A New 2D Analytic Threshold-Voltage Model for Fully Depleted Short-Channel SOI MOSFET's

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Abstract—The exact solution of the 2D Poisson’s equation for the fully depleted SOI MOSFET’s is derived by using a three-zone Green’s function solution technique. Based on the derived 2D potential distribution, the front and back surface potential distributions in the Si film are analytically obtained and their accuracy verified by 2D numerical analysis. The calculated minimum surface potential and its location are used to analyze the drain-induced barrier-lowering effect and further to develop an analytic threshold-voltage model. Comparisons between the developed analytic threshold-voltage model and the 2D numerical analysis are made. It is shown that excellent agreements are obtained for wide ranges of device structure parameters and applied biases.

NOMENCLATURE

- $\varepsilon_{si}$ (or $\varepsilon_{so2}$) Dielectric permittivity of Si (SiO$_2$).
- $q$ Elementary charge.
- $n_i$ Intrinsic carrier concentration of semiconductor.
- $t_{si}$ Thickness of Si film.
- $t_{so2}(t_{ob})$ Thickness of front (bottom)-gate oxide.
- $L$ Effective channel length.
- $k'_m$ Eigenvalue of Region $i$ ($i = I, II, III$) ($k'_m = [n - 1/2] \pi / t_{si}$ for Region I, $k''_m = n \pi / t_{ab}$ for Region II, and $k'''_m = [n - 1/2] \pi / t_{ob}$ for Region III).
- $k_m$ Eigenvalue in all regions ($k_m = m \pi / L$).
- $N_B f(y)$ Doping profile in the Si film, where $f(y)$ is a doping profile function with $f(y) = 1$ for uniform doping concentration.
- $V_{bi}(y)$ Built-in potential of the source/drain/body junctions in Region II.
- $V_{gs}$ Gate-source voltage.
- $V_{ds}$ Drain-source voltage.
- $V_{bs}$ Back gate-source voltage.
- $V_{fb}, f(V_{fb}, b)$ Flatband voltage of the front (back) gate.

$V'_{bs}(V'_{bs}) = V_{gs} - V_{fb}, f(V_{bs} - V_{fb}, b)$.

$D_{xf}(x)(D_{xb}(x))$ Electric displacement at the front (back) Si-SiO$_2$ interface.

$\Phi_i(x, y)$ 2D potential distribution in Region $i$ ($i = I, II, III$).

$E_{x}(x, y)$ 2D vertical electric field distribution in Region $i$ ($i = I, II, III$).

$\Phi^n_x(x)(\Phi^n_y(x))$ Front (back) surface potential in Region II.

$Q^n_B(Q^n_B)$ Fourier coefficient of the bulk charge density with the integer $n (n = 0)$ in Region II

$Q^n_B = \frac{1}{t_{si}} \int_{0}^{t_{si}} (-qN_B f(y) \cos k^n_m y dy$ and

$Q^n_B = \frac{1}{t_{ob}} \int_{0}^{t_{ob}} (-qN_B f(y) \cos k^n_m y dy$.

$D^n_{xf}(D^n_{xb})$ Fourier coefficient of the electric displacement at the front (back) surface

$D^n_{xf} = \frac{2}{L} \int_{0}^{L} D_{xf}(x) \sin k^n_x x dx$ and

$D^n_{xb} = \frac{2}{L} \int_{0}^{L} D_{xb}(x) \sin k^n_x x dx$.

$A^n_x(A^n_y)$ Fourier coefficient of the boundary potential at the source (drain) side in Region I

$A^n_x = \frac{2}{t_{xf}} \int_{0}^{t_{xf}} \phi(0, y) \cos k^n_x y dy$ and

$A^n_y = \frac{2}{t_{yb}} \int_{0}^{t_{yb}} \phi(0, L, y) \cos k^n_y y dy$.

$B^n_x(B^n_y)$ Fourier coefficient of the source boundary potential with the integer $n (n = 0)$ in Region II

$B^n_x = \frac{2}{t_{xf}} \int_{0}^{t_{xf}} V_{bx}(y) \cos k^n_x y dy$ and

$B^n_y = \frac{1}{t_{ys}} \int_{0}^{t_{ys}} V_{by}(y) dy$.
when the channel length of the device is very short. Moreover, the effect of the nonuniformly doped profile in the Si film cannot be taken into account by this kind of analysis. Recently, based on the analysis of Ratnakumar and Meindl [6], Woo et al. [7] separated the 2D Poisson's equation into a 1D Poisson's equation and a 2D Laplace equation. Moreover, the continuity of the electric field across the Si-bottom oxide interface was assumed instead of the continuity of the electric displacement. This assumption may cause a large error for short-channel SOI MOSFET's and an additional series is introduced to improve the accuracy of the potential distribution. Due to the complicated 2D potential distribution, it was difficult for Woo et al. [7] to derive an analytic threshold-voltage model for short-channel SOI MOSFET's.

In order to analytically model the 2D characteristics of short-channel thin film SOI MOSFET's, the 2D Poisson's equation must be solved by incorporating the suitable boundary conditions. The Green’s function technique may give an exact solution for the 2D Poisson’s equation including the nonuniform doping profile. This advantage was first demonstrated in solving the 2D potential distribution of a bulk MOSFET by Lin and Wu [8]. Recently, the multizone solution using the Green’s function technique has been successfully used to describe the electrical characteristics of short gate-length MESFET’s by Chin and Wu [9]. In order to avoid the complexity in dealing with the equivalent charge densities between the regions of different dielectric materials, the multizone solutions with mixed boundary conditions are used in this work to analytically solve the 2D Poisson’s equation in three regions with different dielectrics.

In Section II, the Green’s function solution technique for solving the 2D Poisson’s equation in all regions is introduced and the boundary conditions corresponding to each region are also described. The analytic 2D potential distribution in the silicon region is derived exactly and verified by 2D numerical analysis. In Section III, the derived 2D potential distribution is further used to develop the threshold-voltage model, in which the drain-induced barrier lowering is calculated by the derived 2D potential distribution. In addition, the results of the derived threshold-voltage model are compared with those of 2D numerical analysis. It is shown that excellent agreement between the developed threshold-voltage model and the 2D numerical analysis are obtained, and these verify the applicability of the boundary conditions used in our analysis. At last, a concluding remark is given in Section IV.

II. The Basic Analysis

The basic structure of a thin-film SOI MOSFET for 2D numerical simulation is shown in Fig. 1, where the simplified domain for analytically solving the 2D Poisson’s equation is highlighted by the bolded lines and the boundary conditions used are also listed. In order to avoid the complexity in calculating the equivalent charge density between the regions of different dielectrics, the domain for solving the 2D Poisson’s equation is further divided

\[
B_n^e(B_n^o) = \frac{2}{t_{so}} \int_0^{t_{so}} \left[ V_n(y) + V_{so} \right] \cos k_{n}^2 y \, dy
\]

and

\[
B_o^e = \frac{1}{t_{so}} \int_0^{t_{so}} \left[ V_n(y) + V_{so} \right] \, dy.
\]

\[
C_n^s(C_n^d) = \frac{2}{t_{ob}} \int_0^{t_{ob}} \phi_{III}(0, y) \cos k_{n}^s y \, dy
\]

and

\[
C_o^s = \frac{2}{t_{ob}} \int_0^{t_{ob}} \phi_{III}(L, y) \cos k_{n}^s y \, dy.
\]

\[
\phi_{f, inv} \quad \text{Front surface potential at strong inversion condition}
\]

\[
\phi_{f, inv} = 2\phi_f
\]

and

\[
\phi_f = \frac{k_b T}{q} \cdot \ln \left( \frac{N_b}{n_i} \right)
\]

\[x_{min} \quad \text{Location of the minimum surface potential at the front Si surface.}\]
into three subdomains, as shown in Fig. 1, in which Region I is the front gate-oxide region; Region II represents the Si film; and Region III is the bottom oxide region. Note that the boundary potentials in the y direction in Regions I and III are assumed to vary linearly [7]. The possible errors caused by these assumptions will be discussed later. It is noted that the effects of impact ionization are neglected for simplicity.

Although the 2D Poisson's equations in the front and back oxide regions are reduced to the 2D Laplace equations, the Green's function solution technique is still implemented to solve the 2D potential distribution. Due to different types of boundary conditions and geometries of the regions, the Green's function solution in each region is used and summarized in Table I.

Substituting the Green's function solutions listed in Table I into the Green's theorem [9] and neglecting the free carriers, the general form of the 2D potential distribution in each region can be derived as follows:

\[
\Phi^{(x,y)}(x, y) = \frac{Q^0}{2\epsilon_{\text{si}}} x(L - x) + \sum_{n=1}^{\infty} \frac{Q^n_{\text{si}} \cos k_{\text{si}}(L - x)}{\epsilon_{\text{si}} (k_{\text{si}}^2 + 1)} \sin k_{\text{si}}y \\
+ B_{\text{si}}^{(L - x) + y} + B_{\text{si}}^{(L - x)} + \sum_{m=1}^{\infty} \frac{D_{\text{si}}^{(m)}}{k_{\text{si}} \cosh k_{\text{si}}L} \left[ D_{\text{si}}^{(m)} \sinh k_{\text{si}}L \right]
\]

where the Fourier coefficients have been given in the Nomenclature.
at \( y = 0 \) and \( \Phi_i^I(x, y) \) and \( \Phi_i^III(x, y) \) with respect to \( y \) at \( y = t_{Si} \), we obtain the continuities of the electric displacement densities at the interfaces is thus avoided. To obtain the exact potential distribution, \( D_{Si}^I \) and \( D_{Si}^II \) have to be solved first. By equating (1) and (2) at \( y = t_{Si} \), (3) at \( y = t_{Si} \), \( D_{Si}^I \) and \( D_{Si}^II \) can be obtained. Note that the above equations are exact and the arbitrary doping profile in the Si film can be treated. It is clearly seen that the second term on the right-hand side of (2) demonstrates the charge-coupling effect between the boundaries of source and drain sides, which was overlooked in [7].

In the following analysis, the uniformly doped Si film is assumed for simplicity. In this case, \( f(y) = 1 \), and the Fourier coefficients \( Q_i^B, B_i^I, \) and \( B_i^II \) would vanish except for \( n = 0 \), i.e., \( Q_i^B = - qN_{Bi} \), \( B_i^I = V_{Bi} \), and \( B_i^II = V_{Bi} + V_{ds} \). The 2D potential distribution in Region II is our major concern and can be written as

\[
\Phi_i^II(x, y) = \frac{qN_{Bi}}{2\pi} x (L - x) + V_{Bi} + \frac{y}{L} \cdot V_{ds} + \sum_{m=1}^{\infty} \frac{\sin k_m x}{\epsilon_{Si} k_m} D_{Si}^I \cos(k_m(t_{Si} - y)) - D_{Si}^II \cos(k_m y)
\]

where \( D_{Si}^I \) and \( D_{Si}^II \) have been deduced by equating (4) and (1) at \( y = 0 \) and (4) and (3) at \( y = t_{Si} \), which are expressed as

\[
D_{Si}^I = \frac{\epsilon_{Si} k_m}{d_0 (\sinh k_m t_{Si})} \left[ \frac{d_y^I}{\sinh k_m y} - d_y^I \left( \frac{1}{\tanh k_m t_{Si}} + \frac{\epsilon_{Si}}{\epsilon_{ox}} \right) \tanh k_m t_{ef} \right]
\]

\[
D_{Si}^II = \frac{\epsilon_{Si} k_m}{d_0 (\sinh k_m t_{Si})} \left[ d_y^II \left( \frac{1}{\tanh k_m t_{Si}} + \frac{\epsilon_{Si}}{\epsilon_{ox}} \right) \tanh k_m t_{ef} \right]
\]

in which

\[
d_y^I = -\phi_i^I + \frac{2V_{Bi}}{m\pi} \frac{1 - (-1)^m}{\cosh k_m t_{ef}} + h_i^I
\]

\[
d_y^II = -\phi_i^II + \frac{2V_{Bi}}{m\pi} \frac{1 - (-1)^m}{\cosh k_m t_{ef}} + h_i^III
\]

\[
\phi_i^II = \frac{2}{m\pi} \left( 1 - (-1)^m \right) \frac{qN_{Bi}}{\epsilon_{Si}} + 1 - (-1)^m \frac{qN_{Bi}}{\epsilon_{Si}}
\]

and

\[
\phi_i^III = \frac{1}{m\pi} \left( 1 - (-1)^m \right) \frac{qN_{Bi}}{\epsilon_{Si}} + 1 - (-1)^m \frac{qN_{Bi}}{\epsilon_{Si}}
\]
\[
\begin{align*}
\Phi^I_n(x) &= -\frac{qN_B}{2\epsilon_{Si}} x(L-x) + V_{bh} + \frac{x}{L} V_{ds} \\
&+ \sum_{m=1}^{\infty} \frac{\sin k_m x}{\epsilon_{Si}k_m} (D^I_{m} - D^I_{-m}) \cosh k_m t_{so}.
\end{align*}
\]

From (4), the potential distribution along the front surface of Si film can be obtained as

\[
W_w = \frac{V}{\epsilon_{Si}} \tanh k_m t_{so}.
\]

In order to check the accuracy of the derivations, the calculated results using (5) are compared with those calculated by a 2D numerical simulator [10], in which a new discretized Green's theorem was implemented in the simulator to directly solve the Poisson's equations in different dielectric regions and the finite difference scheme was used to discretize the current continuity equations in the semiconductor region.

Fig. 2(a) shows the calculated front surface potential as a function of the normalized distance from the source edge with the effective channel length as a normalization parameter. It is clearly seen that excellent agreements can be obtained even for a device with the effective channel length as short as 0.3 \( \mu \)m. Note that the number of terms \( (m) \) used in (5) can be as small as 10 to attain such an accuracy. Moreover, the calculated results using Young's model [5] are also shown by the dashed lines in the figure for comparisons. It is clearly seen that the parabolic potential distribution along the y direction is insufficient to describe the effects of the source(drain)/body junctions for short-channel SOI MOSFET's. Obviously, the model developed by Young [5] underestimates the drain-induced barrier-lowering effect. Similarly, the back surface potential can also be obtained from (4) and is expressed by

\[
\Phi^B_n(x) = -\frac{qN_B}{2\epsilon_{Si}} x(L-x) + V_{bh} + \frac{x}{L} V_{ds} \\
+ \sum_{m=1}^{\infty} \frac{\sin k_m x}{\epsilon_{Si}k_m} (D^B_{m} - D^B_{-m}) \cosh k_m t_{so}.
\]

The back surface potential as a function of the normalized distance from the source edge with the effective channel length as a normalization parameter is shown in Fig. 2(b). It is clearly seen that excellent agreements between (6) and 2D numerical analysis are also obtained. The effects of the drain bias are shown in Fig. 3, in which Fig. 3(a) shows the front and back surface potentials of a device.
Fig. 3. The surface potential as a function of the normalized distance from the source edge with the drain bias as a parameter (a) Front and back surface potentials for a 0.6-μm SOI MOSFET operated with different drain biases from 0.05 to 4.5 V. (b) Front and back surface potentials for a 0.3-μm SOI MOSFET operated with different drain biases from 0.05 to 3.0 V.

Lines: Our Model
Symbols: 2-D Numerical Analysis
Solid Lines and Circles: Front Surface Potential
Dashed Lines and Squares: Back Surface Potential
$L_{eff} = 0.6 \mu m$
$V_d = 4.5 V$

$V_d = 3.0 V$

$V_d = 0.05 V$

Fig. 4. Vertical potential distribution in the Si film for different channel lengths and drain biases. Shown in Fig. 4 and excellent agreements can be seen. These indicate that the errors caused by the artificial boundary potentials in Region I and Region III are negligibly small. In our analytic modeling, the effects of free carriers are neglected in the subthreshold region and this assumption is proven to be valid for short-channel thin-film SOI MOSFETs even when the drain bias is large.

III. THE THRESHOLD-VOLTAGE MODEL

From Section II, the surface potential distribution at the front interface can be further rewritten in terms of terminal voltages as

$$\Phi_f^j(x) = -\frac{qN_h}{2e_d} x(L - x) + V_{bi} + \frac{x}{L} V_d + \sum_{n=1}^{\alpha} (V_{gs} \cdot G_f^n + V_{bs} \cdot G_b^n + P^n) \sin k_m x$$

where

$$G_f^n = \left[ 1 - (-1)^n \right] \frac{R_f^g}{d_0^{\alpha}} \left[ \frac{2}{m \pi \cosh k_m L_{eff}} + \sum_{n=1}^{\alpha} \frac{r_{mn}^{\alpha}}{(n - 0.5) \pi} \left[ (-1)^{n+1} - \frac{1}{(n - 0.5) \pi} \right] \right]$$

$$G_b^n = \left[ 1 - (-1)^n \right] \frac{R_b^g}{d_0^{\alpha}} \left[ \frac{2}{m \pi \cosh k_m L_{eff}} + \sum_{n=1}^{\alpha} \frac{r_{mn}^{\alpha}}{(n - 0.5) \pi} \left[ (-1)^{n+1} - \frac{1}{(n - 0.5) \pi} \right] \right]$$
The threshold voltage can be analytically expressed as

\[ V_{th} = \frac{qN_F L^2}{2\epsilon_s} \frac{4(R_n^m + R^m)}{(m\pi)^2} \left[ (1 - (-1)^m) + \sum_{n=1}^{\infty} \left[ \frac{\epsilon_{\alpha} \tanh k_m t_{ob}}{\epsilon_{\alpha} \tanh k_m t_{si}} + \frac{V_{gs}' \sum_{m=1}^{\infty} G^n_m \sin k_m x_{\min}}{L V_{ds}} + \frac{V_{gs} \sum_{m=1}^{\infty} G^n_m \sin k_m x_{\min}}{L V_{ds}} + \sum_{m=1}^{\infty} P^m \sin k_m x_{\min} \right] \right] \]

Differentiating (7) with respect to \( t \), the position of the minimum surface potential can be obtained. Note that the location of the minimum surface potential (\( x_{\min} \)) can only be solved by iteration since no explicit solution for \( x_{\min} \) can be obtained. The calculated \( x_{\min} \) is then substituted into (7) and the minimum surface potential is obtained as

\[ \Phi_{f,\min}^{\|} = \frac{qN_F}{2\epsilon_s} x_{\min} (L - x_{\min}) + V_{th} + \frac{x_{\min}}{L} V_{ds} + V_{gs}' \sum_{m=1}^{\infty} G^n_m \sin k_m x_{\min} + \frac{V_{gs} \sum_{m=1}^{\infty} G^n_m \sin k_m x_{\min}}{L V_{ds}} + \sum_{m=1}^{\infty} P^m \sin k_m x_{\min}. \]  

It is clearly seen that the relation between the minimum surface potential and the external bias can be deduced from (8) when the minimum surface potential reaches the inversion condition \( \Phi_{f,\text{inv}} \) as the applied gate bias is equal to the threshold voltage. Setting \( \Phi_{f,\min}^{\|} = \Phi_{f,\text{inv}} \), the threshold voltage can be analytically expressed as

\[ V_{th} = V_{th,0} + \left\{ \frac{qN_F}{2\epsilon_s} x_{\min} (L - x_{\min}) \right\} \left\{ \sum_{m=1}^{\infty} G^n_m \sin k_m x_{\min} \right\}^{-1} - \frac{x_{\min}}{L} V_{ds} \left\{ \sum_{m=1}^{\infty} G^n_m \sin k_m x_{\min} \right\}^{-1} + \sum_{m=1}^{\infty} (V_{gs}' \cdot G^n_m + P^m) \sin k_m x_{\min} \cdot \sum_{m=1}^{\infty} G^n_m \sin k_m x_{\min} \right\}^{-1}. \] 

Note that an initial guess for the threshold voltage is used to calculate the location of the minimum surface potential, then \( x_{\min} \) is substituted into (9) to solve the threshold voltage, and the iteration procedure is continued until convergence is reached. It is found that the number of iterations is small and the computation time used is short.

In order to compare the results of the developed analytic threshold-voltage model with those of 2D numerical analysis, the threshold voltage derived by the 2D numerical analysis is defined by the drain current as follows: the threshold voltage of a long-channel (10-\( \mu \)m) SOI MOSFET is extracted by checking the \( \Phi_{f,\min}^{\|} - V_{gs} \) relation corresponding to the \( I_{ds}-V_{gs} \) characteristics from 2D numerical analysis. The normalized drain current corresponding to \( \Phi_{f,\min}^{\|} = 2 \phi_{gs} \) is taken as the reference current, then the threshold voltage of a shorter-channel device is extracted by equating the normalized drain current to the specified reference current.

The calculated threshold voltages using (9) for a SOI MOSFET with 70-nm Si film, 15-nm front gate oxide, and 320-nm bottom oxide are compared with the 2D numerical analysis in Fig. 5(a) with the back-gate bias as a parameter. It is clearly seen that excellent agreements between our model and the 2D numerical analysis are obtained and poor agreements for Young's model are demonstrated. Comparisons of the calculated threshold voltages as a function of the effective channel length with the drain as a parameter are shown in Fig. 5(b), in which the thickness of the front gate oxide, the Si film, and the bottom oxide remain the same. Fig. 5(b) shows that good agreements are also obtained for devices with channel length as short as 0.4 \( \mu \)m. However, the discrepancy for shorter channel length devices (for example, \( L_{eff} = 0.3 \mu m \)) operated with larger drain bias is mainly due to the serious drain-induced barrier-lowering effect at the bottom Si/SiO\(_2\) interface, as checked by 2D numerical analysis. It is clearly seen that the underestimation of the drain-induced barrier-lowering effect in [5] results in a smaller threshold-voltage roll-off for shorter effective channel lengths.

The calculated threshold voltages using our model as a function of the effective channel length with the thickness of the front gate oxide as a parameter are compared with those using 2D numerical analysis in Fig. 6. It is easily seen that the developed threshold-voltage model agrees well with the 2D numerical analysis. Similar good agreement can also be observed from Fig. 7, where the threshold voltage is plotted as a function of the effective channel length with the thickness of the Si film as a parameter. The slight discrepancy for shorter channel devices is attributed to the easy penetration of the source/drain built-in electric fields at the back Si surface. From Figs. 6 and 7, it is clearly seen that our analytic model agrees with 2D numerical analysis for different doping concentrations in the Si film. It is concluded that the penetration of the source/drain electric fields is more likely to occur at the back Si surface when the doping concentration in the Si film is lower or when the channel length is shorter. This
The threshold behavior of fully depleted short-channel SOI MOSFET's with different structure parameters under different applied biases.

**IV. CONCLUSION**

The exact solution of the 2D Poisson's equation has been analytically derived by a three-zone Green's function solution technique with the suitable boundary con-
phenomena in different zones. The accuracy of the derived 2D potential distribution in the Si film has been verified by 2D numerical analysis. This indicates that the uses of the artificial linear potentials at the oxide edges are sufficient for solving the 2D Poisson’s equation. Based on the accurate 2D potential distribution, the analytic potential distribution at both front and back surfaces in the Si film are derived. It is shown that the location of the minimum surface potential can only be solved iteratively and the computation effort taken by the iteration procedure is small. Moreover, an analytic threshold-voltage model is derived and compared with the 2D numerical analysis. It is clearly shown that good agreements are obtained between the developed threshold-voltage model and the 2D numerical analysis for wide ranges of device structure parameters and applied biases. The accurate analytic threshold-voltage model can provide a fast physical analysis of the short-channel effect and further give the scaling rule for deep-submicrometer thin-film SOI MOSFET’s in ULSI.

REFERENCES