3D monochromatic image synthesized with vertical area-partitioned recording of master hologram in multiple-exposure holography

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Abstract

We report for the first time theoretical analysis and experimental results of a vertical-area partition method for recording master holograms in multiple-exposure rainbow holography to synthesize monochromatic 3D image from a series of medical tomograms. In this novel method, the master hologram is area partitioned into elementary master holograms, which are recorded in a periodic arrangement along the vertical direction. Under the white-light reconstruction, a 3D monochromatic image composed of a series of medical tomograms can be synthesized with wide viewing angle, high resolution, and low color blur. © 2001 Published by Elsevier Science B.V.

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1. Introduction

Two-step multiple-exposure rainbow holography can be employed to synthesize a 3D image from a series of 2D tomograms, where horizontal-area partition (HAP) method is used to record the master hologram with the multiple-exposure rainbow holography. That is, the master hologram is horizontally area-partitioned into a series of slitted elementary master holograms oriented in the vertical direction. On each of which, a tomogram placed at the relative position correspondent to that in the original object is recorded. During the final white-light reconstruction, a 3D image can be seen through the simultaneous read-out of the multiple holograms [1–4]. Due to the dispersion, however, the reconstructed images from the different elementary holograms will appear in different colors and in different positional recovery, if no special treatment is accompanied. This kind of image distortion can easily result in false gray level and perspective judgments to human vision [5]. Some efforts have been made on to partially alleviate these disadvantages [6–8]. A method of inclining the master hologram [4,9] at the recording
was proposed to eliminate this effect, and a dispersion compensation grating [10,11] has been employed to obtain achromatic reconstruction. All these methods need either complicated calculations or critical experimental conditions.

In this paper we report for the first time theoretical and experimental results of synthesizing monochromatic 3D images from a series of medical tomograms with vertical-area partition (VAP) method for recording the master hologram in the two-step multiple-exposure rainbow holography. By employing this VAP method, a 3D image, which consists of a series of monochromatic images of 2D tomograms, can be read-out simultaneously during the final white-light reconstruction. The monochromatic 3D image has the characteristic of wide viewing angles, high resolution, and little color blur.

In Section 2, a comparison between the constructions of HAP and VAP method for recording the master hologram will be presented, which is followed by the analysis of the difference of the viewing effect in Section 3. In Section 4, further analysis from the viewpoint of the dispersion windows of the both methods is introduced. Section 4 discusses about the resolution limit and the color blur of the VAP method. Experimental results of the 3D synthesis are given in Section 5. Finally, conclusions are made in Section 6.

2. HAP and VAP

We first briefly review the conventional HAP method and our VAP method in the first step for recording master hologram in a two-step multiple-exposure rainbow holography to synthesize 3D images.

In the HAP method, the master hologram is area-partitioned into a series of vertical slitted elementary holograms sitting side by side horizontally R1 (along Y-direction, see Fig. 1(a)), with each slit recording a different tomogram, and the neighboring slitted elementary master hologram recording the successive frame of the 2D tomogram. Fig. 1(a) shows the optical layout of the HAP [3,12]. The master hologram H1 is located in the XY-plane. A narrow vertical slit R2 (it has its length in X-direction) can be shifted in the Y-direction, which sets the region of the recording for each elementary master hologram.

In our VAP method, instead, the master hologram is area-partitioned into a series of horizontal slitted elementary master holograms along the vertical direction, by using a horizontal slit to set the recording regions of the elementary master holograms. The slit has its length along the Y-direction, and can be shifted in the X-direction during the successive recording of the elementary tomograms. The schematic diagram for the VAP method of recording the master hologram is shown in Fig. 1(b).

The rest of the procedures in both of these two methods are exactly the same. During the second recording step, (see Fig. 2), R1′, a conjugated reference beam to R1, is used to reconstruct the images of the tomograms recorded on H1. H2 is placed at the central position of the reconstructed real images of the tomograms and illuminated by a recording reference beam R2. Therefore, the slits on H1 and the reconstructed real images of the tomograms from H1 will be recorded on H2. By the final white-light reconstruction, H2 is illuminated by a collimated white-light beam on a conjugate direction and all the reconstructed tomograms can be viewed simultaneously through the respective real images of the slits. A 3D image composed of a series of tomograms is thus synthesized. R3: Besides, each of the reconstructed real images of the slits during the white-light reconstruction also sets a limit to the viewing field for observing the respective reconstructed image recorded on each elementary hologram.

3. Different viewing results

The optical setup systems recording the master holograms, R4: both in the case of HAP and VAP, are with the identical instrumental arrangement. The distance between the input objects (i.e., the 2D tomograms) and each of the elementary master holograms is much larger than the dimensions of each slit. Hence, the wave vector $\vec{k}$ of the object wave can be considered approximately to be mainly along the Z-direction. The plane wave
reference beam, $R_1$, for recording all the tomograms is perpendicular to the $X$-axis (see Fig. 1(a) and (b)). Consequently, the orientation of the interference fringes on the master hologram $H_1$ is mainly along the $X$-direction. The spacing of the interference fringes on the $H_1$ will, therefore, only depend on the angle between the normal of the reference wave vector and the $Z$-axis. Since the conditions of the optical waves for recording hologram $H_2$ is similar to that for recording the master hologram $H_1$, the fringe orientation on the second hologram $H_2$ are similar to that of $H_1$. Therefore, the dispersion of the reconstructed images during the final white-light reconstruction, R5: either in the case of HAP or VAP, will spread mainly along the $Y$-direction.

It is well known that the read-out of rainbow holography is based on the dispersion effect during the white-light reconstruction. By multiple-exposure rainbow hologram, the construction of the multiple slits on the $H_1$ should also be taken into consideration, for the orientation of the slits will also affect the viewing effect, and thus affect the final simultaneous read-out of the different 2D tomograms.

During the final white-light reconstruction step of the multiple-exposure rainbow hologram, R6: for both HAP and VAP all the real images of the slits on $H_1$ are reconstructed simultaneously in a form of observation window. Due to the dispersion effect, the observation window will disperse into a series of observation windows with different

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Fig. 1. Recording of slitted elementary master holograms: (a) HAP, (b) VAP ($H_1$: master hologram, $S$: slits, $R_1$: collimated recording reference beam, $\theta_1$: angle between reference and object beam, $D$: diffuser, $O$: object beam, $T$: tomogram).
colors. Depending on the orientation of the area-partition in recording the elementary master holograms, by HAP or VAP, however, the synthesized tomograms will have different viewing effects.

This can be analyzed quantitatively by using the Champagne’s imaging equations [13], which here-on express the relation between the coordinates of a point real image reconstructed from H₁ or that of the central point of the related elementary master hologram on H₁ in the second recording, \((x₀, y₀, z₀)\), and that of the corresponding image reconstructed from H₂ \((x, y, z)\) during the final white-light reconstruction:

\[
x_i = x_0 \tag{1}
\]

\[
y_i = y_0 - z_0 \sin \theta'_2[(\lambda_0/\lambda_i) - 1] \tag{2}
\]

\[
z_i = (\lambda_0/\lambda_i)z_0 \tag{3}
\]

R7: where \(\theta'_2\) is the angle to Z-axis of the final reconstructing beam complementary to that of the final recording. \(\theta_2\), \(\lambda₀\) is the optical wavelength used for the recordings and \(\lambda_i\) represents the wavelength included in the white-light reconstruction. It can be seen from Eqs. (1)–(3) that, both in the case of HAP and VAP, with different optical reconstruction wavelengths, the center position of a reconstructed slit or the tomogram recorded within it will shift along the Y and the Z-directions, but remain the same in the X-direction. This effect is demonstrated in Figs. 3 and 4, respectively, where each observation window is composed of the reconstructed slits with the same value of \(\lambda_i\) (same \(x₀\) and different \(y₀\) in Fig. 3 for HAP, or same \(y₀\) and different \(x₀\) in Fig. 4 for VAP). It is noticed that different windows in different wavelengths shift along the directions of Y and Z, according to Eqs. (2) and (3). The locus of the centers of the observation windows in different wavelengths traces a straight line at an angle \(\alpha\) to Z-axis. By substituting Eq. (3) in Eq. (2), the \(\alpha\) can be calculated:

\[
\tan \alpha = \sin \theta'_2 \tag{4}
\]

This expression is similar to that of the achromatic angle in achromatic holographic stereogram reported by Benton [9].

Thus, by the HAP approach, the whole series of the reconstructed images of the tomograms have to be viewed simultaneously through the respective real images of the slits in different viewing windows, along, or with a angle much less than \(\alpha\), to Z-axis (see Fig. 3). As a result, the rainbow effect and false positional recovery will be seen among the depth of the synthesized 3D image. This may easily result in a false judgement of gray level and perspective recovery to human eyes, due to the human photopic eye response [5].

By the VAP approach, the dispersion effect of the reconstructed images will also remain along the Y-direction and Z-direction. However, the direction of the dispersion is vertical to that of the area-partition, the X-direction. Therefore, each of the dispersed windows can be viewed at different viewing angles within the image field of the reconstructed tomograms, other than \(\alpha\), to Z-axis without overlapping, and all the reconstructed 2D tomograms can be viewed simultaneously with one wavelength through a single viewing window (see Fig. 4). Consequently, the rainbow effect as well as the false positional recovery will not appear among the different depths of the synthesized 3D images.
Fig. 3. Observation windows in white-light reconstruction and viewing effect of HAP: (a) top view, (b) perspective view. By the white-light reconstruction of H₂, all the real images of the slits on the master hologram H₁ will be simultaneously reconstructed in the form of observation window, which will be dispersed into a series of windows with different wavelengths along a straight line at an angle z to Z-axis. A series of corresponding reconstructed images of the tomograms can be only viewed through different observation windows with different wavelengths, resulting in a rainbow distribution among the depth of the synthesized 3D image. Besides, false recovery of position will appear generally, due to the relative distribution of the dispersed windows shown in the illustration, as an example (H₂: holograms for reconstruction, z: dispersion angle, C: white-light for reconstruction, W: construction of the observation window, Ts: reconstructed images, R₉: viewed along –Z direction, Z₀: distance between H₂ and the observation window of λ₉, λ₀: wavelength same as in the recordings, λ₁: longer wavelength than in the recordings, λ₉: shorter wavelength than in the recordings).
Fig. 4. Observation windows in white-light reconstruction and viewing effect of VAP: (a) top view, (b) perspective view. By the white-light reconstruction of $H_2$, all the real images of the slits on the master hologram $H_1$ will be simultaneously reconstructed in the form of observation window, which will be dispersed into a series of windows with different wavelengths along a straight line at an angle $z$ to $Z$-axis. All the reconstructed images of the tomograms can be viewed through only a single window with the identical wavelength. No rainbow effect or false positional recovery will be observed at any possible viewing angle, other than $z$, R10: which ensures a monochromatic synthesis of the 3D image with better resolution. A series of reconstructed images, $T_s$, is shown in the top of the figure observed along the $-Z$ direction, as an example ($H_2$: holograms for reconstruction, $z$: dispersion angle, C: white-light for reconstruction, W: construction of the observation window, $T_s$: reconstructed images viewed along $-Z$ direction, $Z_0$: distance between $H_2$ and the observation window of $\lambda_0$, $\lambda_0$: wavelength same as in the recordings, $\lambda_C$: longer wavelength than in the recordings, $\lambda_s$: shorter wavelength than in the recordings).
4. Resolution of the reconstructed images by VAP

It is known that the resolution and the color blur of a rainbow hologram are generally related to the slit diffraction [14]. For the VAP approach, the vertical diffraction is limited by \( w \), the width of the slits on \( H_1 \). The resolution limit of the reconstructed image can be approximately expressed by [15]

\[
E_i = \frac{\lambda_i(Z_0 - Z_0)}{w}
\]

where \( Z_0 \) is the distance between \( H_2 \) and \( H_1 \), \( Z_{0i} \) is the distance between \( H_2 \) and the real image of the \( i \)th tomogram reconstructed from \( H_1 \), at the second recording, and \( \lambda_i \) is one of the wavelength components included in the white light for reconstruction. By substituting the experimental conditions of VAP, \( \lambda_i = 550 \text{ nm} \), \( Z_0 = 500 \text{ mm} \), \( |Z_{0i}| \leq 30 \text{ mm} \), and \( w = 3 \text{ mm} \), into Eq. (5), the minimal resolvable spacing is calculated as \( E_i \leq 0.1 \text{ mm} \). Since the physical limitation of human vision is not as good as 0.1 mm, hence the diffraction effect caused by the slitted elementary holograms can be neglected, as long as the width of the slit \( w \) is at least 3 mm.

The color blur due to the horizontal dispersion is affected by \( l \), the length of the slit. Assuming that the eye is located at the real image of the slit and that the eye pupil is much smaller than \( l \), Yu [15] has shown that the length of the spectrally dispersed image in \( Y \)-direction of one object point can be given by:

\[
\Delta y \approx Z_0/l/Z_0
\]

which can be considered as the resolution limit of the image. Eq. (6) implies that a higher resolution along the horizontal direction can be obtained by reducing the slit length \( l \), at the cost of losing brightness of the reconstructed images.

A balance between the resolution and the brightness of the reconstructed images can be achieved by optimizing the width and the length of the slitted elementary master holograms. Besides, periodic groups of slitted elementary holograms can be recorded repeatedly in a cycle structure, so that the effective total recording area and, consequently, the brightness of the reconstructed images is enlarged.

5. Experimental results

In the first step of the experiment, the VAP approach is employed to record the slitted elementary master holograms on \( H_1 \). To record \( H_2 \) in the second step, \( H_1 \) is illuminated by a reference beam conjugate to that used for recording \( H_1 \). Thus, a series of real images of the 2D tomograms were obtained. The holographic plate for recording \( H_2 \) is positioned at \( Z_0 \), the center of these real images, so that quasi-image holograms and an image-hologram at the position of the plate, of the real images, as well as the slits on \( H_1 \) as objects, are recorded on \( H_2 \).

Coherent light of the wavelength 532 nm from a Verdi laser was used for all the holographic recordings. All the holograms were recorded with silver-halide emulsion plates of type TJ-III made in Tianjin, China. In the recordings, the hologram plates were placed on the \( X-Y \) plane and the reference beam will be kept at an angle of \( \theta_2 = 28^\circ \) to \( Z \)-axis or complementary to that. At the second recording, \( H_2 \) was recorded at a distance of \( Z_0 = 500 \text{ mm} \) away from the master hologram \( H_1 \), which was at the same value of the distance between \( H_1 \) and the center of the tomograms series in the first recording. In this way, different series of medical tomograms are used to synthesize 3D monochromatic images by VAP approach. A circulation technique is employed to periodically record the slitted elementary master hologram. That is, to synthesize a series of 20 pieces of human knee-joint tomograms with this technique, the recording area of the master holographic plate of \( 240 \times 30 \text{ mm}^2 \) is area-partitioned into 80 slitted elementary holograms, which are divided into four periodic groups along the vertical direction with 20 slits in each period. In this way, each tomogram will be repeatedly recorded in four respective elementary master holograms, which offers a larger effective recording area, so that higher brightness of the reconstructed images and wider viewing angles of the final read-out in the white-light.
reconstruction can be achieved. For the series of 15 pieces of human brain tomograms, a master holographic plate of $180 \times 30 \text{ mm}^2$ is area-partitioned in the same way.

Figs. 5 and 6 show, respectively, the monochromatic 3D images synthesized from 15 pieces of human brain tomograms and 20 pieces of human knee-joint tomograms. As can be seen from the figures, there is no distortion in gray levels and in position recovery. Also the brightness and resolution appear to be satisfactory for the medical inspection.

6. Discussions and conclusions

We have reported and demonstrated a VAP method for recording master holograms in the multiple-exposure rainbow holography. This technique has been successfully used to synthesize monochromatic 3D images from a series of medical 2D tomograms by a white-light reconstruction. Our theoretical and experimental results show an advantage of the VAP method over the conventional HAP method that any false recovery in gray levels or position will not be induced within the viewing field of the synthesized mono-chromatic 3D image, without carrying out any extra experimental arrangement or complicated theoretical calculations.

In order to alleviate the horizontal color blur during the white-light reconstruction for higher resolution, the slit for recording the master hologram should be maintained within a certain length. The reduction in the brightness can be compensated for by a circulation technique of periodic recording. This technique can further provide a wider viewing angle in the vertical direction.

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