Phenomenology of a TeV right-handed neutrino and the dark matter model

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In a model of a TeV right-handed (RH) neutrino by Krauss, Nasri, and Trodden, the sub-eV scale neutrino masses are generated via a three-loop diagram with the vanishing seesaw mass forbidden by a discrete symmetry, and the TeV mass RH neutrino is simultaneously a novel candidate for cold dark matter. However, we show that with a single RH neutrino it is not possible to generate two mass-square differences as required by the oscillation data. We extend the model by introducing one more TeV RH neutrino and show that it is possible to satisfy the oscillation pattern within the modified model. After studying in detail the constraints coming from the dark matter, lepton flavor violation, the muon anomalous magnetic moment, and the neutrinoless double beta decay, we explore the parameter space and derive predictions of the model. Finally, we study the production and decay signatures of the TeV RH neutrinos at TeV $e^+e^-/\mu^+\mu^-$ colliders.

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I. INTRODUCTION

One of the most natural ways to generate a small neutrino mass is via the seesaw mechanism [1]. There are very heavy right-handed neutrinos, which are gauge singlets of the standard model (SM), and so they could have a large Majorana mass $M_R$. After electroweak symmetry breaking, a Dirac mass term $M_D$ between the right-handed and the left-handed neutrinos can be developed. Therefore, after diagonalizing the neutrino mass matrix, a small Majorana mass $\sim m_\nu^2/M_R$ for the left-handed neutrino is obtained. This is a very natural mechanism, provided that $M_R \sim 10^{11} - 10^{13}$ GeV. One drawback of this scheme is that these right-handed neutrinos are too heavy to be produced in any terrestrial experiments. Therefore, phenomenologically there are not many channels to test the mechanism. Although it could be possible to get some hints from the neutrino masses and mixing, it is rather difficult to reconstruct the parameters of the right-handed neutrinos using the low energy data [2].

Another natural way to generate a small neutrino mass is via higher loop processes, e.g., the Zee model [3], with some lepton number violating couplings. However, these lepton number violating couplings are also subject to experimental constraints, e.g., $\mu \rightarrow e \gamma$, $\tau \rightarrow e \gamma$. In the Zee model, there are also extra scalars whose masses are of electroweak scale, and so can be observed at colliders [4].

On the other hand, recent cosmological observations have established the concordance cosmological model where the present energy density consists of about 73% of cosmological constant (dark energy), 23% (nonbaryonic) cold dark matter, and just 4% of baryons. To clarify the identity of the dark matter remains a prime open problem in cosmology and particle physics. Although quite a number of promising candidates have been proposed and investigated in detail, other possibilities can never be neglected.

Recently, Krauss, Nasri, and Trodden [5] considered an extension to the SM, similar to the Zee model, with two additional charged scalar singlets and a TeV right-handed neutrino. They showed that with an additional discrete symmetry the Dirac mass term between the left-handed and right-handed neutrinos are forbidden and thus avoiding the seesaw mass. Furthermore, the neutrino mass can only be generated at three-loop level, and sub-eV neutrino masses can be obtained with the masses of the charged scalars and the right-handed neutrino of order of TeV. Phenomenologically, this model is interesting because the TeV right-handed neutrino can be produced at colliders and could be a dark matter candidate.

In this work, we explore in detail the phenomenology of the TeV right-handed (RH) neutrinos. We shall extend the analysis to three families of left-handed neutrinos and explore the region of the parameters that can accommodate the present oscillation data. In the course of our study, we found that the model in Ref. [5] with a single RH neutrino cannot explain the oscillation data, because it only gives one mass-square difference. We extend the model by adding another TeV RH neutrino, which is slightly heavier than the first one. We demonstrate that it is possible to accommodate the oscillation pattern. We also obtain the relic density of the RH neutrino, and discuss the possibility of detecting them if they form a substantial fraction of the dark matter. We also study the lepton number violating processes, the muon anomalous magnetic moment, and production at leptonic colliders. In particular, the pair production of $N_1N_2,N_2^*N_2^*$ at $e^+e^-/\mu^+\mu^-$ colliders gives rise to very interesting signatures. The $N_2$ so produced will decay into $N_1$ plus a pair of charged leptons inside the detector. Thus, the signature would be either one or two pairs of charged leptons plus a large missing energy.

The organization is as follows. We describe the model in the next section. In Sec. III, we explore all the phenomenology associated with the TeV RH neutrino. In Sec. IV, we discuss the signatures in collider experiments. Section V is devoted to a conclusion.
II. REVIEW OF THE MODEL

The model considered in Ref. [5] has two extra charged scalar singlets $S_1, S_2$ and a right-handed neutrino $N_R$. A discrete $Z_2$ symmetry is imposed on the particles, such that all SM particles and $S_1$ are even under $Z_2$ but $S_2, N_R$ are odd under $Z_2$. Therefore, the Dirac mass term $L\phi N_R$ is forbidden, where $\phi$ is the SM Higgs boson. The seesaw mass is avoided.

In the present work, we extend the model a bit further by adding the second TeV right-handed neutrino, which also has odd $Z_2$ parity. The reason for this is because with only 1 TeV RH neutrino, it is impossible to obtain two mass-square differences, as required by the oscillation data. However, with two TeV RH neutrinos it is possible to accommodate two mass-square differences with the corresponding large mixing angles. We will explicitly show this result in the next section.

The most general form for the interaction Lagrangian is\(^1\)

$$\mathcal{L} = f_{\alpha \beta} L^T_{C_i} \tau_2 L_{\bar{\rho}} S_1^{\alpha} + g_{1a} N_1 S_1^a \epsilon_{\alpha R} + g_{2a} N_2 S_2^a \epsilon_{\alpha R} + V(S_1, S_2) + \text{h.c.} + M_N \frac{N_1^T C N_1 + M_N^2 N_2^2 C N_2}{2}, \quad (1)$$

where $\alpha, \beta$ denote the family indices, $C$ is the charge-conjugation operator, and $V(S_1, S_2)$ contains term $\lambda_4 (S_1 S_2^T)^2$. Note that $f_{\alpha \beta}$ is antisymmetric under interchange of the family indices. Note that even with the presence of the first term in the Lagrangian it cannot give rise to the one-loop Zee diagrams for neutrino mass generation, because there is no mixing term between the Zee charged scalar $S_1^+$ and the standard model Higgs doublet that can generate the charged lepton mass.

If the masses of $N_1, N_2, S_1, S_2$ are arranged such that $M_{N_1} < M_{N_2} < M_{S_1} < M_{S_2}$, $N_1$ would be stable if the $Z_2$ parity is maintained. The $N_1$ could be a dark matter candidate provided that its interaction is weak enough. Also, $N_1, N_2$ must be pair produced or produced associated with $S_1$ because of the $Z_2$ parity. The $N_2$ produced would decay into $N_1$ and a pair of charged leptons. The decay time may be long enough to produce a displaced vertex in the central detector. The $S_2$, if produced, would also decay into $N_1, N_2$, and a charged lepton. We will discuss the phenomenology in details in the next section.

III. PHENOMENOLOGY

A. Neutrino masses and mixings

The goal here is to find the parameter space of the model in Eq. (1) such that the neutrino mass matrix so obtained can accommodate the maximal mixing for the atmospheric neutrino, the large mixing angle for the solar neutrino, and the small mixing angle for $\theta_{13}$ [6]:

$$\Delta m_{atm}^2 = 2.7 \times 10^{-3} \text{ eV}^2, \quad \sin^2 2 \theta_{atm} = 1.0,$$

$$\Delta m_{sol}^2 = 7.1 \times 10^{-5} \text{ eV}^2, \quad \tan^2 \theta_{sol} = 0.45,$$

$$\sin^2 2 \theta_{13} \leq 0.1. \quad (2)$$

The three-loop Feynman diagram that contributes to the neutrino mass matrix has been given in Ref. [5]. The neutrino mass matrix ($M_{\nu}$) is given by

$$M_{\nu} = \frac{1}{(4 \pi)^2} \frac{1}{M_{S_2}} \lambda_s f_{\alpha \beta} m_{e} \rho \sigma_s m_{\mu} f_{\alpha \beta}, \quad (3)$$

where $\alpha, \beta$ denote the flavor of the neutrino. Note that in the Zee model, the neutrino mass matrix entries are proportional to $f_{\alpha \beta}$ such that only off-diagonal matrix elements are nonzero. It is well known that the Zee model gives bimaximal mixings, which have some difficulties with the large-mixing angle solution of the solar neutrino [6]. Here in Eq. (3) we do not have the second Higgs doublet to give a mixing between the SM Higgs doublet and $S_1^+$, and therefore the one-loop Zee-type diagrams are not possible. However, the mass matrix in Eq. (3) allows for nonzero diagonal elements, which may allow the departure from the bimaximal mixings.

The mixing matrix between flavor eigenstates and mass eigenstates is given as

$$U_{ei} = \begin{pmatrix} c_{13} c_{12} & s_{12} c_{13} & s_{13} \\ -s_{12} c_{23} - s_{23} s_{13} c_{12} & c_{23} c_{12} - s_{23} s_{13} s_{12} & s_{23} c_{13} \\ s_{23} s_{12} - s_{13} s_{23} c_{12} & -s_{23} c_{12} - s_{13} s_{23} s_{12} & c_{23} c_{13} \end{pmatrix}, \quad (4)$$

where we have ignored the phases. The mass eigenvalues are given by

$$U^T M U = M_{\text{diag}} = \text{diag}(m_1, m_2, m_3). \quad (5)$$

The mass-square differences and mixing angles are related to oscillation data by

$$\Delta m_{\text{sol}}^2 = \Delta m_{\text{atm}}^2 = m_2^2 - m_1^2,$$

$$\theta_{\text{sol}} = \theta_{12}, \quad \theta_{\text{atm}} = \theta_{23}. \quad (6)$$

From Eq. (3) the neutrino mass matrix is rewritten as
\[
(M_\nu)_{\alpha\beta} \sim \frac{\lambda_\nu}{(4\pi^2)^3 M\nu_2} \left( \begin{array}{ccc}
(fmg)_e^2 & (fmg)_e(fmg)_\mu & (fmg)_e(fmg)_\tau \\
(fmg)_\mu(fmg)_e & (fmg)_\mu^2 & (fmg)_\mu(fmg)_\tau \\
(fmg)_\tau(fmg)_e & (fmg)_\tau(fmg)_\mu & (fmg)_\tau^2
\end{array} \right),
\]

where \( (fmg)_a = \sum_p f_a \alpha m_\mu g_\rho \), the mass eigenvalues are given by
\[
m_1 = m_2 = 0, \quad m_3 = \frac{\lambda_\nu}{(4\pi^2)^3 M\nu_2} \left[ (fmg)_e^2 + (fmg)_\mu^2 + (fmg)_\tau^2 \right].
\]

This model obviously cannot explain the neutrino oscillation data because of the vanishing \( \Delta m_{23}^2 \).

Hereafter we would like to discuss a possibility to improve this shortcoming. The reason that this model predicts two vanishing mass eigenvalues is the proportionality relation in the mass matrix (7). Therefore it is necessary to break the proportionality relation. Although one way to improve the mass matrix might be to add small perturbations to the original mass matrix, we, however, found that this approach cannot resolve the difficulty. Instead, we consider a modification of the right-handed neutrino sector. As mentioned before, we employ two TeV RH neutrinos, the mass matrix (7) is replaced by
\[
(M_\nu)_{\alpha\beta} \sim \frac{1}{(4\pi^2)^3 M\nu_2} \frac{1}{\lambda_\nu} \sum_{i=1,2} (fmg_1)_a g_\mu m_f \beta, \quad (10)
\]

\[
2\lambda_\nu = 1 + w^2 + t^2 + c^2 + d^2 \pm \sqrt{(1 + w^2 + t^2 + c^2 + d^2)^2 - 4(d^2 + c^2 w^2 + d^2 w^2 - 2 c d t + c^2 t^2)},
\]

and each of the mixing angles is given by
\[
t_{23} = \frac{w(\lambda_+ - c^2 - d^2)}{\sqrt{\lambda_+ - c^2 + t c d}}, \quad (15)
\]
\[
s_{13} = \frac{\lambda_- - d^2 - t c d}{\sqrt{(\lambda_+ - c^2 - d^2)^2 + (1 + t^2_{23})w^2(\lambda_+ - c^2 - d^2)^2}}, \quad (16)
\]
\[
c_{12} = \frac{d w}{c_{13} \sqrt{(c^2 + d^2) w^2 + (c t - d)^2}}, \quad (17)
\]

where \( \lambda_+ \) denotes the two RH neutrinos.

If we assume \( (fmg_2)_\mu \ll (fmg_1)_\mu \), Eq. (10) is rewritten as
\[
(M_\nu)_{\alpha\beta} \sim \frac{\lambda_\nu (fmg_1)_e^2}{(4\pi^2)^3 M\nu_2} \left( \begin{array}{ccc}
1 + c^2 & w & t + c d \\
w & w^2 & w t \\
t + c d & w t & t^2 + d^2
\end{array} \right), \quad (11)
\]

\[
w = (fmg_1)_\mu / (fmg_1)_e, \quad t = (fmg_1)_\tau / (fmg_1)_e, \quad c = (fmg_2)_\mu / (fmg_1)_e, \quad d = (fmg_2)_\tau / (fmg_1)_e,
\]

and has one zero and two nonzero eigenvalues:
\[
m_\pm \sim \frac{\lambda_\nu (fmg_1)_e^2}{(4\pi^2)^3 M\nu_2} \lambda_\ne, \quad (13)
\]

where
\[
\sin^2 2 \theta_{13} = \frac{2}{w^2} \left( 1 - \frac{t c d}{\lambda_+} \right) \frac{2}{1 + t^2_{23}} \leq 0.1, \quad (18)
\]

we obtain \( w^2 \approx 20 \). Since Eq. (17) is rewritten as
\[
t_{12}^2 = \frac{c^2 w^2 + (c w - d)^2}{d^2 w^2}, \quad (19)
\]

where \( c_{13} \approx 1 \) and \( w = t \) are used, we obtain
\[
\frac{c^2}{d^2} - \frac{1}{4}, \quad (20)
\]

by comparing with Eq. (2). From the mass-square differences,
\[ \frac{\Delta m_{\text{sol}}^2}{\Delta m_{\text{atm}}^2} = \left( \frac{\lambda_+}{\lambda_-} \right)^2 = \left( \frac{2e^2w^2 + d^2w^2 - 2cdw}{4w^4} \right)^2 \]
\[ = \frac{3e^2}{2w^2} - 10^{-2}, \quad (21) \]

we find
\[ e^2 \sim \pm \frac{4}{3} \left( \frac{w^2}{20} \right). \quad (22) \]

Finally, \( \Delta m_{\text{atm}}^2 = m_3^2 = m_{\beta}^2 \) is rewritten as
\[ 2.7 \times 10^{-3} \text{ eV}^2 \approx -\frac{40\lambda_s (fm g_1)_{\beta}}{(4\pi)^3 M_{S_2}} \left( \frac{w^2}{20} \right)^2, \quad (23) \]

where we used \( \lambda_s = 2w^2 \). In Sec. III E, we find some parameter space that leads to correct mixing angles and mass-square differences, after considering also the constraints from the dark matter relic density and lepton flavor violation.

**B. Neutrinoless double beta decay**

A novel feature of the Majorana neutrino is the existence of neutrinoless double beta decay, which essentially requires a nonzero entry \((M_\nu)_{ee}\) of the neutrino mass matrix. Its non-observation has put an upper bound on the size of \((M_\nu)_{ee} \leq 1 \text{ eV} \) [7].

In the model with two RH neutrinos, \((M_\nu)_{ee}\) is estimated to be
\[ (M_\nu)_{ee} \sim -\frac{\lambda_s}{(4\pi)^3 M_{S_2}} [ (fm g_1)_{\beta} + (fm g_2)_{\beta} ] \]
\[ \sim 3 \times 10^{-3} \left( \frac{1 \pm \frac{3}{4} (20/w^2)}{2} \right) \text{ eV} \quad (24) \]

by using Eqs. (22) and (23). Thus, we find that this model is consistent with the current experimental bound. Such a small \((M_\nu)_{ee}\) may still be within reach of the GENIUS neutrinoless double beta decay experiment [8].

**C. Dark matter: Density and detection**

The lightest RH neutrino is stable because of the assumed discrete symmetry. Here we consider the relic density of the lightest RH neutrino, and the relic density must be less than the critical density of the Universe. First of all, we verify that the second lightest RH neutrino is of no relevance here because of the short decay time. The heavier RH neutrino will decay into the lighter one and two right-handed charged leptons, \( N_2 \rightarrow N_1 l_\alpha^+ \ell_\beta^- \) (\( \alpha, \beta \) denote flavors), and its decay width is given by
\[ \Gamma_{N_2} = \frac{M_{N_2}^2}{512\pi^3} |g_{1\alpha\beta} g_{2\alpha}|^2 \frac{1}{2M_{l}} \left[ 2(1 - \mu_s)(\mu_1 - \mu_s)(\mu_1 + \mu_s) \right. \]
\[ \left. + \mu_1 \mu_3 - 3 \mu_2^2 \right] \log \left( \frac{\mu_s - 1}{\mu_s - 1} \right) + (1 - \mu_s) \mu_s (2\mu_1 - 5\mu_s) \]
\[ - 5\mu_1 \mu_3 + 6 \mu_2^2 - 2 \mu_1^2 \log \mu_1 \right], \quad (25) \]

where \( \mu_1 = M_{N_1}^2 / M_{N_2}^2, \mu_3 = M_{S_2}^2 / M_{N_2}^2 \). In the worst case when \( M_{N_2} \) is very close to \( M_{N_1} \), say, they are both of order 1 TeV but differ by 1 GeV only, and we set \( g_1 \sim 0.1 \). In this case, the decay width is then of order \( 10^{-5} \text{ s}^{-1} \), i.e., the decay time is still many orders smaller than the age of the present Universe. Therefore, the presence of \( N_2 \) will not affect the relic density of \( N_1 \).

The relevant interactions for the annihilation is \( N_1 N_1 \rightarrow l_\alpha^+ l_{\beta R} \) through charged scalar \( S_2^- \) exchange. The corresponding invariant matrix element is given by
\[ |M|^2 = \frac{|g_{1\alpha\beta} g_{2\beta}|^2}{4} \left[ \begin{array}{c} (2q_1 \cdot p_1) (2q_2 \cdot p_2) (u - M_{S_2}^2) \\ (t - M_{S_2}^2) \end{array} \right] \]
\[ = \frac{2M_{N_1}^2 (2p_1 \cdot p_2)}{(t - M_{S_2}^2) (u - M_{S_2}^2)}, \quad (26) \]

where \( q_i \) and \( p_i \) are four-momenta of the incoming \( N_1 \) particles and the outgoing leptons, respectively. Then, we obtain

\[ 2q_1^0 q_2^0 \sigma v = \frac{d^3 p_1}{(2\pi)^2} \frac{d^3 p_2}{(2\pi)^2} \frac{d^3 p_2}{(2\pi)^2} (2\pi)^2 |M|^2 a^4 (q_1 + q_2 - p_1 - p_2) \]
\[ = \frac{1}{8\pi} \left. \left| g_{1\alpha\beta} g_{2\beta} \right|^2 \frac{m_{l_\alpha}^2 + m_{l_\beta}^2}{2} \left( \frac{s}{2} - M_{N_1}^2 \right) \right. \]
\[ + \frac{8}{3} \left( \frac{M_{N_2}^2 - M_{N_1}^2}{(M_{S_2}^2 + s/2 - M_{N_1}^2)^2} + (s/2)(M_{S_2}^2 - M_{N_1}^2) + s^2/8 \right) \frac{s}{4} \frac{s}{4} \left( \frac{s}{4} - M_{N_1}^2 \right), \quad (28) \]
where $m_{1a}$ is the lepton mass. We expanded $|M|^2$ in powers of the three-momenta of these particles and integrated over the scattering angle in the second line. Following Ref. [9], the thermal averaged annihilation rate is estimated to be

$$\langle \sigma v \rangle = \left[ \frac{M_{N_1}^2 T}{2 \pi^2} K_2 \left( \frac{M_{N_1}}{T} \right) \right]^{-2} \frac{T}{4(2 \pi)^3} \int_{4M_{N_1}^2}^{\infty} ds \times \sqrt{s-4M_{N_1}^2} K_1(\sqrt{s/T})(2q_1^0 2q_2^0 \sigma v)$$

$$= \sum f \left[ g_{1a} g_{1b} \right]^2 \frac{M_{N_1}^4}{32 \pi} \left( \frac{M_{N_1}^2}{M_N^2 + M_{N_1}^2} \right)^4 M_{N_1}$$

$$= \sigma_0 \left( \frac{T}{M_{N_1}} \right),$$

(29)

where $\Sigma_f$ denotes the summation over lepton flavors, and we have omitted the contributions from the $S$-wave annihilation terms, which are suppressed by the masses of the final state leptons. The relic mass density is given by

$$\Omega_{N_1} h^2 = 1.1 \times 10^9 \left( \frac{M_{N_1}}{T_d} \right) \frac{2(M_{N_1}/T)}{\sqrt{g_\ast} M \rho_0 (av)} \text{GeV}^{-1},$$

(30)

where $T_d$ is the decoupling temperature, which is determined as

$$\frac{M_{N_1}}{T_d} = \ln \left[ \frac{0.152}{\sqrt{g_\ast(T_d)} M \rho_0 (av)} \right]$$

$$- \ln \left[ \frac{0.152}{\sqrt{g_\ast(T_d)} M \rho_0 (av)} \right],$$

(31)

and $g_\ast$ is the total number of relativistic degrees of freedom in the thermal bath [10].

By comparing with the recent data from the Wilkinson Microwave Anisotropy Probe (WMAP) [11], we find

$$\Omega_{DM} h^2 = 0.113 = 2.2 \times 10^{-13} \frac{M_{N_1}}{10^3 \text{GeV}} \left( \frac{T_d}{M_{N_1}} \right)^2 \frac{(M_{N_1}/T_d)}{\sqrt{g_\ast} M \rho_0 (av)}.$$

(32)

We can calculate $\sigma_0$ from Eqs. (31) and (32), and we obtain

$$\sigma_0 = 1.4 \times 10^{-7} \left( \frac{10^3}{g_\ast(T_d)} \right)^{1/2} \left( 1 + 0.07 \ln \left( \frac{M_{N_1}}{10^3 \text{GeV}} \right) \right)$$

$$\times \left( \frac{10^3}{g_\ast(T_d)} \right) \text{GeV}^{-2},$$

(33)

if we ignore the second term in Eq. (31). Indeed, we can confirm the validity of this assumption within about 10% error by using Eq. (33). Actually, Eq. (31) is evaluated to be

$$\frac{M_{N_1}}{T_d} = \ln(2.5 \times 10^{13}) - \frac{3}{2} \ln(2.5 \times 10^{13})$$

$$= 31 - 5.1 = 26.$$

(34)

Our result of $\langle \sigma v \rangle$ is consistent with a previous estimation [12]. Equations (29) and (33) read

$$\sum_f |g_{1a} g_{1b}|^2$$

$$= 1 \left( \frac{M_{N_1}}{1.3 \times 10^2 \text{GeV}} \right)^2 \left( \frac{1 + M_{S_2}^2 / M_{N_1}^2}{1 + 2} \right)^4 \left( \frac{1 + 2^2}{1 + M_{S_2}^4 / M_{N_1}^4} \right).$$

(35)

It is obvious that the RH neutrino must be as light as $10^2$ GeV and at least one of $g_{1a}$ should be of order of unity, such that the relic density is consistent with the dark matter measurement. As the mass difference between $M_{S_2}$ and $M_{N_1}$ becomes larger, the upper bound on $M_{N_1}$ becomes smaller provided that we keep $g \leq 1$.

The detection of the RH neutrinos as a dark matter candidate depends on its annihilation cross section and its scattering cross section with nucleons. Conventional search of dark matter employs an elastic scattering signal of the dark matter with the nucleons. We do not expect that the $N_1$ dark matter would be easily identified by this method, given its very mild interaction. In addition, because of the Majorana nature the annihilation into a pair charged lepton at the present velocity ($v_w \rightarrow 0$) is also highly suppressed by the small lepton mass, even in the case of the tau lepton. However, one possibility was pointed out by Baltz and Bergstrom [12] that the annihilation $N_1 N_1 \rightarrow l^+ l^- \gamma$ would not suffer from helicity suppression. The rate of this process is approximately $\overline{d} \tau$ times the annihilation rate at the freeze-out. As will be indicated later, the dominant mode would be $\mu^+ \mu^- \gamma$. There is a slight chance to observe the excess in positron, but, however, the energy spectrum is softened because of the cascade from the muon decay. However, the chance of observing the photon spectrum is somewhat better [12].

D. Lepton flavor changing processes and $g - 2$

There are two sources of lepton flavor violation in Eq. (1). The first one is from the interaction $f_{a'b'} T_a C i/2 L_{b'} L_{b'}$. This one is similar to the Zee model. (However, the present model would not give rise to neutrino mass terms in one loop because of the absence of the $S_{1+}^+ - \phi$ mixing.) The flavor violating amplitude of $\ell_a \rightarrow \ell_b$ via an intermediate $\nu_{b'}$ would be proportional to $|f_{a'b'}|$. The second source is from the term $g_{1a} N_1 S_{1+}^+ \ell_{aR}$ in the Lagrangian (1). The flavor violating am-

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2Krauss et al. [5] claimed that $M_{N_1} \sim 1$ TeV and $g^2 \sim 0.1$ is consistent with the dark matter constraint, but in their rough estimation a numerical factor of $(T_d/M_N) \sim 200$ is missing from the equation of $\langle \sigma v \rangle$. 

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plitude of $\ell_{\alpha} \rightarrow \ell_{\beta}$ via an intermediate $N_\tau$ would be proportional to $|g_{\ell_{1}\alpha\ell_{1}\beta}|$. We apply these two sources to the radiative decays of $\ell_{\alpha} \rightarrow \ell_{\beta} \gamma$ and the muon anomalous magnetic moment.

The new contribution to the muon anomalous magnetic moment can be expressed as

$$\Delta a_{\mu} = \frac{m_{\mu}^2}{96\pi^2} \left[ \left( |f_{\mu\tau}|^2 + |f_{\mu\tau'}|^2 \right) \left( \frac{2 \times 10^2 \text{ GeV}}{M_{S_1}} \right)^2 + \frac{6 |g_{1\mu}|^2}{M_{S_2}} \right] F_2(M_{S_1}^2/M_{S_2}^2)$$

$$+ \frac{6 |g_{2\mu}|^2}{M_{S_2}^2} \left( \frac{2 \times 10^2 \text{ GeV}}{M_{S_2}} \right)^2 F_2(M_{S_1}^2/M_{S_2}^2),$$

(36)

where $F_2(x) = (1 - 6x + 3x^2 + 2x^3 - 6x^2 \ln x)/(6(1 - x)^4)$. The function $F_2(x) \rightarrow 1/6$ for $x \rightarrow 0$, and $F_2(0.25) \approx 0.125$. We naively set $F_2(x) = 1/6$ for a simple estimate. Therefore, we obtain

$$\Delta a_{\mu} = 3 \times 10^{-10} \left[ \left( |f_{\mu\tau}|^2 + |f_{\mu\tau'}|^2 \right) \left( \frac{2 \times 10^2 \text{ GeV}}{M_{S_1}} \right)^2 + \left( |g_{1\mu}|^2 + |g_{2\mu}|^2 \right) \left( \frac{2 \times 10^2 \text{ GeV}}{M_{S_2}} \right)^2 \right] \lesssim 10^{-9},$$

(37)

which implies that $f_{\mu\tau}, f_{\mu\tau'}, g_{1\mu}, g_{2\mu}$ can be as large as $O(1)$ for $O(200 \text{ GeV}) S_1^3, S_2^4$ without contributing in a significant level to $\Delta a_{\mu}$.

Among the radiative decays $\mu \rightarrow e \gamma$ is the most constrained experimentally, $B(\mu \rightarrow e \gamma) < 1.2 \times 10^{-11}$ [13]. The contribution of our model is

$$B(\mu \rightarrow e \gamma) = \frac{\alpha v^4}{384\pi} \left[ \left( |f_{\mu\tau}|^2 + |f_{\mu\tau'}|^2 \right) \left( \frac{2 \times 10^2 \text{ GeV}}{M_{S_1}} \right)^4 + \frac{36 |g_{1\mu}|^2}{M_{S_2}^2} \right] F_2(M_{S_1}^2/M_{S_2}^2)$$

$$+ \frac{36 |g_{2\mu}|^2}{M_{S_2}^2} \left( \frac{2 \times 10^2 \text{ GeV}}{M_{S_2}} \right)^4 F_2(M_{S_1}^2/M_{S_2}^2),$$

(38)

where $v = 246 \text{ GeV}$. Again we take $F_2(x) = 1/6$ and $O(200 \text{ GeV})$ mass for $S_1^3, S_2^4$ for a simple estimate:

$$B(\mu \rightarrow e \gamma) = 1.4 \times 10^{-5} \left[ \left( |f_{\mu\tau}|^2 + |f_{\mu\tau'}|^2 \right) \left( \frac{2 \times 10^2 \text{ GeV}}{M_{S_1}} \right)^4 + \left( |g_{1\mu}|^2 \right) \left( \frac{2 \times 10^2 \text{ GeV}}{M_{S_2}} \right)^4 + \left( |g_{2\mu}|^2 \right) \left( \frac{2 \times 10^2 \text{ GeV}}{M_{S_2}} \right)^4 \right] < 1.2 \times 10^{-11},$$

(39)

which implies that

$$|f_{\mu\tau}| < 1 \times 10^{-3},$$

$$|g_{1\mu}| < 1 \times 10^{-3},$$

$$|g_{2\mu}| < 1 \times 10^{-3}. \quad (40)$$

This is in contrast to a work by Dicus et al. [14]. In their model, the couplings $g_i$'s are much larger than $f_{ij}$'s.

E. An example of consistent model parameters

Here we summarize the constraints from previous subsections, and illustrate some allowed parameter space. The prime constraints come from neutrino oscillations. The maximal mixing and the mass-square difference required in the atmospheric neutrino and the small $\theta_{13}$ read

$$f_{\tau\mu} m_{\mu} g_{1\mu} \approx f_{\tau\mu} m_{\tau} g_{1\tau} \Rightarrow f_{\mu\tau} m_{\mu} g_{1\mu} + f_{e\mu} m_{e} g_{1e}$$

$$\sim \sqrt{\frac{1}{\lambda_s} \left( \frac{M_{S_2}}{10^2 \text{ GeV}} \right)} \text{MeV}, \quad (41)$$

where the terms $f_{\tau\mu} m_{e} g_{1e}$ and $f_{\mu\tau} m_{e} g_{1e}$ have been omitted because these terms are suppressed by electron mass. The large mixing angle and the mass-square difference required in the solar neutrino are given by

$$f_{\tau\mu} m_{e} g_{2\tau} + f_{\tau\mu} m_{e} g_{2\mu} \approx (f_{e\mu} m_{e} g_{2\mu} + f_{e\mu} m_{e} g_{2\tau})$$

$$\Rightarrow f_{\mu\tau} m_{e} g_{2\tau} + f_{\mu\tau} m_{e} g_{2\mu}, \quad (42)$$

$$f_{\tau\mu} m_{\mu} g_{1\mu} \approx f_{e\mu} m_{e} g_{1\mu}$$

$$\Rightarrow \frac{f_{\mu\tau} m_{\mu} g_{1\mu}}{2} \approx 3 \times 10^{-11}. \quad (43)$$

On the other hand, the dark matter constraint requires at least one of the $g_{1\mu}, g_{1\tau}, g_{1\tau}$ to be of order of unity. While the muon anomalous magnetic moment does not impose any strong constraints, lepton flavor violating processes, especially $B(\mu \rightarrow e \gamma)$, give the following strong constraints:

$$|f_{\mu\tau}| \lesssim 1 \times 10^{-3},$$

$$|g_{1\mu}|^2, |g_{2\mu}|^2 \lesssim 1 \times 10^{-3}. \quad (45)$$

Now, let us look for an example of consistent parameters. From Eq. (41), we obtain $|m_{\mu} g_{1\mu}| \approx |m_{e} g_{1e}|$, in other words $|g_{1\mu}| \approx |g_{1e}|$, and

$$f_{\tau\mu} \approx f_{\mu\tau}, \quad (46)$$

Since either $g_{1\mu} \approx g_{1e}$ must be of order of unity from the dark matter constraint, we take $g_{1\mu} = 1$. From Eqs. (42) and (43) with $g_{2\tau} = 0$, we obtain

$$f_{\tau\mu} = 2 f_{\mu\tau}, \quad |m_{\mu} g_{2\mu}| \approx |m_{e} g_{2e}|$$

and

$$g_{2\mu}^2 \approx 8/3 \times 10^{-1} g_{2e}^2 \approx 0.27 (g_{1\mu}/1)^2. \quad (48)$$

Equations (46) and (47) can be rewritten as

$$\frac{1}{2} \approx \frac{f_{\tau\mu}}{f_{\mu\tau}}. \quad (49)$$
where we find that a mild cancellation between $f_{e\mu}$ and $f_{\tau\mu}$ is necessary. For instance, $f_{\tau\mu}/f_{\tau\mu} = -1/3$. The strong cancellation corresponds to the small $\theta_{13}$. However, a cancellation with too high accuracy would require a $\lambda_\tau$, which is too big by Eq. (41). Therefore, one can say that this model predicts a relatively large mixing in $\theta_{13}$. Now we obtain an example set of parameters that makes this model workable and it is

\[
|g_{1\tau}| < 1 \times 10^{-3}, \quad |g_{1\mu}| < 1, \quad |g_{1\ell}| = 0.06, \\
|g_{2\ell}| < 2 \times 10^{-3}, \quad |g_{2\tau}| = 0.5, \quad |g_{2\mu}| < 10^{-2}, \quad (50)
\]

IV. PRODUCTION AT $e^+e^-, \mu^+\mu^-$ COLLIDERS

The decay of $N_2$ may have an interesting signature, a displaced vertex, in colliders. Depending on the parameters, $N_2$ could be able to travel a typical distance, e.g., mm, in the detector without depositing any kinetic energy, and suddenly decay into $N_1$ and two charged leptons. The signature is very striking.

The $N_1N_1$, $N_2N_2$, and $N_1N_2$ pairs can be directly produced at $e^+e^-$ colliders. The differential cross section for $e^+e^- \rightarrow N_1N_1$, $I=1,2$, is given by

\[
\frac{d\sigma}{d\cos \theta} (e^+e^- \rightarrow N_1N_1) = \frac{g_{1e}^2}{64\pi s} \frac{2(y_i-x_i)^2+x_i}{-2y_i^3+y_i(6y_i-1) - 2x_i y_i (3x_i+2) + x_i^2 (1+x_i) (1+y_i)} \left[ \beta_i (-2y_i+2x_i+1) + 2((y_i-x_i)^2+x_i) \log \left( \frac{2y_i-2x_i+\beta_i-1}{2y_i-2x_i-\beta_i-1} \right) \right], \quad (51)
\]

where $y_i = M_{N_i}^2/s$ and $x_i = M_{S_i}^2/s$. For $N_1N_2$ production the differential cross section is given by

\[
\frac{d\sigma}{d\cos \theta} (e^+e^- \rightarrow N_1N_2) = \frac{|g_{1e}g_{2e}|^2}{128\pi s} \frac{\beta_{12}}{s} \left[ \frac{(t-M_{N_1}^2)(t-M_{N_2}^2)}{(t-M_{S_1}^2)^2} + \frac{(u-M_{N_1}^2)(u-M_{N_2}^2)}{(u-M_{S_2}^2)^2} - \frac{2M_{N_1}M_{N_2}s}{(t-M_{S_1}^2)(u-M_{S_2}^2)} \right], \quad (53)
\]

and the integrated cross section is

\[
\sigma(e^+e^- \rightarrow N_1N_2) = \frac{|g_{1e}g_{2e}|^2}{128\pi s} \frac{\beta_{12}}{s} \frac{4}{\beta_{12}s(-1+x_1+x_2-2x_i)(-1+\beta_{12}+x_1+x_2-2x_1)(1+\beta_{12}+x_1+x_2-2x_2)} \times \left[ \beta_{12}s(-1+x_1+x_2-2x_i)(-1+\beta_{12}+x_1+x_2-2x_1)(1+\beta_{12}+x_1+x_2-2x_2) \times s(2\sqrt{x_1x_2}+(x_1+x_2)(x_1+x_2-4x_i+1)+2x_i(2x_i+1)) \left[ \beta_{12}^2-(-1+x_1+x_2-2x_i)^2 \right] \log \left( \frac{-1-\beta_{12}+x_1+x_2-2x_i}{-1+\beta_{12}+x_1+x_2-2x_i} \right) \right], \quad (54)
\]

where $\beta_{12}=\sqrt{(1-x_1-x_2)^2-4x_1x_2}$. The above cross section formulas are equally valid for $\mu^+\mu^-$ collisions. Since the constraints from the last section restrict $g_{1e}$ and $g_{2e}$ to be hopelessly small, we shall concentrate on using $g_{1\mu}$ and $g_{2\mu}$.

The production cross sections for the $N_2N_2$ and $N_1N_2$ pairs are given in Figs. 1(a) and 1(b), respectively, for $\sqrt{s} = 0.5,1,1.5$ TeV and for $M_{N_2}$ from 150 to 800 GeV, and we have set $g_{1\mu} = 1, g_{2\mu} = 0.5$ [see Eq. (50)]. In the curve for $N_1N_2$, we set $M_{N_1} = M_{N_2} = 50$ GeV. We are particularly in-
terested in the \( N_1N_2, N_2N_2 \) production, because of its interesting signature.

As we have calculated the decay width of \( N_2 \) in Eq. (25), the \( N_2 \) can decay into \( N_1 \) plus two charged leptons, either promptly or after traveling a visible distance from the interaction point. It depends on the parameters involved, mainly the largest of \( g_{1\mu}g_{2\alpha} \). As seen in Eq. (50) the largest is \( |g_{1\mu}g_{2\mu}| \sim 0.5 \), and so the decay of \( N_2 \) is prompt. Therefore, in the case of \( N_1N_2 \) production, the signature would be a pair of charged leptons plus missing energies, because the \( N_1 \)'s would escape the detection. The charged lepton pair is likely to be on one side of the event. In case of \( N_2N_2 \) production, the signature would be two pairs of charged leptons with a large missing energy. Note that in the case of \( N_1N_1 \) production, there is nothing in the final state that can be detected. From Fig. 1 the production cross sections are of order \( O(10 - 100 \text{ fb}) \), which implies plenty of events with \( O(100 \text{ fb}^{-1}) \) luminosity.

One may also consider \( S^+_S S^-_S \) pair production. The \( S^+_S \) so produced will decay into \( S^+_S \to N_1 \ell^- R \) or \( N_2 \ell^- R \), where \( \ell_a = e, \mu, \tau \). However, the constraints on the parameter space require the mass of \( M_{S^+_S} \) substantially heavier than \( M_{N_1} \) and \( M_{N_2} \), and therefore the \( S^+_S S^-_S \) pair production cross section is relatively much smaller.

V. CONCLUSIONS

In this paper, we have discussed a model that explains the small neutrino mass and dark matter in the Universe at the same time. Such a model was proposed by Krauss et al. as a modification of Zee model. However, our study revealed that their original model is unfortunately not capable of explaining the neutrino oscillation pattern.

We have extended the model by introducing another right-handed neutrino. We succeed in showing that such an extension is possible to achieve the correct neutrino mixing pattern. A prediction of this model is the normal mass hierarchy. In addition, the undiscovered mixing angle \( \theta_{13} \) is relatively large, because of the requirement of a mild cancellation between the parameters for a small \( \theta_{13} \) and a sensible coupling of the charged scalar, \( \lambda_\nu \).

The relic density of the lightest right-handed neutrino has also been revisited. Under the constraint by WMAP we found that the mass of the right-handed neutrino cannot be as large as TeV but only of order \( 1 \times 10^2 \text{ GeV} \), after a careful treatment of the calculation. In addition, other constraints including the muon anomalous magnetic moment, radiative decay of muon, and neutrinoless double beta decay have also been studied. With all the constraints we are still able to find a sensible region of parameter space.

Finally, our improved model has an interesting signature at leptonic colliders via pair production of right-handed neutrinos, in particular \( N_1N_2 \) and \( N_2N_2 \). The \( N_2 \) so produced will decay into \( N_1 \) plus two charged leptons. Thus, the signature is either one or two pairs of charged leptons with a large missing energy. Hence, this model can be tested not only by neutrino experiments but also by collider experiments.

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