The anomalous Hall effect viewed from the time-dependent Ginzburg–Landau equations for d-wave superconductors

Tzong-Jer Yang*, Mei-Cheng Dai

Department of Electrophysics, National Chiao Tung University, Hsinchu, Taiwan 30050, People’s Republic of China

Abstract

From time-dependent Ginzburg–Landau theory of d-wave superconductors, the rate of increase of the total free energy plus the rate of dissipation are demonstrated to be equal to the inflow of energy current. The equation of motion of a single vortex for s- and d-wave order parameters is derived. The imaginary parts of the relaxation time for s-wave order parameter complex relaxation time $g$ for mixed s- and d-wave superconductors. Their results are cumbersome and hard to solve them. Now we follow Dorsey [1] and Kopnin et al. [2,3] approach to get equation of motion for a single vortex in the low-field regime. Our results show that imaginary parts of two complex relaxation time and the mixed gradient terms play an important role to change the sign of the Hall effect.

We follow the procedure made in Refs. [2,3] to derive energy theorem for mixed s- and d-wave superconductors. The total free energy of this system consists of the energy of the metal in the normal state $F_n$, the energy of the electromagnetic field $F_{em}$, and the free energy $F_{sn}$ for the transition to the superconducting state and for the interaction of the superfluid current with the electromagnetic field. The continuity equation $\nabla \cdot j = 0$ is needed. $j = j_n + j_s + \delta_n \cdot E$ is the total current density. $j_n$ is the normal current density and $j_s = - \delta F_{sn}/\delta A$ is the supercurrent density. The TDGL equations for the s- and d-wave order parameters are also required and expressed as

$$
\eta_s (h\tilde{\chi} + 2i\phi)s = - \delta F_{sn}/\delta s^*, 
$$

(1)

$$
\eta_s (h\tilde{\chi} + 2i\phi)d = - \delta F_{sn}/\delta d^*. 
$$

(2)

and complex-conjugate equations for $s^*$ and $d^*$. Here $\phi$ is a scalar potential, $\eta_s$ and $\eta_d$ are the dimensionless order parameter complex relaxation times: $\eta_d \equiv \eta_{d1} + i\eta_{d2}$, $\eta_s \equiv \eta_{s1} + i\eta_{s2}$. Then Maxwell’s equations are joined together with the total time derivative of the total free energy $dF/dt = dF_n/dt + dF_{em}/dt + dF_{sn}/dt$. The first term in $dF/dt$ is $dF_n/dt \equiv \nabla \cdot (\mu_{\parallel}/e) \cdot d\mu$, where $\mu$ is the chemical potential and generally small enough to be neglected. The second term $dF_{em}/dt = - \nabla \cdot (S + j \cdot E) \cdot d\mu$, where $S$ is Poynting vector. The third term

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*Corresponding author.

E-mail address: yangtj@cc.nctu.edu.tw (T.-J. Yang)
part of complex relaxation time. If \( \varepsilon_2 < 0 \), and \( |\varepsilon_2| > \varepsilon_1 \), then \( \tan \theta_H \) will change sign. From the equation of motion of vortex and Faraday's law, we have the longitudinal conductivity and the transverse or Hall conductivity. Thus \( \tan \theta_H \) is independent of magnetic field near \( H_{c1} \). Xu et al. [5,6] show that the transition temperature for s-wave can only be affected by magnetic impurities, while the transition temperature for d-wave is affected by non-magnetic impurities. In the high scattering strength, one may find that the range of the sign change of the Hall angle depends on the concentration of non-magnetic impurities.

In conclusion, the energy balance theorem is verified in the mixed d- and s-wave superconductivity. The imaginary part of complex relaxation time and the non-magnetic impurities may affect the sign change of the Hall angle.

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References