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A new type of optical heterodyne polarimeter

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Received 21 May 2002, in final form 29 October 2002, accepted for publication 5 November 2002
Published 26 November 2002
Online at stacks.iop.org/MST/14/55

Abstract
Phase variations occur when a circularly polarized light beam either passes through a chiral solution or is reflected from a non-absorbing material. They can be measured accurately with a circular heterodyne interferometry. These data are substituted into the special equations derived from Jones calculus. The chiral parameter and the average refractive index of a chiral solution can be estimated simultaneously in just one optical configuration. This polarimeter has the merits of both the common-path interferometer and the heterodyne interferometer. The proposed device was validated in this work.

Keywords: chiral parameter, refractive index, heterodyne interferometry, standards and calibration

1. Introduction
A chiral solution is often used in biological technology, medicine, and pharmaceutics [1, 2]. Its chiral parameter and average refractive index are very important characteristics for the understanding of its quantum structure and geometric configuration [3].

Polarimeters are commonly used to measure the chiral parameter. This is related to the measurement of light intensity variations [4]. Thus, the stability of the light source, the scattering light, and the internal reflections influence the measurement accuracy. Chou et al [5, 6] presented a heterodyne polarimeter by introducing the heterodyne interferometry into a Mach–Zehnder interferometric configuration. Because of its two-path optical configuration, it is susceptible to air turbulence and it can become unstable. Other methods for measuring the average refractive index have been presented based on the effect of either the total internal reflection [7] or Brewster angle [8]. These methods produced good measurement results. To the author’s knowledge, there is no reference reported for measuring the chiral parameter and the average refractive index simultaneously in one optical configuration. To improve on this, a new type of optical heterodyne polarimeter is presented in this paper. It has the merits of both the common-path interferometer and the hereodyne interferometer.

2. Principles [8, 9, 10]

Figure 1 schematically illustrates the improved optical heterodyne polarimeter. For convenience, the +z-axis is chosen to be along the light propagation direction and the x-axis is along the direction perpendicular to the paper plane. A light coming from a heterodyne light source having an angular frequency difference \( \omega \) between s- and p-polarizations passes through a quarter-waveplate Q. If the fast axis of Q is located at 45° to the x-axis, then the Jones vector of the light can be written as

\[
E_i = \frac{1}{2} \begin{pmatrix} 1 \\ i \end{pmatrix} \exp(i \frac{\omega t}{2}) + \frac{1}{2} \begin{pmatrix} 1 \\ -i \end{pmatrix} \exp(-i \frac{\omega t}{2} + i \frac{\pi}{2}).
\]  

(1)

From equation (1), we can see that the right- and the left-circular polarizations have frequency shifts \( \omega/2 \) and \(-\omega/2\), respectively. Thus there is an angular frequency difference \( \omega \) between them. The light is incident on a beam splitter BS, and is divided into two parts: the reflected light and the transmitted light. The reflected light passes through an analyser AN, and enters a photodetector Dr. If the transmission axis of AN is located at 45° to the x-axis, then the Jones vector of the light arriving at Dr is

\[
E_r = \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} \exp(-i \frac{\omega t}{2}) \\ 0 \\ \exp(i \frac{\omega t}{2}) \end{pmatrix}.
\]
and

\[
E_1 = \frac{1}{2} \begin{pmatrix}
1 & 1 \\
1 & 1
\end{pmatrix} \begin{pmatrix}
\cos \theta_1 \\
\sin \theta_1
\end{pmatrix} \begin{pmatrix}
t_p \\
r_s
\end{pmatrix} + \frac{1}{2} \begin{pmatrix}
1 & -i \\
-1 & 1
\end{pmatrix} \exp\left(-i \frac{\pi}{2} + \frac{\pi}{2} \right) \begin{pmatrix}
1 \\
1
\end{pmatrix},
\]

and

\[
E_2 = \frac{1}{2} \begin{pmatrix}
1 & 1 \\
1 & 1
\end{pmatrix} \begin{pmatrix}
\cos \theta_2 \\
\sin \theta_2
\end{pmatrix} \begin{pmatrix}
t_p' \\
r_s'
\end{pmatrix} \begin{pmatrix}
0 \\
0
\end{pmatrix} + \frac{1}{2} \begin{pmatrix}
1 & -i \\
-1 & 1
\end{pmatrix} \exp\left(-i \frac{\pi}{2} + \frac{\pi}{2} \right) \begin{pmatrix}
1 \\
1
\end{pmatrix}.
\]

respectively, where

\[
A = |r_p'(\cos \theta_1 + \sin \theta_1)|^2 + |r_s'(\cos \theta_1 - \sin \theta_1)|^2,
\]

\[
\phi_1 = \tan^{-1}\left[\frac{r_p'(\cos \theta_1 - \sin \theta_1)}{r_s'(\cos \theta_1 + \sin \theta_1)}\right],
\]

\[
\theta_1 = (k_r - k_s)d_1 = \frac{2\pi}{\lambda} (n_r - n_l)d_1,
\]

\[
r_p = \frac{n_l^2 - \sqrt{2n_r^2 - 1}}{n_l^2 + \sqrt{2n_r^2 - 1}},
\]

\[
r_s = 1 - \sqrt{2n_r^2 - 1} + \sqrt{2n_r^2 - 1},
\]

\[
B = \sqrt{[t_p't_s'(\cos \theta_2 + \sin \theta_2)]^2 + [t_s't_p'(\cos \theta_2 - \sin \theta_2)]^2},
\]

\[
\phi_2 = \tan^{-1}\left[\frac{t_s't_p'(\cos \theta_2 - \sin \theta_2)}{t_p't_s'(\cos \theta_2 + \sin \theta_2)}\right],
\]

\[
\theta_2 = (k_r - k_s)d_2 = \frac{2\pi}{\lambda} (n_r - n_l)d_2,
\]

\[
t_p = \frac{1}{n'}(r_p + 1),
\]

\[
t_s = r_s + 1,
\]

\[
t_p' = \frac{2n'' \cos \alpha}{n_l^2 \cos \alpha + \sqrt{n''^2 - \sin^2 \alpha}},
\]

\[
t_s' = \frac{2 \cos \alpha}{\cos \alpha + \sqrt{n''^2 - \sin^2 \alpha}},
\]

\[
\alpha = \sin^{-1}\left(\frac{1}{\sqrt{2n''}}\right),
\]

\[
n' = n_r/n, \quad n'' = (n')^{-1}.
\]

Consequently the intensities measured by D₁ and D₂ are

\[
I_1 = |E_1|^2 = \frac{1}{8} A^2 [1 + \cos(o\omega + \psi_1)],
\]

and

\[
I_2 = |E_2|^2 = \frac{1}{8} B^2 [1 + \cos(o\omega + \psi_2)],
\]

where

\[
\psi_1 = 2\phi_1 + \phi - \frac{\pi}{2},
\]

\[
\psi_2 = 2\phi_2 + \phi - \frac{\pi}{2},
\]

and

\[
\phi = (k_r - k_s)d = \frac{2\pi}{\lambda} (n_r - n_l)d = \frac{4\pi}{\lambda} gd.
\]
Two pairs of signals \((I_1, I_1')\) and \((I_2, I_2')\) are sent to a phase meter PM. Their phase differences \(\psi_1\) and \(\psi_2\) are obtained.

From equations (9)–(12), (14)–(20) and (25), it is obvious that \(\psi_1\) and \(\psi_2\) can be experimentally measured under the condition that \(d, d_1, d_2\), and \(n_e\) are specified. Hence a set of simultaneous equations

\[
\psi_1 = \psi_1(n, g, d, d_1, d_2, n_e),
\]

\[
\psi_2 = \psi_2(n, g, d, d_1, d_2, n_e).
\]

are obtained. These simultaneous equations are solved using the Runge–Kutta method [12], where the parameters \(n\) and \(g\) can be estimated.

3. Experiments and results

In order to show the feasibility of this method, we measured the average refractive indices and the chiral parameters of glucose solutions and sucrose solutions at 20 \(^\circ\)C. The heterodyne light source [8] consisting of a He–Ne laser with a wavelength of 632.8 nm and an electro-optic modulator was used. The frequency difference between the left- and right-circular polarizations was 1 kHz. A quartz glass plate with a refractive index 1.4570 was inserted into the chiral solution, and \(d, d_1,\) and \(d_2\) were 50, 10 and 10 mm, respectively. The rectangular glass box containing the test chiral solution and the glass plate \(G\) was mounted on a high-precision rotation stage (model M-URM100PP, Newfocus) with an angular resolution of 0.001\(^\circ\). A phase meter with resolution 0.01\(^\circ\) was self-made. In addition, a personal computer was employed to record and analyse the data. The experimental results and their corresponding reference values are summarized in table 1, where \(\delta_{ref}\) is derived from the reference values and the definition of the specific rotation [1, 13]. It is clear that they show good agreement.

4. Discussion

From equation (25) we get

\[
|\Delta g| = \frac{\lambda}{4\pi d} |\Delta \phi| + \frac{\lambda \phi}{4\pi d^2} |\Delta d|,
\]

where \(\Delta g\), \(\Delta \phi\) and \(\Delta d\) are the errors of \(g\), \(\phi\) and \(d\), respectively. Considering the angular resolution of the phase meter, the second-harmonic error and the polarization-mixing error, \(|\Delta \phi| \cong 0.03^\circ\) can be estimated in our experiments [14]. In addition, the condition \(|\Delta d| = 0.01\) mm is also assumed in our experiments. Substituting these data and the experimental conditions into equation (28) gives \(|\Delta g| \cong 5.5 \times 10^{-10}\).

From equations (23) and (24), we have

\[
\psi' = \psi_2 - \psi_1 = 2 \tan^{-1}\left[\frac{I_{1}'(\cos \theta_2 - \sin \theta_2)}{I_{1}(\cos \theta_2 + \sin \theta_2)}\right]
\]

\[
= 2 \tan^{-1}\left[\frac{r_p(\cos \theta_1 - \sin \theta_1)}{r_p(\cos \theta_1 + \sin \theta_1)}\right]
\]

\[
= 2 \tan^{-1}\left[\frac{(\cos \theta_2 - \sin \theta_2)(\cos \theta_1 + \sin \theta_1)r_{pt_1}t_{p'}'}{(\cos \theta_2 + \sin \theta_2)(\cos \theta_1 + \sin \theta_1)r_{pt_1}t_{p'}'}\right]
\]

\[
+ (\cos \theta_2 - \sin \theta_2)(\cos \theta_1 - \sin \theta_1)(r_{ipt_1}t_{p}')(29)
\]

From equation (29), we obtain

\[
\Delta n = \frac{|\sec^2(\frac{\psi'}{2})\Delta \psi'| + |\frac{\Delta \psi}{\Delta \theta_1}| + |\frac{\Delta \psi}{\Delta \theta_2}| + |\frac{\Delta \psi}{\Delta \theta_3}|}{|\psi'|},
\]

where \(\Delta n, \Delta \psi', \Delta \theta_1, \Delta \theta_1\) and \(\Delta \theta_2\) are the errors of \(n, \psi', \theta_1, \theta_1\) and \(\theta_2\), respectively.

In the above equation, \(|\Delta \theta_1|, |\Delta \theta_2|\), and \(h\) can be expressed as

\[
|\Delta \theta_1| = \frac{4\pi}{\lambda} (n_e - n_1) \Delta d_1 = \frac{4\pi}{\lambda} g \Delta d_1,
\]

\[
|\Delta \theta_2| = \frac{4\pi}{\lambda} (n_e - n_1) \Delta d_2 = \frac{4\pi}{\lambda} g \Delta d_2,
\]

and

\[
h = h(n, \theta_1, \theta_1, \theta_2)
\]

\[
m_1 = \cos \theta_1 + \sin \theta_1,
\]

\[
m_1 = \cos \theta_1 - \sin \theta_1,
\]

\[
m_2 = \cos \theta_2 + \sin \theta_2,
\]

\[
m_2 = \cos \theta_2 - \sin \theta_2
\]

respectively; where

\[
m_1 = \cos \theta_1 + \sin \theta_1,
\]

\[
m_1 = \cos \theta_1 - \sin \theta_1,
\]

\[
m_2 = \cos \theta_2 + \sin \theta_2,
\]

\[
m_2 = \cos \theta_2 - \sin \theta_2
\]
the signal

Hence a beam-splitting
cube cannot be introduced to
dependent to the average refractive index of the test chiral
solution. Its reflected light is
because the reflective thin film in the beam-splitting cube does
can divide the incident light into two parts, the reflected light
index of the chiral solution under test. A beam-splitting cube
plate G should be chosen so that it is different from the average

Electronic noise and the data of

Substituting our experimental conditions

\[ n_2 = \cos \theta_2 - \sin \theta_2, \]  
(37)

\[ r_p(\theta_1) = \frac{n^2 \cos \theta_1 - \sqrt{n^2 - \sin^2 \theta_1}}{n^2 \cos \theta_1 + \sqrt{n^2 - \sin^2 \theta_1}}, \]  
(38)

\[ r_s(\theta_1) = \frac{\cos \theta_1 - \sqrt{n^2 - \sin^2 \theta_1}}{\cos \theta_1 + \sqrt{n^2 - \sin^2 \theta_1}}, \]  
(39)

\[ t_p(\theta_1) = \frac{1}{n} \left[ r_p(\theta_1) + 1 \right], \]  
(40)

\[ t_s(\theta_1) = r_s(\theta_1) + 1, \]  
(41)

\[ t_p'(\theta_1) = \frac{2n^2 \cos \alpha}{n^2 \cos \alpha + \sqrt{n^2 - \sin^2 \alpha}}, \]  
(42)

\[ t_s'(\theta_1) = \frac{2 \cos \alpha}{\cos \alpha + \sqrt{n^2 - \sin^2 \alpha}}, \]  
(43)

\[ \alpha = \sin^{-1} \left( \frac{\sin \theta_1}{n'} \right). \]  
(44)

Substituting our experimental conditions \(|\Delta \psi'| \approx 0.03^\circ\),

From equations (8), (11), (12) and (21) it can be seen that
the signal \( I_2 \) is too weak to be measured, as \( n' \) is nearly equal to
one. To avoid this drawback, the reflective index of the glass
plate G should be chosen so that it is different from the average
index of the chiral solution under test. A beam-splitting
cube can divide the incident light into two parts, the reflected light
and the transmitted light, as the glass plate G does. But
because the reflective thin film in the beam-splitting cube does
not contact with the test chiral solution, its reflected light is
independent to the average refractive index of the test chiral
solution. Hence a beam-splitting cube cannot be introduced to
our experimental setup to replace the glass plate G.

If the function generator is used to generate the electrical
reference signal, then the optical setup can be simplified [8].
Note, however, that the reference signal is influenced by
the electronic noise and the data of \( \Delta \phi \) will fluctuate. In our
experiments, both the reference signal and the test signal are
affected by the electronic noise. Their fluctuations tend to
cancel out such that the data of \( \Delta \phi \) can be read directly in a
stable manner.

5. Conclusions

Phase variations occur when a circularly polarized light beam
either passes through a chiral solution or is reflected from
non-absorbing materials. They can be measured accurately
with circular heterodyne interferometry. The chiral parameter
and the average refractive index of a chiral solution can be
measured simultaneously in just one optical configuration.

Because of the common-path configuration and heterodyne
phase measurement, this polarimeter has the merits of simple
structure, easy operation, rapid measurement and high stability.
The prototype was set up to demonstrate its validity. The
estimated accuracy of the chiral parameters for the glucose or
sucrose solutions is \( 5 \times 10^{-10} \), and the estimated accuracy
of the average refractive index is \( 5 \times 10^{-4} \).

Acknowledgment

This study was supported in part by the National Science
Council, Taiwan, Republic of China, under contract no NSC
90-2215-E-009-077.

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Table 1. Experimental results and the corresponding reference data.

<table>
<thead>
<tr>
<th>Solutions</th>
<th>( \psi_1 ) (deg)</th>
<th>( \psi_2 ) (deg)</th>
<th>( n_{ref} \times 10^8 ) (at 632.8 nm)</th>
<th>( n )</th>
<th>( n_{ref} \times 10^8 ) (at 589.3 nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Glucose (w = 5%)</td>
<td>-258.84</td>
<td>2.44</td>
<td>4.01(^a)</td>
<td>1.3394</td>
<td>1.3402(^b)</td>
</tr>
<tr>
<td>Glucose (w = 10%)</td>
<td>-257.12</td>
<td>5.31</td>
<td>8.17(^a)</td>
<td>1.3472</td>
<td>1.3477(^b)</td>
</tr>
<tr>
<td>Sucrose (w = 5%)</td>
<td>-258.25</td>
<td>3.18</td>
<td>5.11(^c)</td>
<td>1.3396</td>
<td>1.3403(^b)</td>
</tr>
<tr>
<td>Sucrose (w = 10%)</td>
<td>-256.01</td>
<td>6.77</td>
<td>10.43(^c)</td>
<td>1.3483</td>
<td>1.3478(^b)</td>
</tr>
</tbody>
</table>

\(^a\) Reference [1].
\(^b\) Reference [13].
\(^c\) Estimated data from [13] with a curve fitting technique.