when \( \kappa \) is less than 1, the value of \( x' \) when \( \sqrt{(D^*_a/D_x)(D^*_p/D_x)} = 1 \) is equal to 1, but when \( \kappa = 0.01 \), the region where 1D analysis is valid delays to \( x' > 30 \). Physically, \( \kappa \) represents the ratio of heat wave propagation speeds in directions perpendicular and parallel to the sample surface. A large value of \( \kappa \) indicates that \( D_y \) is much larger than \( D_x \) and heat propagates slowly in the \( x \)-direction. Consequently, heat waves are rapidly conducted to the bottom of the sample and the thickness effect rapidly diminishes. It quickly approaches 1D analysis near the edge of the heated region. At smaller values of \( \kappa \), the heat propagates faster in the
ous optical thicknesses, the heating energy can penetrate into opaque samples. This phenomenon occurs because for semi-transparent materials, the heating energy can penetrate into the sample medium and is directly absorbed within it, i.e., the medium is more uniformly heated across its depth within the irradiated region and thus the 1D heat propagation holds from a position close to the edge of the heated region for samples with a smaller optical thickness. Based on the analytical results for the two-dimensional temperature wave propagation, Takahashi et al.\(^8\) proposed the maximum thickness required to determine thermal diffusivity for a material heated at the surface, provided that the error of the slope of the phase or amplitude distributions is within 1% at \(x' = 0.5 \pi /kd\). According to our results, the ac calorimetric method can be applied to semitransparent samples with thicknesses larger than those of samples calculated by Takahashi et al.\(^8\) within the same error. For example, Takahashi et al. found the maximum thickness of a diamond sample to be 6.3 mm at \(f = 1\) Hz. The absorption coefficient \(\alpha\) of diamond is 0.11 cm\(^{-1}\) under a heating light with a wavelength of 0.4358 \(\mu\)m.\(^12\) By considering the transparency, this study derives the maximum thickness of a diamond sample to be 13.8 mm within the same error as Takahashi et al. Readers may refer to ref. 12 for the absorption coefficients of various materials. For isotropic semitransparent samples, the ac calorimetric method for measuring thermal diffusivity can also avoid the photo-heating problems that occurred in the flash method\(^13\) because the thermocouple is located outside the heating beam. The effects of boundary reflectivity, \(R\), are also tested, indicating that this does not significantly influence the region of 2D temperature wave propagation. However, \(R\) will influence the magnitudes of the temperature waves since it affects energy absorption.

Regarding the measurement of \(D_y\), Yang et al.\(^9\) and Kato et al.\(^10\) have proposed ac calorimetric methods for measuring the cross-plane thermal diffusivity. In a 1D model by Yang et al.\(^9\) the thermal diffusivity perpendicular to the sample surface is determined from the constant slope of the phase vs the square root of the frequency in the high-frequency region. The present study has noted that the ac light heating method is not suitable for determining the \(D_y\) of semitransparent samples if the thermocouples are placed under the light beam. In such conditions, photo-heating problems occur with the thermocouples. For media with sufficiently large optical thicknesses, this study assumes optical thickness \(\tau = 1000\), and the heating is then within a very small depth near the front surface and the above-mentioned linear relation may apply for parameter estimation. However, the slopes of these linear relationships depend on the value of \(\kappa\). Figure 3 shows the slopes, i.e., the partial derivatives of \(\phi(-a'/2, \pi, f')\), with respect to \(\sqrt{f'}\) as a function of \(\kappa\). Figure 3 also displays the values of the partial derivatives of \(\phi(-a'/4, \pi, f')\), \(\phi(0, \pi, f')\) with respect to \(\sqrt{f'}\). Notably, the constant slope of the phase vs the square root of the frequency does not vary with \(x'\) as \(x'\) is under the heating beam. Thus, the value of \(\kappa\) can be determined from Fig. 3. With derived \(\kappa\), \(D_y\) can readily be obtained by multiplying \(D_y\) with \(\kappa\). The relations between the slope and \(\kappa\), as shown in Fig. 3, can be mathematically described as

\[
\alpha_d = 5.568 / \sqrt{\kappa} = \sqrt{\pi^3 / \kappa},
\]

where \(\alpha_d\) is the constant slope of the phase vs the square root of the frequency. The result of \(\alpha_d\) for isotropic material (\(\kappa = 1\)) in this study is consistent with \(\sqrt{\pi^3}\) derived by Yang et al.\(^9\) Equation (18) indicates that the anisotropic effect results in a multiplying factor of \(1 / \sqrt{\kappa}\) on \(\alpha_d\). In a real experiment,
the plot of the $\phi(-a/2, \pi, f')$ phase is derived as a function of the square root of the dimensional frequency $f$ and the slope of the plot in a high-frequency region, for example, $\alpha_3$. Incorporating eqs. (18) and (7) produces the result of the ratio of thermal diffusivity components, $\kappa = \pi a'^2 / D_x a_3'^2$. This step then leads to the cross-plane thermal diffusivity, $D_y = \pi a'^2 / a_3'^2$. This result is consistent with the results of Yang et al.\(^9\) for isotropic materials. The above analysis shows that anisotropic and 2D effects on the measurement of cross-plane thermal diffusivity are insignificant as long as the detector is located under the heating beam.