Analysis of Passively $Q$-Switched Lasers With Simultaneous Modelocking

Yung-Fu Chen, Jian-Lung Lee, Hung-Dau Hsieh, and Sheng-Wei Tsai

Abstract—Simultaneous $Q$-switching and modelocking in a diode-pumped Nd:YAG/Cr$^{4+}$: YAG laser is experimentally demonstrated. A general recurrence is derived for the analysis of the temporal shape of a single $Q$-switched envelope with modelocked pulse trains. With the developed model, the modelocked pulse energy and the total $Q$-switched pulse energy can be calculated. Excellent agreement was found between the present results and detailed theoretical computations.

Index Terms—Mode locking, passively $Q$-switched laser, saturable absorber, solid-state laser.

I. INTRODUCTION

Simultaneous $Q$-switching and mode locking is of great interest for the generation of high peak power and ultrashort pulses. The high peak power from the simultaneously $Q$-switched and modelocked laser is beneficial to wavelength conversion in an external nonlinear crystal [1]. Moreover, the pulse trains contained in $Q$-switched and modelocked lasers can advantageously be used to study dynamic optical nonlinearities in the 10~1000-ns time interval [2]. The non-linear absorption of saturable absorbers was first successfully employed for the simultaneous $Q$-switching and modelocking of solid-state lasers in 1965 [3], [4]. In early technologies, dyes were commonly employed in pulsed, modelocked solid-state lasers. Recently, the generation of passively $Q$-switched and modelocked pulses has been accomplished in diode-pumped Nd:YVO$_4$ lasers with a Cr$^{4+}$: YAG saturable absorber [5]–[7].

Several recent theoretical investigations [8]–[10] were proposed to optimize the performance of $Q$-switched lasers. Degnan [8] derived the key parameters of an energy-maximized, passively $Q$-switched laser as functions of two variables and generated several design curves. More recently, Xiao and Bass [9] and Zhang et al. [10] followed Degnan’s approach to include the effect of excited-state absorption (ESA) of the saturable absorber into the analysis. However, modeling for the passively $Q$-switched laser with simultaneous modelocking has not been reported.

In this work, we present the results of experiments in which a diode-pumped Nd:YAG/Cr$^{4+}$: YAG laser has been simultaneously $Q$-switched and modelocked. A general model including the influence of ESA was developed to analyze the modelocked pulse energy within a single $Q$-switched envelope. Using the hyperbolic secant square function to model the modelocked pulse, the temporal shape of a single $Q$-switched pulse can be reconstructed. The theoretical calculations have shown fairly good agreement with the experimental results.

II. EXPERIMENTAL RESULTS

The present setup is basically similar to the previous cavity for the Nd:YVO$_4$/Cr$^{4+}$: YAG passively $Q$-switched and modelocked laser [6], as shown in Fig. 1. The pump power is a 30-W fiber-coupled diode-laser array with the output wavelength of the lasers at 25°C, ranging from 807 to 810 nm. The fibers were drawn into round bundles of 0.8-mm diameter with a numerical aperture of 0.18. A focusing lens with 20-mm focal length and 85% coupling efficiency was used to re-image the pump beam onto the laser crystal. The waist diameter of the pump beam was about 600 µm. The 1.0-at.% Nd$^{3+}$, 10-mm-long Nd: YAG crystal was coated for antireflection at 1064 nm ($R < 0.2\%$) on both end surfaces. The laser crystal was wrapped with indium foil, and mounted in a water-cooled copper block. The water temperature was maintained at 20°C. The cavity was designed to easily allow mode matching with the pump beam. Several Cr$^{4+}$: YAG crystals with different initial transmissions were used in the experiment. Both sides of the Cr$^{4+}$: YAG crystal were antireflection coated at 1064 nm. The resonator consisted of three highly reflective (at 1064 nm) mirrors—M1, M2, and M3—and one output coupler. Mirror M1 is a flat mirror, and the radii of curvature for M2 and M3 are 50 and 10 cm, respectively. M2 and M3 were separated by 60 cm. The flat output coupler was 1.0° wedged. The total cavity length was approximately 1 m. The radius of the cavity mode was about 300 µm. The ratio between the effective area in the gain medium and in the saturable absorber...
can be easily changed by changing the position of the Cr<sup>4+</sup> : YAG crystal. Moving the Cr<sup>4+</sup> : YAG crystal away from the output coupler decreases the ratio $A/A_0$ in the present cavity. The experimental results show that a ratio greater than 1.0 is sufficient for the effective $Q$-switching process. The pulse temporal behavior was recorded by a LeCroy 9362 oscilloscope (500-MHz bandwidth) and a fast Si p-i-n photodiode with a rise time of $\sim0.35$ ns.

When diode-pumped, passively $Q$-switched lasers are operated in the CW-pumped condition [11]–[15], the pulse repetition rate usually increases with an increase of the pump power. However, the interpulse time jittering is generally greater than 5%. In this work, we modulated the laser diode with a 200-μs pump–pulse duration to avoid the time jitter in the pulse trains. First, we used an output coupler with $R = 0.74$ to generate simultaneous $Q$-switching and modelocking. The experimental results reveal that the time jitter of the pulse trains is less than 0.5% for modulation frequencies lower than 3.5 kHz. The threshold pump energies for $Q$-switched behavior are 1.2, 2.0, 2.8, 3.1, and 3.4 mJ for saturable absorbers of $0.95$, $0.82$, $0.65$, $0.50$, and $0.35$, respectively. The output pulse energies of the $Q$-switched pulse train at the pump threshold are 0.05, 0.14, 0.20, 0.25, and 0.3 mJ for saturable absorbers of $0.95$, $0.82$, $0.65$, $0.50$, and $0.35$, respectively. The pulse-to-pulse amplitude fluctuation of the $Q$-switched pulse train was found to be less than $\pm5\%$. Experimental results show that a nearly 100% depth of modelocking was achieved for all cases. The expanded temporal shapes of a single $Q$-switched pulse for saturable absorbers of $0.65$, $0.50$, and $0.35$ are shown in Fig. 2. Note that the modelocked pulse duration shown in Fig. 2 was limited by the response time of the Si p-i-n diode and oscilloscope. The modelocked pulse duration inside the $Q$-switched pulse was measured using an autocorrelator (KTP type-II interaction) in collinear configuration [6]. Note that we only measured the average pulse duration because it is difficult to characterize single modelocked pulses underneath the $Q$-switched envelope. The temporal duration of the modelocked pulses is nearly independent of $T_0$ and the value is measured to be about 200 ps. For $T_0 = 0.35$, the peak power near the maximum of the $Q$-switched envelope was greater than 200 kW.

Using an output coupler with $R = 0.59$, a 100% depth of modelocking could also be obtained. The threshold pump energies for $Q$-switched behavior are 2.2, 2.8, 3.4, 3.7, and 4.0 mJ for saturable absorbers of $0.95$, $0.82$, $0.65$, $0.50$, and $0.35$, respectively. The output pulse energies of the $Q$-switched pulse train at the pump threshold are 0.03, 0.13, 0.26, 0.32, and 0.40 mJ for saturable absorbers of $0.95$, $0.82$, $0.65$, $0.50$, and $0.35$, respectively. It was found experimentally that changing the output reflectivity evidently did not affect the temporal duration of the modelocked pulses. This result indicates that ultrashort pulse formation is nearly independent of $T_0$ and $R$. On the contrary, the characteristics of $Q$-switched pulse formation mainly depends on $T_0$ and $R$. The peak power near the maximum of the $Q$-switched envelope was greater than 300 kW for $T_0 = 0.35$. This result is three times greater than the peak power obtained in the Nd : YVO<sub>4</sub>/Cr<sup>4+</sup> : YAG laser with the same $T_0$ and $R$.

According to the fluctuation mechanism, the picture of ultrashort pulse formation is the following. Fluctuations in intensity arise due to the interference of a great number of modes having a random phase distribution so that the radiation consists of a chaotic collection of ultrashort peaks in the linear stage of generation. The most intensive fluctuation peaks are compressed and amplified faster than all the weaker ones in the nonlinear stage. Once the pulse intensity rises beyond the saturation intensity range of the absorber, the preferred pulse will not be much further shortened on subsequent roundtrips. Therefore, we consider that the photon intensity shape has the form

$$\phi(t) = \sum_{m=0}^{\infty} \Phi_m f(t - t_m)$$

where $t_m = mt_p$, $\Phi_m$ is the relative amplitude of the modelocked pulses at the $m$th roundtrip and $f(t)$ is the modelocked pulse evolving from the noise such that

$$\int_{-\infty}^{\infty} c\sigma f(t) \, dt = 1.$$
Here, \( f(t) \) is assumed to be a sharp pulse centered at \( t = 0 \) which falls off rapidly in a time short compared to the resonator roundtrip transit time \( t_r \).

Considering the ESA effect in a four-level saturable absorber such as was considered by Hercher [16], the relative amplitude of the modelocked pulses at time \( t_m = nt_r \), after an additional roundtrip through the cavity is given by [17]

\[
\Phi_m = \Phi_{m-1} \exp \left\{ -2\sigma \pi(n(t_m)) l - \left[ 2\sigma_{gs} n_{gs}(t_m) l \right] s + 2\sigma_{es} n_{es}(t_m) L \right\} \left[ \ln \left( \frac{1}{R} \right) + L \right] \]

where

- \( \sigma \) stimulated emission cross section of the gain medium;
- \( n(t_m) \) population density of the gain medium at the \( m \)th roundtrip;
- \( l \) length of the gain medium;
- \( \sigma_{gs} \) ground-state absorption (GSA) in the saturable absorber;
- \( \sigma_{es} \) ESA cross section in the saturable absorber;
- \( n_{gs}(t_m) \) absorber ground-state population density at the \( m \)th roundtrip;
- \( n_{es}(t_m) \) absorber excited-state population density at the \( m \)th roundtrip;
- \( R \) reflectivity of the output mirror;
- \( L \) nonsaturable intracavity roundtrip dissipative optical loss.

Introducing the variable \( \beta = \sigma_{es}/\sigma_{gs} \) and using the condition \( n_{gs}(t_m) + n_{es}(t_m) = n_{so} \), (3) can be rewritten as

\[
\Phi_m = \Phi_{m-1} \exp \left\{ -2\sigma \pi(n(t_m)) l - \left[ 2(1 - \beta)\sigma_{gs} n_{gs}(t_m) l \right] s + \beta \ln \left( \frac{1}{T_0} \right) + \left[ \ln \left( \frac{1}{R} \right) + L \right] \right\} \frac{1}{\left( \frac{1}{T_0} \right) + L} \]

(4)

where \( n_{so} \) is the total density of the absorber and \( T_0 = \exp(-\sigma_{gs} n_{so} l) \) is the initial transmission of the saturable absorber. Note that the condition \( n_{gs}(t_m) + n_{es}(t_m) = n_{so} \) is an assumption introduced by Hercher [16] and adopted by Xiao and Bass [9] to simplify the analysis of passive saturable absorbers. This condition assumes: 1) that the upper terminal level of the GSA relaxes infinitely fast (relative to the temporal duration of the optical pulse) to the lower level of the ESA and 2) that the upper terminal level of the ESA behaves similarly. Namely, it is assumed that the saturable absorber atomic populations are totally contained in either the ground or excited states during the interaction with the optical pulse. These approximations may not be valid for very short modelocked pulsewidths.

Since the \( Q \)-switched laser output pulses are much shorter than both the spontaneous lifetime and the pump period (time between output pulses), spontaneous relaxation and pumping can be safely neglected during the development of the output pulse. Therefore, the equation for the time rate of change of the population inversion density can be expressed as

\[
\frac{dn}{dt} = -\gamma \sigma n \phi_n
\]

(5)

where \( c \) is the speed of light and \( \gamma \) is the inversion reduction factor. Note that the parameter \( \gamma \) can only be treated as a constant when all of the Stark sublevel thermalization rates and multiplet relaxations in the gain medium or absorber are either very fast or very slow relative to the photon decay time of the resonator [18].

Dividing (5) by \( n \), using (1) and (2), and integrating over time from zero to \( t_m \), \( n(t_m) \) is given by

\[
n(t_m) = n(0) \prod_{k=0}^{m-1} \exp(-\gamma \Phi_k) \]

(6)

where \( n(0) \) is the initial population inversion density in the gain medium. \( n(0) \) can be determined from the condition that the roundtrip gain is exactly equal to the roundtrip losses just before the \( Q \)-switch opens. Thus

\[
n(0) = \frac{\ln \left( \frac{1}{T_0} \right) + \ln \left( \frac{1}{R} \right)}{2\sigma l}.
\]

(7)

The equation for the time rate of change of the absorber ground state population density is given by

\[
\frac{dn_{gs}}{dt} = -\frac{A}{A_s} \sigma_{gs}/n_{gs}
\]

(8)

where \( A/A_s \) is the ratio of the effective area in the gain medium and in the saturable absorber. Dividing (5) by (8) and integrating gives

\[
n(0) = \frac{n(0)}{n(0)} \left[ \frac{\ln \left( \frac{1}{T_0} \right) + \ln \left( \frac{1}{R} \right) + L}{2\sigma l} \right] \left[ \frac{1}{\left( \frac{1}{T_0} \right) + L} \right]
\]

(9)

where

\[
\alpha = \frac{1}{\gamma} \frac{\sigma_{gs} A}{\sigma_{gs} A_s}
\]

(10)

The parameter \( \alpha \) indicates how fast the saturable absorber is bleached. The larger the parameter \( \alpha \), the faster the saturable absorber is bleached. Substituting (6), (7), and (9) into (4), the recurrence relation for \( \Phi_m \) is given by

\[
\Phi_m = \Phi_{m-1} \exp \left\{ \prod_{k=0}^{m-1} \exp(-\gamma \Phi_k) - 1 - \left[ \ln \left( \frac{1}{R} \right) + L \right] + \left[ \ln \left( \frac{1}{T_0} \right) \right] \right\}
\]

(11)

For a given initial value \( \Phi_0 \), \( \Phi_m \) can be solved by recurrence (11). Fig. 3 shows the calculated result for a Nd: YAG/Cr: YAG passively \( Q \)-switched laser when using the following parameters: \( \sigma = 6.5 \times 10^{-19} \) cm\(^2\) [19], \( \sigma_{gs} = 70 \times 10^{-19} \) cm\(^2\) [20], \( \beta = 0.28 \) [20], \( \gamma = 1 \), \( A = 0.123 \) mm\(^2\), \( A/A_s = 1 \), \( L = 0.01 \), \( R = 0.74 \), \( T_0 = 0.6 \), and \( \Phi_m = 10^{-3} \). The inversion reduction factor is assumed to be \( \gamma = 1 \), so as to obtain a best fit for the output energy.
In terms of $\phi(t)$, the instantaneous power coupled from the output mirror is given by [17]

$$P(t) = \frac{h\nu A}{t_v} \ln \left( \frac{1}{R} \right) \phi(t)$$

(12)

where $h\nu$ is the laser photon energy and $A$ is the cavity volume occupied by the photons. Substituting (1) into (12), the output power can be expressed as

$$P(t) = \frac{h\nu A c}{2} \ln \left( \frac{1}{R} \right) \sum_{m=0}^{\infty} \Phi_m f(t - t_m).$$

(13)

There have been several theoretical works about the pulse formation in a passively mode-locked laser [1], [21], [22]. A computer simulation of the evolution of a mode-locked pulse train from the noise is shown in [22]. For simplicity, the intensity shape of the mode-locked pulse evolution from the noise is assumed to be the hyperbolic secant square function $f(t) \propto \text{sech}^2(t/\tau_p)$, where the parameter $\tau_p$ is related to the FWHM mode-locked pulsewidth by $\tau(FWHM) = 1.76\tau_p$ [23]. The mode-locked pulsewidth mainly depends on the cavity length and the effective lasing bandwidth. Substituting the $\text{sech}^2$ functional form into (13) and using (11), $\tau_p = 120$ ps, $R = 0.74$, and the parameter values for the present Nd: YAG/Cr$^{4+}$/YAG passively $Q$-switched laser, the temporal shapes of a single $Q$-switched pulse for saturable absorbers of $T_0 = 0.65$, 0.50, and 0.35 were calculated and shown in Fig. 4. It can be seen that the calculated results are in good agreement with the experimental data shown in Fig. 2.

Integrating (13) over time from zero to infinity, the output energy is given by

$$E = \frac{h\nu A}{2\sigma} \ln \left( \frac{1}{R} \right) \sum_{m=0}^{\infty} \Phi_m.$$

(14)

Note that the total pulse energy of the $Q$-switched envelope depends on $\sum_{m=0}^{\infty} \Phi_m$, and not on the mode-locked pulse shape $f(t)$. Using (11), (14), and $A = 0.123$ mm$^2$, the output pulse energy of a single $Q$-switched envelope can be calculated. The value of the beam area $A$ was determined by the $ABCD$ matrix approach with the thermal lensing effect for the present cavity configuration. Fig. 5 depicts the experimental and theoretical results for the dependence of the $Q$-switched pulse energy on initial transmission for $R = 0.74$ and $R = 0.50$. It can be seen that the theoretical calculation agrees very well with the experimental results.
Finally, it is worthwhile to mention that the use of a saturable absorber as a passive model locker in a solid-state laser may have two different modes: CW mode locking and Q-switched mode locking. The criterion for the transition between the regimes of CW mode locking and Q-switched mode locking has been investigated [24], [25]. According to this criterion, a solid-state laser with a Cr$^{4+}$: YAG saturable absorber usually operates in the regimes of Q-switched mode locking because of the large absorber saturation energy.

IV. CONCLUSION

We have demonstrated the use of a Cr$^{4+}$: YAG crystal to obtain a high-peak-power diode-pumped Nd: YAG laser in a Q-switched modelocked mode. The peak power of a single pulse near the maximum of the Q-switched envelope was greater than 300 kW. A general model by including the ESA effect has been developed to reconstruct the temporal shape of a single Q-switched pulse with simultaneous modelocking. The modelocked pulse energy and the total Q-switched pulse energy can be also computed from the derived recurrence. The theoretical calculations show a good agreement with the experimental results.

REFERENCES


Yung-Fu Chen was born in Lukang, Taiwan, R.O.C., in 1968. He received the B.S. degree in electronics engineering in 1990 and the Ph.D. degree from the Institute of Electronics in 1994, both from National Chiao Tung University (NCTU), Hsinchu, Taiwan, R.O.C.

Since October 1994, he has been with Precision Instrument Development Center (PIDC), National Science Council, Taiwan, R.O.C., where his research mainly concerns the development of diode-pumped solid-state laser as well as quantitative analysis in surface electron spectroscopy. Since August 1999, he has been Associate Professor in the Electrophysics Department of NCTU. His main research includes diode-pumped visible lasers, Q-switched lasers, mode-locked lasers, and transverse pattern formation in microchip lasers.

Dr. Chen is a member of the Optical Society of America and the IEEE Lasers and Electro-Optics Society.
Jian-Lung Lee was born in I-Lan, Taiwan, R.O.C., in 1976. He received the B.S. degree in electrophysics in 2000 from National Chiao Tung University (NCTU), Hsinchu, Taiwan, R.O.C., where he is currently working toward the M.S. degree in the Institute of Electrophysics. His research involves passively $Q$-switched and modelocked solid-state lasers.

Hung-Dau Hsieh was born in Taipei, Taiwan, R.O.C., in 1978. He received the B.S. degree in physics from National Center University, Taiwan, R.O.C., in 2000. He is currently working toward the M.S. degree at the Institute of Electrophysics, National Chiao Tung University (NCTU), Hsinchu, Taiwan, R.O.C. His research is focused on passively $Q$-switched and CW modelocked solid-state lasers.

Sheng-Wei Tsai was born in Taipei, Taiwan, R.O.C., in 1969. He received the B.S. degree in physics from National Sun Yat-Sen University, Taiwan, R.O.C., in 1993 and the M.S. degree in electro-optical engineering from National Chiao Tung University (NCTU), Hsinchu, Taiwan, R.O.C., where he is currently working toward the Ph.D. degree at the Institute of Electro-Optical Engineering. His research includes laser technology and laser physics.