Performance prediction of a small-sized herringbone-grooved bearing with ferrofluid lubrication considering cavitation

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The small-sized herringbone groove journal bearing (HGJB), i.e., so-called “magnetic bearing,” filled with Newtonian ferrofluid lubrication is investigated via finite difference analysis (FDA), with consideration of cavitation zones in HGJB. The FDA starts with constructing the mass flux equations of the HGJB filled with ferrofluid. Discretization for FDA is next performed over the bearing clearance domain, from which algebraic finite difference equations based on the mass flow balance over the clearance domain are derived. Solving the equations, rotordynamic coefficients, cavitation zones, and side leakage rate are successfully predicted to show effectiveness in enhancing bearing performance by ferrofluid. © 2009 American Institute of Physics.

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I. INTRODUCTION

Small-sized herringbone groove journal bearings (HGJBs), so-called “magnetic bearings,” are popular to support motor spindles in hard disk drives. Figure 1 shows a photo of HGJB and its assembly with the rotor. Early analyses of HGJBs with gas lubricated, carried out mainly in the 1960s and 1970s, were based on the narrow groove theory. Osman et al.1 conducted research on ferrofluid-lubricated bearings with the focus on wire-generated magnetic fields. Chao and Huang3 used the finite difference method to calculate rotordynamic coefficients of a HGJB with ferrofluid. However, the effects of cavitation on the performance of magnetic bearing have never been considered.

The present study pays effort to model cavitation zones in magnetic HGJB. The investigation starts with building theoretical HGJB models, and followed by finite differencing and soling for solutions.

II. FINITE DIFFERENCE ANALYSIS

For a ferrofluid under magnetic field, the induced magnetic force per unit volume can be described by

\[ f_m = \nabla \times (\text{curl} \vec{H}_m) + \mu_0 M^* \nabla \times (\text{curl} \vec{H}_m), \]

\[ \nabla \times (\text{curl} \vec{H}_m) = \nabla \times (\text{curl} \vec{H}_m) + \mu_0 M^* \nabla \times (\text{curl} \vec{H}_m), \]

where \( h_m \) represents the induced free current, \( \mu_0 \) is the permeability of free space, and \( X_m \) satisfies \( M^* \equiv X_m h_m \), where \( M^* \) is the magnetization vector and strength of the ferrofluid. The ferrofluid are nonconductive and no free currents are induced. The first term in Eq. (1a) can then be canceled. For linear behavior of the magnetic fluid, i.e., \( M^* \equiv X_m h_m \), Eq. (1a) can be reduced to

\[ f_m = \mu_0 X_m h_m \nabla h_m. \]

Starting from the basic Navier–Stokes equation, using the coordinates defined in Fig. 2 \((x, y, z)\) along the circumference while \( z \) along the axis of the bearing) and seeing the magnetic force as an external body force, the momentum equations become

\[ \frac{\partial p}{\partial x} = \frac{\partial}{\partial y} \left( \mu \frac{\partial u}{\partial y} \right) + \mu_0 M^* \frac{\partial h_m}{\partial x}, \]

\[ \frac{\partial p}{\partial y} = 0, \]

\[ \frac{\partial p}{\partial z} = \frac{\partial}{\partial y} \left( \mu \frac{\partial w}{\partial y} \right) + \mu_0 M^* \frac{\partial h_m}{\partial z}. \]

Equation (2) is next integrated twice along the ferrofluid film. The fluid velocities can be obtained as

\[ u(y) = \frac{1}{2\mu} \left[ \frac{\partial p}{\partial x} - f_{mx} \right] y(y-h) + \frac{y}{h} R^2 \omega \]

FIG. 1. (Color online) Photo of the small-sized HGJB and (b) assembly of the rotor and HGJB.

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where $f_{m1} = \mu_0 X_m h_m (\partial h_m / \partial \bar{x})$ and $f_{m2} = \mu_0 X_m h_m (\partial h_m / \partial z)$ are magnetic body forces. The mass flux per unit width, that is, fluxes in the $x$ and $z$ directions, can be calculated by $M_x = \rho f_{m1}(\bar{y}) d\bar{y}$ and $M_z = \rho f_{m2}(\bar{y}) d\bar{y}$. Substituting the obtained flux into continuity equation, a modified Reynolds equation is obtained.

The analytical technique developed by Elrod is employed to predict cavitation in HGJB. The cavitation index, $g$, is adopted, which is generally called a switch function physically while mathematically a unit step function. Furthermore, Elrod introduced a variable $\theta$, which is termed the fractional film content and defined as the ratio of the density of the lubricant to the density at the cavitation pressure, i.e.,

$$\theta = \frac{\rho_c}{\rho}.$$  \hfill (5)

Elrod related the density and the fractional film content to the film pressure through the bulk modulus as follows:

$$\beta = \frac{d\rho}{dp}.$$  \hfill (6)

Integration on the above yields

$$p = p_c + g \beta \ln \theta_c,$$  \hfill (7)

where $g=1.0$ when $\theta \geq 1$ (full film region) and $g=0.0$ when $\theta < 1$ (cavitating region). Substituting Eq. (7) into Eq. (4) and nondimensionalizing yield

$$\frac{1}{4\pi} \frac{\partial (\theta H)}{\partial \bar{x}} + \frac{\alpha (L/D)^2}{12} \frac{\partial (H^2 \theta H_m \theta)}{\partial \bar{x}} + \frac{\alpha}{12} \frac{\partial (H^2 H_m \theta)}{\partial \bar{z}} = \frac{\beta_h}{48\pi} \frac{\partial (g \theta H_{Hm} \theta_h)}{\partial \bar{x}} + \frac{\beta_h}{48\pi} \frac{\partial (g \theta H_{Hm} \theta_h)}{\partial \bar{z}},$$  \hfill (8)

where dimensionless parameters were designed as follows:

$$H = h/C, \quad \alpha = (h_{m0} \gamma X_m h_{m0} C)/\mu_0 L^2, \quad H_m = h_m/h_{m0}, \quad \beta_h = \rho h_{m0}, \quad \theta_c = \rho_c, \quad \bar{x} = x/2\pi R, \quad \bar{z} = z/L.$$

The pressure-induced flow is taken care of next for finite differencing, which is

$$\frac{\partial (g \theta H_{Hm} \theta_h)}{\partial \bar{x}} = \frac{\partial (g \theta H_{Hm} \theta_h)}{\partial \bar{x}}.$$  \hfill (12)

This pressure-induced flow exists in the full film zone ($\theta \geq 1$), but vanishes in the cavitated zone ($\theta < 1$). Equation (12) can be treated in a variety of ways, which ensure that it is central differenced in the full film and vanish in the cavitated zone. Consider
where the last term at the RHS vanishes for all values of \( \theta_c \) since the spatial derivative of \( g \) vanishes for all \( \theta_c \) except at \( \theta_c = 1 \). Inserting Eq. (13) into Eq. (12) and central differencing yield

\[
\frac{\partial^2 \theta_c}{\partial \xi^2} = \frac{g}{\Delta \xi^2} \left( \frac{\partial g}{\partial \xi} \right)_{\xi+1/2, j} \delta_{i+1,j}(\theta_c - 1, j - 1) - (\theta_c - 1) \frac{\partial g}{\partial \xi^2}.
\] (13)

Equations (12) and (15) are ready to be inserted into the original governing Eq. (9) in coordinates \((\xi, \eta)\), then resulting in the final finite difference equations in the form of

\[
A_{i,j}(\theta_c)_{i,j} = A_{i+1,j}(\theta_c)_{i+1,j} + A_{i,j+1}(\theta_c)_{i,j+1} + A_{i-1,j}(\theta_c)_{i-1,j} + S,
\]

(15)

where the coefficients \( A_{i,j} \), \( A_{i+1,j} \), \( A_{i,j+1} \), and \( A_{i-1,j} \) involve the contributions in \( \xi \) and \( \eta \) directions and \( S \) is the source term. The above system can be solved by Gaussian elimination method.

IV. RESULTS AND DISCUSSION

Efforts are paid in this section to examine the effects of choosing ferrofluid as media on various performance indices of a HGJB. The parameter values of bearings and magnetic fields considered herein, which are those for a typical small-sized fluid bearing used in precision data-storage drives with three different applied magnetic fields, are shown in Fig. 3. The magnetic field is assumed generated by a finite long wire concentric with the HGJB. The results of the proposed magnetic field models and analysis are obtained for a bearing with a length-to-diameter ratio equal to 1.5. Figure 3 shows the computation results in terms of bearing performance index for different magnetic force coefficient \( \alpha \). As \( \alpha \) increases, (1) cavitation zone is moderately decreased, (2) load capacity is significantly increased, and (3) side leakage is significantly decreased. The above results all attribute a better bearing performance.


