A flux estimation method for a permanent-magnet synchronous motor

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Abstract

This paper deals with the flux estimation of a permanent-magnet synchronous motor (PMSM). Contrary to the conventional no-load test, the proposed method needs no extra servomotor. It is simply to drive the PMSM in a single-phase mode with the currents large enough to make the PMSM rotate in the same direction. Under such a condition, the flux of the PMSM can be easily estimated.

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1. Introduction

The permanent-magnet synchronous motor (PMSM) has been used broadly in industry, because of its easy controllability and fast response. An accurate estimation of the flux is useful for the design and the servo control of a PMSM. Several researches have dealt with the estimation of the flux of a PMSM [1–3]. It is of no doubt that the no-load test method [1] has become the most popular one. In this method, an auxiliary motor is required to drive the PMSM at constant speed. The windings of PMSM are at open-circuit so that the flux can be estimated by the emf of the PMSM.

This paper tries to propose an alternative estimation method, which does not require the aid of an auxiliary servomotor. The proposed method is simply to drive the PMSM by exciting a single phase, say, let \(i_a = -i_b\), and \(i_c = 0\), where \(i_a\), \(i_b\), and \(i_c\) are the currents of phases a, b, c, respectively. The flux is then easily estimated using the measurement of three terminal voltages \(v_a\), \(v_b\), and \(v_c\).

A spindle motor in a 50 × XCD-ROM driver is taken as an example to compare the estimation results between the proposed method and the conventional no-load test. It will be shown that the experiment results of both methods are very close.
2. Proposed method

Consider a PMSM with three-phase and Y-connected windings, whose models can be described as \[4\]

\[
\begin{bmatrix}
    v_{as} \\
v_{bs} \\
v_{cs}
\end{bmatrix} =
\begin{bmatrix}
    r_s & 0 & 0 \\
    0 & r_s & 0 \\
    0 & 0 & r_s
\end{bmatrix}
\begin{bmatrix}
i_a \\
i_b \\
i_c
\end{bmatrix}
+ \begin{bmatrix}
    L_s & -M & -M \\
    -M & L_s & -M \\
    -M & -M & L_s
\end{bmatrix}
\begin{bmatrix}
    \frac{di_a}{dt} \\
    \frac{di_b}{dt} \\
    \frac{di_c}{dt}
\end{bmatrix}
+ \omega_r \lambda_r
\begin{bmatrix}
    \cos \theta_r \\
    \cos \left( \theta_r - \frac{2\pi}{3} \right) \\
    \cos \left( \theta_r + \frac{2\pi}{3} \right)
\end{bmatrix},
\] (1)

where \(v_{as}, v_{bs}, \) and \(v_{cs}\), are the terminal voltages with respect to the neutral point \(s\) (see Fig. 1(a)), \(i_a, i_b, \) and \(i_c\) are the currents of phases a, b, \(c\), respectively, \(r_s\) is the winding resistance per phase, \(L_s\) is the self-inductance per phase, \(M\) is the mutual inductance per phase, \(\omega_r\) is the electric angular speed of the rotor, \(\lambda_r\) is the maximum flux induced by the rotor magnet, and \(\theta_r\) is the rotational electrical angle of the rotor. Note that \(\theta_r = 0\) is the position where the intersection line of the N–S and the S–N magnet is in alignment with the centerline of a tooth of phase a (see Fig. 1(b)). Furthermore, the output torque \(T_e\) of the motor can be obtained as

\[
T_e = \frac{P}{2} \lambda_r \left( i_a - i_b - i_c \right) \cos \theta_r + \frac{\sqrt{3}}{2} \left( i_b - i_c \right) \sin \theta_r
\]

\[
= \frac{2J d\omega_r}{dt} + \frac{2B_0}{P} \omega_r + T_L,
\] (2)

where \(J\) is the inertia, \(P\) is the number of poles of the magnet, \(B_m\) is the damping ratio, and \(T_L\) is the loading torque. Suppose that the PMSM is operated in a single-phase mode, i.e., phase \(c\) is open, such that \(i_a = -i_b\), and \(i_c = 0\). Such a kind of operation can be achieved by manipulating four transistors on legs, Leg2 and Leg3, in Fig. 1(a), while turning those on Leg1 always open. Under this situation, Eq. (1) is simplified in the form of

\[
v_{as} = v_a - v_s = r_s i_a + L_s \frac{di_a}{dt} + M \frac{di_b}{dt} + \omega_r \lambda_r \cos \theta_r,
\]

\[
v_{bs} = v_b - v_s = -r_s i_a - L_s \frac{di_a}{dt} - M \frac{di_c}{dt}
\]

\[
+ \omega_r \lambda_r \cos \left( \theta_r - \frac{2\pi}{3} \right),
\]

\[
v_{cs} = v_c - v_s = \omega_r \lambda_r \cos \left( \theta_r + \frac{2\pi}{3} \right),
\] (3)

where \(v_a, v_b, \) and \(v_c\) are the terminal voltages, and \(v_s\) is the neutral voltage. Similarly, Eq. (2) is also simplified as

\[
T_e = \frac{\sqrt{3}P}{2} \lambda_r i_a \cos \left( \theta_r + \frac{\pi}{6} \right)
\]

\[
= \frac{2J d\omega_r}{dt} + \frac{2B_0}{P} \omega_r + T_L.
\] (4)

According to Eq. (4), we can make the motor rotate continuously in one direction by simply

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**Fig. 1.** (a) A three-phase PMSM with Y-connected windings and its driver. (b) The relative position between the stator and the rotor at \(\theta_r = 0\).
assigning the current \( i(t) \) in phase with \( \cos(\theta + \pi/6) \), and large enough to ensure \( T_c > T_L \). Since the values of \( v_{as}, v_{bs}, \) and \( v_{cs} \) are not available and only \( v_a, v_b, \) and \( v_c \) can be measured, we introduce a new variable \( v_o(t) \) defined as

\[
v_o(t) = \frac{-(v_a + v_b - 2v_c)}{3} = \omega_r \dot{\theta}_r \cos\left(\theta_r + \frac{2\pi}{3}\right).
\]

(5)

It follows from Eq. (5) and \( \omega_r = d\theta_r/dt \) that

\[
v_o(t) = \frac{\omega_r \dot{\theta}_r \cos\left(\theta_r + \frac{2\pi}{3}\right)}{3} d\theta_r.
\]

(6)

Integrating both sides of Eq. (6) leads to

\[
\Phi(t) = \int_0^t v_o(\tau) d\tau = \int_{\theta_r(0)}^{\theta_r(t)} \dot{\theta}_r \cos\left(\theta_r + \frac{2\pi}{3}\right) d\tau
\]

\[
= \dot{\theta}_r \left( \theta_r(t) + \frac{2\pi}{3} \right) - \dot{\theta}_r \left( \theta_r(0) + \frac{2\pi}{3} \right)
\]

\[
= \dot{\theta}_r \left( \theta_r(t) - \theta_r(0) + \frac{2\pi}{3} \right) + \Phi_0
\]

(7)

where \( \Phi_0 \) is a constant. Consequently, the flux estimation method is that the PMSM is operated in a single-phase mode with the current in phase with \( \cos(\theta_r + \pi/6) \) and large enough to overcome the load \( T_L \), and the measured histograms of \( v_a, v_b, \) and \( v_c \) are used to calculate \( v_o(t) \) in Eq. (5) and then \( \Phi(t) = \int_0^t v_o(\tau) d\tau \). The flux \( \dot{\theta}_r \) of the PMSM is then the amplitude of the sinusoidal part of \( \Phi(t) \) according to Eq. (7).

3. Implementation

A three-phase, 12-pole, and nine-slot DC brushless motor used as a spindle motor of a 50 \( \times \) XCD-ROM is taken as an example. The motor with \( \dot{\theta}_r = 7.9 \times 10^{-4} \) Wb-turn has the surface-mounted NdFeB magnet rotor. There are three Hall elements \( H_a, H_b, \) and \( H_c \) mounted on the stator (see Fig. 1(b)), each of which generates a pair of differential signals, e.g., \( H_a^+ \) and \( H_a^- \) as the feedback signals for the motor driver. Now, we want to establish a driving system that drives the PMSM only in a single-phase mode as described in Section 2. First, the winding labeled c in Fig. 1(a) should be floated from the transistors.

When transistors Tr4 and Tr5 in Fig. 1(a) are on and Tr3 and Tr6 are off, the currents of the PMSM are \( i_a = -i_b, \) and \( i_c = 0 \) as shown in Fig. 2(c). On the other hand, the currents are reversed when Tr3 and Tr6 are on and Tr4 and Tr5 are off as shown in Fig. 2(b). Both cases of the single-phase mode can be implemented with a BA6849 chip of the Rohm Company.

The logic table of the BA6849 is shown in Table 1. It is apparent that the states 1 and 4 meet the requirements of the circuits of Fig. 2(b) and (c), respectively. In these two states, the inputs \( H_a^+, H_b^+, H_b^-, H_c^+, H_c^-, \) and \( H_c^- \) should be retained at the level of \( M = 2.5 \text{ V} \), while those of \( H_a^+ \) and \( H_c^+ \) are in opposite levels, i.e., \( H_c^+ = L, \) when \( H_a^+ = H \). The implementation circuit is then connected to the input pins \( H_a^+, H_b^+, H_b^-, H_c^+, H_c^-, \) and \( H_c^- \) to a power source of the voltage level 2.5 V, and to pass the signal of \( H_a^+ \) also to pin \( H_c^+ \) via a NOT device (see Fig. 2(a)). The output signals of the Hall sensor \( H_a \) are operated by a comparator to generate the signal for the input pin \( H_c^+ \) of the BA6849. If the positive end of the sensor \( H_a \) has a higher voltage level than the negative end, the output of the comparator is 2.6 V. If the positive end has a lower voltage level, then the output of the comparator is 2.4 V. The rotor flux linkage measured by the Hall sensor \( H_a \) is so arranged to be proportional
to \( \sin(\theta_r + 2\pi/3) = -\cos(\theta_r + \pi/6) \). Thus, the input signal of \( H_a^+ \) is in phase of \(-\cos(\theta_r + \pi/6)\), so that the current \( i \) is in phase of \( \cos(\theta_r + \pi/6) \).

An experiment is conducted by the implemented system for the PMSM described above. The current \( i \) generated by the BA6849 is measured and shown in Fig. 3(a), while the input signal of pin \( H_a^+ \) is shown in Fig. 3(b). It is apparent that the current \( i \) is positive when the state of \( H_a^+ \) is \( L \), and negative when that of \( H_a^+ \) is \( H \). The voltages of \( v_a \), \( v_b \), and \( v_c \) are also measured to compute \( v_o \) by Eq. (5) and thus \( \Psi \) by Eq. (7), whose results are shown in Fig. 4(a) and (b), respectively.

The constant part of \( \Psi \) in Fig. 4(b) is about \( \Psi_0 = -7.8 \times 10^{-4} \) Wb-turn, while the amplitude of the sinusoidal part of \( \Psi \) is \( 7.7 \times 10^{-4} \) Wb-turn. Thus, \( \lambda_e = 7.7 \times 10^{-4} \) Wb-turn. The conventional method with no-load test uses a servomotor to drive a PMSM at a constant speed. Since the PMSM is free to run, the three phase currents of the PMSM are all zero, i.e., \( i_a = i_b = i_c = 0 \).

Under such a situation, the emf \( v_a \) and \( v_b \) of phases a and b are measured to compute \( v_{ab} = v_{ab} - v_{bs} = v_a - v_b = \sqrt{3}\lambda_e \omega_0 \cos(\theta_r + \pi/6) \), which follows from Eq. (1) for \( i_a = i_b = i_c = 0 \). Because the speed of the PMSM is known, \( \lambda_e \) is then \((1/\sqrt{3}\omega_0)\) times the amplitude of the sinusoidal \( v_{ab} \).

\[ H = 2.6 \text{ V}, \quad M = 2.5 \text{ V}, \quad L = 2.4 \text{ V}. \]
conventional method, which allows us to calculate out $\dot{\lambda}_r \approx 7.62 \times 10^{-4}$ Wb-turn. This verifies that the proposed method is reliable.

4. Conclusion

This paper proposes a flux estimation method for a PMSM. The PMSM is operated in a single-phase mode and the currents are controlled so that the PMSM rotates in the same direction. Under such a condition, calculating out $\Psi(t)$ in Eq. (7) allows us to obtain the flux $\dot{\lambda}_r$ as the amplitude of the sinusoidal part of $\Psi(t)$. An implemented system with a BA6849 chip is established to verify the proposed method.

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References