Injection-locked coupled microstrip leaky-mode antenna array

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Abstract: The paper presents a novel design for an injection-locked microstrip leaky-mode antenna array. It demonstrates that the coupling parameters can be obtained numerically, leading to the analysis of the steady-state phase relationships of the coupled oscillators based on Van der Pol equations for determining the excitation signals of the array. In particular, it applies the coupled-mode approach to investigating the clear and considerable mutual coupling effect on the radiation characteristics of the microstrip leaky-mode array. This, in turn, produces a very efficient and accurate assessment of the radiation far-field patterns. Finally, a proof-of-concept design using a two-element injection-locked microstrip leaky-mode array is presented for experimental verification, showing excellent agreement between theoretical data and measured results.

1 Introduction

Matured MMIC (monolithic microwave integrated circuit) and quasi-optical techniques offer the prospect of reducing construction costs for active phased arrays [1], making the application of such arrays a rapidly growing area of research [2-4]. However, the complexity of the distribution network required increases as the element number of the phased array system grows. Thus a challenge confronts us in designing the signal distribution for large phased array systems. Given the large size, high loss and dispersion in conventional transmission mediums, designers are seeking improved methods for high combination efficiencies using quasi-optical spatial power combining techniques [5, 6] and a simpler distribution network layout [7]. From this perspective, the microstrip leaky-mode antenna based array becomes attractive because it employs a linear array to achieve the pencil beam that otherwise can only be obtained by conventional two-dimensional arrays incorporating patch resonators or other means, greatly reducing component count. This paper describes a new design for beam scanning array based on the concept of quasi-optical power combining using the microstrip leaky-mode antenna array. As Fig. 1 shows, an injection signal from an external stable source feeds into one end of the active linear array with a unit element comprised of a free-running oscillator and a microstrip leaky-mode antenna. The injection signal couples into and locks the nearest active devices of the microstrip leaky-mode antenna. The latter then synchronizes with the next sequential unit of the active antenna element, and so on. As the frequency of the external signal varies, the beam-scanning characteristic is derived from the nature of the leaky-wave antenna array.

1 CPW (coplanar waveguide) quasi-optical oscillator on back side of substrate
2 Microstrip leaky-mode antennas with length L, width W and gap 3 on top side of substrate

2 Theoretical analysis

2.1 Mode coupling of complex-wave and coupling parameters

As Fig. 1 shows, microstrips are placed as close as possible to achieve sufficient coupling strength for an acceptable locking bandwidth. However, coupling between the adjacent microstrip lines alters the modal spectra of the microstrip's $EH_0$ mode, which is the first higher order of the microstrip mode leaking power away in the form of a space wave. By rigorous full-wave analysis [8], one can always obtain both the undisturbed (before coupled) $EH_0$ mode of a single microstrip line and all coupled $EH_0$ modes of the microstrip array with fewer elements. Fig. 2 plots the theoretical results of a 3-element array, clearly indicating the
effect of mode coupling of complex waves inherent in a microstrip leaky-mode array. The coupled-mode approach [9, 10] states that mode coupling of complex waves inherent in N-element leaky lines is governed by a system of linear differential equations and expressed as follows:

\[
dl(z)/dz = -\gamma I(z) + \sum_{j=1}^{N} C_{ij} I_j(z)
\]

where \(I_j(z)\) is the modal current vector on microstrip \(i\) and \(C_{ij}\) is the mutual coupling interaction between the \(i\)th and \(j\)th elements. Using matrix notation, eqn. 1 is abbreviated as

\[
dl(z)/dz = \overline{B} \cdot \overline{I}
\]

where \(\overline{B} = [\text{diag}(\gamma) - [C_{ij}]]\) and \(\overline{I} = [I_1, I_2, ..., I_N]^T\). When all the coupled microstrips are in equal width, \(\gamma\) must be equal to \(\gamma\), where \(\gamma\) is the complex propagation constant of the EH1 leaky mode of a single microstrip. On the other hand, N coupled leaky-mode solutions should exist in the N-element array, denoted as \(\lambda_1, \lambda_2, ..., \lambda_N\). For a specific mode with complex propagation constant \(\lambda_j\), the modal solution mandates

\[
dl(z)/dz = \gamma_j \cdot I_j
\]

Substituting eqn. 3 into eqn. 2 for \(i = 1, 2, ..., N\), we obtain

\[
\overline{A} \cdot \overline{I} = 0
\]

where \(\overline{A} = [\text{diag}(\gamma) - [C_{ij}]]\) and \(\overline{I} = [I_1, I_2, ..., I_N]^T\). The source-free modal solutions require nontrivial solutions for the modal current vector \(I(z)\). Therefore eqn. 4 leads to a standard eigenvalue problem by solving

\[
\det(\overline{A}) = 0
\]

Eqn. 4 clearly demonstrates that one can either attain the \(\lambda\) (coupled EH1 modes) given \(C_{ij}\) (the square matrix of coupling parameters) or deduce \(C_{ij}\) given \(\lambda\). Thus, substituting the rigorous data of the coupled EH1 modes (\(\lambda\) shown in the solid line of Fig. 2) into eqn. 4, one can deduce all coupling parameters numerically. The Appendix (Section 8) carries out the detailed formulation for deducing all coupling parameters. By using eqn. 13, Fig. 3 plots the theoretical results of the coupling parameters \(C_{ij}\) and \(C_{12}\), revealing that the strength of \(C_{12}\) = coupling due to other-adjacent-element = is much smaller than that of \(C_{12}\), nearest-neighbour coupling.

![Fig. 2](image1.png)

**Fig. 2** Comparison of dispersion characteristics of single microstrip line and those of 3-element coupled microstrip lines, showing that modal spectra of microstrip's EH1 mode is altered by unidirectional coupling of adjacent lines.

- **EH1 mode of single microstrip line**
- **Coupled microstrip line**

\(w = 4.47\,\text{mm}, x = 10.765\,\text{mm}, \theta = 25\,\text{mm}, E_r = 10.2\)

2.2 Coupled oscillator theory

A competent theory of coupled oscillators needs to predict the steady-state phase relationships in the array numerically. York et al. [6] indicated that coupled Van der Pol equations adequately describe the coupled oscillator arrays for power combining. With predicted coupling parameters \(C_{ij}\) given in eqn. 13, the equations describing the amplitude and phase dynamics for an array of \(N\) elements can be achieved. From the theoretical results shown in Fig. 3, this work only considers the nearest-neighbour coupling, with \(\rho_j = 0\) for all \(j \neq \pm 1\). Furthermore, the coupling is reciprocal. Since the oscillators in the linear array are equidistant, all of the coupling terms are identical, and the following simplifications are possible: \(\rho_j = \rho\) and \(\Phi_j = \Phi\). Thus, one can (to first order) ignore the amplitude dynamics. The system is then described by [6] as follows:

\[
\frac{d\theta_i}{dt} = \omega_i - \frac{\rho}{2Q} \sum_{j=1}^{N} \frac{I_j}{I_i} \sin(\Phi + \theta_i - \theta_j)
\]

where \(i = 1, 2, ..., N\), and where \(I_i\) is the instantaneous amplitude, \(\omega_i\) is the free running frequency, and \(\theta_i = \omega t + \phi_i\) is the instantaneous phase of oscillators \(i\). Meanwhile \(Q\) is the Q-factor of the oscillator embedding circuits. Eqn. 6 allows the steady-state phase differences between each oscillator to be solved, given the free-running frequencies and computed coupling parameters by eqn. 13.

![Fig. 3](image2.png)

**Fig. 3** Strength of coupling parameters \(\beta = \log_{10}(C_{ij}, i = 2, 3)\), showing that coupling parameters other than nearest-neighbour lines \((C_{12})\) can be ignored as negligible.

\(\text{Bar} - C_{12}, \circ - C_{13}\)

\(w = 4.47\,\text{mm}, x = 10.765\,\text{mm}, \theta = 25\,\text{mm}, E_r = 10.2\)

2.3 Radiation pattern of an active, coupled, microstrip leaky-mode array

For a coupled microstrip array, each coupled EH1 mode is supported by a particular modal current distribution on the strips. This can be viewed as an eigenvalue corresponding to a specified eigenvector. For instance, another view for distinguishing the two leaky modes of a two-element array is based on the eigenvectors of the modal current distributions on the strips, either in-phase \((1/\sqrt{2}, 1/\sqrt{2})\) for the EH1 even-mode or out-of-phase \((-1/\sqrt{2}, 1/\sqrt{2})\) for the EH1 odd-mode. These in-phase and out-of-phase modal current distributions are two orthogonal eigenvectors of the two-element microstrip array from the perspective of the coupled-mode approach. Thus the excitation signal \(I^{inc}(0)\), which is obtained by eqn. 6 provided that active devices oscillate equal amounts of instantaneous amplitude \(\xi\), may be expressed by superposition of \(N\) eigenvectors based on the eigenfunction approach, i.e.

\[
I^{inc}(0) = \sum_{k=1}^{N} \alpha_k \xi_k
\]

where \(I^{inc}(0)\) is the oscillating signal on the \(z = 0\) plane, \(\xi_k\) is the eigenvector of \(A\), and \(\alpha_k\) represents the modal ampli-
waveguide) he to integrate with a quasi-optical oscillator. Transition, the slotline is transformed into a CPW (coplanar scheme described above. Thus, an L-type matching circuit is established to compensate for the imaginary part of the input impedance of the two-element active array. Antenna design

A two-element microstrip leaky-mode antenna array integrated with a quasi-optical oscillator presents the prototype of a proof-of-concept design. The slotline underneath the microstrip is employed to excite the EH1 mode efficiently [8], and also acts as a short-circuited tuning stub to compensate for the imaginary part of the input impedance of the antenna. Thus, an L-type matching circuit is established in a very compact fashion. Followed by a CPW-to-slotline transition, the slotline is transformed into a CPW (coplanar waveguide) line to integrate with a quasi-optical oscillator. Fig. 1b shows the single antenna impedance matching scheme described above.

3 Proposed proof-of-concept design of a two-element active array

3.1 Antenna design

A two-element microstrip leaky-mode antenna array built on a 25"-thick RT/Duroid 6010 substrate with a thickness of the substrate was optimised to maximise power output for the desirable pointing direction of the antenna. The complete quasi-optical oscillator was built on a 25mm-thick RT/Duroid 6010 substrate with a thickness of 25mm. The quasi-optical oscillator was optimised to maximise power output for the desirable pointing direction of the leaky-mode antenna. Realising the compact antenna design (as Fig. 1b shows) needs the quantitative assessment of the characteristic impedance for the microstrip leaky mode to provide an insightful circuit-domain view of the leaky line. The input impedance of the microstrip leaky-mode antenna with length L is expressed as

\[ Z_{\text{in}}(\omega) = -jZ_c(\omega) \cot(kzL) \] (10)

where \( Z_c \) is the characteristic impedance of the microstrip leaky mode given by [12, 13] as

\[ Z_c = \frac{1}{s} \int E \times H^* \cdot \hat{z} \cdot d\mathbf{a} / |I|^2 \] (11)

where \( I \) is the total current on the metal strip, and \( S \) is the cross-sectional area of the microstrip. It is well known that \( c \) is the free-space wavelength, \( \beta_1 (\phi) \) is the phase (attenuation) constant of the \( i \)th coupled EH1 mode and \( \xi_0 \) represents the \( k \)th element at the \( i \)th eigenvector. Meanwhile, the total far-zone electric field \( E_\phi \) is the superposition of the N-element microstrip leaky-mode array expressed as

\[ E_\phi \approx \sum_{k=1}^N E_{\phi_k} \exp(j k x_k \cos \theta \cos \phi) \] (9)

where \( x_k \) is the location of the \( k \)th microstrip. Thus, the radiation characteristics of the coupled microstrip array can be simulated by the following procedures:

(a) Solving the coupled EH1 modes of a three-element array by using rigorous full-wave analysis makes it possible to deduce the coupling parameters using eqn. 13.

(b) Substituting the computed coupling parameters (step (a)) into the characteristic equation of (\( A \) in eqn. 4 allows the computation of all the coupled EH1 modes (\( \lambda_i, i = 1, 2, ..., N \) and their corresponding eigenvectors (\( \xi_0, i = 1, 2, ..., N \) by solving the standard eigenvalue problem (eqn. 5).

(c) By substituting computed coupling parameters (in step (a)) into eqn. 6, we can solve the steady-state phase difference (\( \theta \)) between oscillators for an empirically determined Q-factor to obtain \( \tilde{Q}(0) \).

(d) With computed \( \tilde{Q}(0) \) and \( \xi_0 \), we obtain \( N \) excited modal amplitudes (\( \alpha_i, i = 1, 2, ..., N \) by solving \( N \) linear independent equations derived by eqn. 7.

(e) Substituting computed values of \( \alpha_i, \lambda_i \) and \( \xi_0 \) into eqn. 8 makes it possible to simulate the far-field pattern of the injection-locked microstrip leaky-mode array.

3.2 Quasi-optical oscillator design

Various oscillator designs share similar design procedure regardless of the problem being solved. We first employ linear analysis to obtain the first-order design parameters of the proposed circuit schematic diagram as shown in Fig. 1b and then apply harmonic balance analysis to predict the oscillation frequency and to optimise the output power. The simulation result indicates the first harmonic at a frequency of 9.4GHz, the value of shunt inductance (\( L_s \)) can be calculated based on circuit theory so that the microstrip leaky-mode antenna is matched to the slotline impedance of \( R_s \) (95\( \Omega \)). The calculated values of \( R_s \) and \( L_s \) allow the dimensions of the slotline of width 15mm and length 142mm to be determined.

![Fig. 4](image-url)
relative dielectric constant of 10.2. For a free-running situation, the near-field pick-up measurement shows that the unlocked source oscillates at 9.415GHz with 11.8dBm power level, which is very close to the simulated result. Meanwhile, the measurement also observes DC-to-RF efficiency of 23% and phase noise of -90dBc/Hz at a 10kHz offset from the carrier.

4 Measurement results

The theoretical prediction of the coupling parameters for the leaky-mode antenna array was first validated experimentally using an imaging technique [6]. A single active microstrip leaky-mode antenna was tuned to a measured free-running frequency of 9.415GHz; the microstrip leaky-mode antenna was then positioned near a vertical ground plane, thus simulating two identical, in-phase (for the odd-symmetrical nature of the microstrip E\textsubscript{H} mode) coupled oscillators. With varying position of the microstrip leaky-mode antenna, a frequency shift was evident which relates to the coupling parameters. Fig. 5 displays the results, and indicates that the theoretical prediction by this approach and by the empirically determined Q-factor of 14.1 compares very favourably with array measurement. Subsequently a prototype, two-element, active array was built for experimental validation.

For a free-running situation, the near-field pick-up measurement shows that the unlocked source oscillates at 9.415GHz with 11.8dBm output power. Then, an injection locking measurement is taken. As an external stable source of 12dBm at 9.426GHz is injected into one end of the array, the oscillator is locked and synchronised to the frequency of the injection signal through the mutual-coupling interaction between antennas. Fig. 6 shows the spectrum of an injection-locked oscillator, demonstrating 23MHz locking ranges. Furthermore, the measured ERP (effective radiated power) of a single- and two-element array is 22.5dBm and 27.3dBm, respectively. Fig. 7 (Fig. 8) plots the measured far-field pattern in the azimuth (elevation) plane cut at the peak value of the main lobe as a comparison with ones by using the rigorous analysis described in Section 2, and good agreement is achieved. As the frequency of the external source sweeps, the antenna will simultaneously scan it by phase control in azimuth and by frequency control in elevation. Fig. 9 plots the measured far-field patterns in the azimuth plane (x-y plane) cut at the peak value of the main beam corresponding to the injected frequencies at 9.406, 9.410 and 9.415GHz, respectively. The antenna beam is scanned to $\phi_{\text{th}}$ (x-y plane) at $36^\circ$, $13.4^\circ$ and $19.5^\circ$ for the respective injection signal frequencies, showing good agreement with measured results.
Fig. 9 Measured far-field beam scanning patterns controlled by frequencies of injection signal
F = (i) 9.406GHz; (ii) 9.41GHz; (iii) 9.415GHz; (iv) 9.41GHz (single element)

5 Conclusion
This work has presented an injection-locked microstrip leaky-mode antenna array in which beam scanning has been achieved without using dedicated phase shifters. The coupled-mode approach is adopted to analyse and design an injection-locked coupled microstrip leaky-mode antenna array that considers the mutual coupling of the leaky lines. Finally, this study experimentally verifies the novel design via a two-element, proof-of-concept design, exhibiting 23MHz locking bandwidth, 27.3dBm ERP and one-sided continuous H-plane beam scanning from 5° to 17° for 10MHz offset from the free-running frequency of 9.415GHz.

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7 References

8 Appendix: Formulation of coupling coefficients
Roots of the characteristics polynomial function of $\bar{A}$ are essentially the complex propagation constants for the coupled microstrips array. Thus we can rewrite eqn. 5 as
$$\det(\bar{A}) = \lambda^N + b_{N-1}\lambda^{N-1} + \cdots + b_1\lambda + b_0 \equiv\prod_{i=1}^{N} (\lambda_i - \lambda)$$
where $\lambda_i (i = 1-N)$ are eigenvalues to be solved and $b_i (i = 1-N)$ are the constant coefficients which are a function of the 'undisturbed' leaky mode ($\gamma$ and coupling coefficients ($C_{ij}$)). Expanding the determinant (det$(\bar{A})$), and comparing order by order at both sides of eqn. 12 for $N = 3$, we obtain the following equations for solving the known coupling $C_{12}$ and $C_{13}$, representing the coupling parameters of adjacent and other-than-adjacent elements, respectively:
$$\gamma = (\lambda_1 + \lambda_2 + \lambda_3)/3$$
$$\gamma^2 - 2C_{12}C_{13} = -C_{12}C_{13} + C_{13}^2 \gamma = \lambda_1\lambda_2 + \lambda_2\lambda_3 + \lambda_3\lambda_1$$
$$3\gamma^2 - 2C_{12}^2 - C_{13}^2 = \lambda_1\lambda_2\lambda_3$$