Femtosecond second-order solitons in optical fiber transmission

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Abstract

A propagation of the femtosecond second-order solitons in an optical fiber is studied. We show that a generalized nonlinear Schrödinger equation well describes the propagation of the second-order soliton even containing only a few optical cycles. The propagations of a 50fs and a 10fs second-order soliton in an optical fiber are numerically simulated. It is found that, for the case of 10fs second-order soliton, the soliton decay is dominated by the third-order dispersion, in contrast to the case of 50fs second-order solitons, where the soliton decay is dominated by the delayed Raman response. It is also found that the exact delayed Raman response form must be used for the propagation of the 50fs or less than 50fs second-order soliton.

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1. Introduction

The higher-order dispersive and nonlinear effects of ultrashort optical pulses are currently investigated by theory and experiment [1–8]. The positive third-order dispersion effect, distort the pulse shape with an oscillatory structure near the trailing edge [2,3]. The delayed Raman response effect causes a continuous downshift of the optical frequency of the pulse propagating along the optical fiber, known as the soliton self-frequency shift [4,5]. The effect of the self-steepening term leads to an optical shock on the trailing edge of the pulse [6–8]. On the other hand, the effects of third-order dispersion, delayed Raman response, and self-steepening term on the propagation of second-order solitons in a single-mode fiber will lead to the soliton decay [9–15]. When the third-order dispersion parameter exceeds a threshold values, the third-order dispersion effect is going to lead to the soliton decay of higher-order solitons [10–12]. The delayed Raman response effect leads to breakup of higher-order solitons into their constituents, the main peak shifts toward the trailing side [13,14]. The shift is due to a decrease in the group velocity occurring as a result of the red shift of soliton spectral peak. Even relatively small delay Raman response effect still leads to the decay of higher-order solitons. The self-steeping effect will break the degeneracy of higher-order solitons and causes the propagating speed of the solitons being different [15].

In the paper, we will research a propagation of the femtosecond second-order solitons in optical fiber. It is shown that a generalized nonlinear Schrödinger equation well describes the propagation of the second-order soliton of pulsewidth down to 10fs. We numerically
investigate that the 50 and 10 fs second-order solitons of propagation in an optical fiber. We find that, for the case of 10 fs second-order soliton, the soliton decay is dominated by the third dispersion and the delayed Raman response has the least effect, in contrast to the case of 50 fs second-order soliton, where the soliton decay is dominated by the delayed Raman response. We also find that the approximation which assumes the Raman gain is linear in frequency is no longer suitable for the propagation of a 50 fs or less than 50 fs second-order soliton pulses. The method overestimates the soliton decay. Therefore, it is necessary to use the exact delayed Raman response form when the propagation of a 50 fs or less than 50 fs second-order soliton pulse is considered.

2. Theory model

We now consider the influence of third-order dispersion, delayed Raman response, and self-steeping on the propagation of the second-order soliton in a single-mode fiber. An accurate wave equation is used to describe the propagation [16]:

$$\frac{\partial A}{\partial z} = \left( -\beta_1 \frac{\partial A}{\partial t} - i \beta_2 \frac{\partial A}{\partial t}^2 + \beta_3 \frac{\partial A}{\partial t}^3 + \frac{i \beta_4}{24} \frac{\partial A}{\partial t}^4 \right) + i \gamma \left( NA + i z_1 \frac{\partial}{\partial t} NA \right) - i \gamma x_2 \frac{\partial^2 NA}{\partial t^2} - i \frac{\gamma \beta_2 A}{\beta_0} \left[ (1 - x) \frac{\partial A(z, t)}{\partial t} \right]^2 + \alpha \int_{-\infty}^t \int_{-\infty}^t df'(t-t') \left| \frac{\partial A(z, t')}{\partial t} \right|^2 \right] \frac{\partial}{\partial t} N A, \tag{1}$$

where $A(z, t)$ is the field envelope, $\gamma = n_2 c / n_{2, eff}$, $n_2$ is the Kerr coefficient, $n_{2, eff}$ is effective fiber cross section, $z_1 = 2 / \omega_0 - \beta_1 / \beta_0 \approx 1 / \omega_0, x_2 = 1 / \omega_0^2 - 2 \beta_2 / \beta_0 \omega_0 + \beta_3 / \beta_0 - 2 \beta_2 / 2 \beta_0 \approx - \beta_2 / 2 \beta_0, \omega_0$ is the angular frequency of the carrier wave, $\beta_0$ is the propagation constant $\beta$ at $\omega_0$, and $\beta_1$ is the reciprocal group velocity. $\beta_2, \beta_3, \text{and} \beta_4$ are the second-, third-, and fourth-order dispersions, respectively.

The delayed response $N(z, t)$ is described by Chen et al. [17]

$$N(z, t) = (1 - x)|A(z, t)|^2 + \alpha \int_{-\infty}^t df'(t-t')|A(z, t')|^2. \tag{2}$$

On the right-hand side of Eq. (2), the first term represents Kerr nonresonant virtual electronic transitions in the order of about 1 fs or less [18], the second term represents delayed Raman response, $f(t)$ is the delayed response function, and $\alpha = 0.18$ parameterizes the relative strengths of Kerr and Raman interactions. In this paper, $f(t)$ is obtained by modeling the Raman gain by 27 Lorentzian lines, centered on the different optical phonon frequencies and

$$f(t) = \sum_{i=1}^{27} \frac{\tau_i}{\tau_i^2 + \gamma} \exp(-t/\tau_i) \sin(t/\tau_i), \tag{3}$$

where the parameters are determined by fitting the imaginary parts of its spectrum to actual Raman gain of fused silica. The fitted Raman gain spectrum is plotted in Fig. 1.

In dimensionless soliton units, Eq. (1) can be rewritten to

$$\frac{\partial}{\partial \zeta} u = \frac{i \gamma^2 u}{\tau_0} + i \frac{u}{\tau_0} + \frac{u^2}{\tau_0^2} + \frac{i \beta_4}{24} \frac{u^4}{\tau_0^4} \frac{\partial^4 u}{\partial \tau^4} + \frac{i N}{\tau_0^2} \frac{u}{\tau_0} + \frac{1}{\tau_0} \frac{\partial}{\partial \tau} N u - \frac{1}{\tau_0^2} \frac{\partial N}{\partial \tau} u \tag{4}$$

where $\zeta = z/L_D, \tau = t - \beta_1 z/T_0, u = N P A / \sqrt{T_0}, \beta = \beta_2 / 6 \beta_0 T_0, L_D = T_0^2 / \beta_2 / \beta_0$ is dispersion length, and $N = N_P N / P_0$. The parameter $N_P = [\gamma T_0^2 / (\beta_2)]^{1/2}, N_P = 1$ for the fundamental soliton, $T_0 = T_w / 1.763, T_w$ is the pulse full-width at the half-maximum, and $P_0$ is peak power of the incident pulse.

When these higher nonlinear terms are negligible, Eq. (4) reduces to

$$\frac{\partial}{\partial \zeta} u = \frac{i \gamma^2 u}{\tau_0} + \frac{u}{\tau_0} + \frac{u^2}{\tau_0^2} + \frac{i \beta_4}{24} \frac{u^4}{\tau_0^4} \frac{\partial^4 u}{\partial \tau^4} + \frac{i N}{\tau_0^2} \frac{u}{\tau_0} + \frac{1}{\tau_0} \frac{\partial}{\partial \tau} N u. \tag{5}$$

![Fig. 1. The imaginary part of the spectrum of delayed Raman response function fitted by 27 Lorentzian lines.](Image 338x85 to 512x246)
It is known as the generalized nonlinear Schrödinger equation.

3. Numerical results

To solve Eqs. (4) and (5), we use the split-step Fourier method and take the typical fiber parameters to be: soliton wavelength $\lambda = 1.55 \mu m$, $\beta_2 = -20 fs^2/mm$, $\beta_3 = 100 fs^3/mm$, $\beta_4 = 0 fs^4/mm$, $n_2 = 2.3 \times 10^{-20} m^2/W$, and $A_{eff} = 50 \mu m^2$.

First, we numerically show the pulse shapes of the 10fs second-order soliton in Fig. 2. Here $z_0$ is soliton period. As the numerical result is obtained without these higher-order nonlinear terms, it is found that the difference between with and without the four higher nonlinear terms is only less than 0.1%. Therefore, for propagation of the second-order soliton of pulsewidth down to 10fs, these higher nonlinear terms are negligible by the numerical result, i.e., the generalized nonlinear Schrödinger equation well describes the propagation of the second-order soliton of pulsewidth down to 10fs.

3.1. The propagation of the 50fs second-order soliton

Fig. 3 shows the power evolution of pulse shapes when only the third-order dispersion effect is considered. It is seen that the soliton decay occurs within two soliton periods. In Fig. 4, we show the power evolution of pulse shapes when only the delayed Raman response effect is considered. The soliton delay occurs within two soliton periods, the main peak shifts toward the trailing side at a rapid rate with a further increase in the distance and the lower-intensity soliton moves on the leading side. With only the self-steeping effect, the power evolution of pulse shapes are shown in Fig. 5. Up to five soliton periods, the pulse shapes have not obvious separation. This means that the self-steeping effect is very small compared to the effect of the delayed Raman response. When the three higher-order effects are considered together, the power evolution of pulse shapes is shown in Fig. 6. It is seen that the delay of the main peak is about 72fs after propagating the distance of five soliton periods. Comparing Fig. 6 with Fig. 4, we can see that they are similar except that the pulse shape of main peak broadens and the delay of the main peak becomes smaller for the case where all three effects are considered. The broadening leads to the delay of the reduction of the main peak. Among the three effects, it is seen from Figs. 3–6 that the soliton decay is dominated by the delayed Raman response and the self-steeping has the least effect. When the delayed Raman response is approximated by assuming that the Raman gain is linear in frequency, in dimensionless soliton units, Eq. (4) reduces to

$$\frac{\partial}{\partial z} u = i \frac{\partial^2 u}{\partial t^2} + \beta_2 \frac{\partial^3 u}{\partial t^3} + \frac{24|\beta_4|}{T_0} \frac{\partial^4 u}{\partial t^4} + i|u|^2 u - \frac{T_R u |u|^2}{T_0} \frac{\partial}{\partial t} (|u|^2 u),$$

where $T_R$ is the slope of the Raman gain profile at the carrier frequency. Here we consider $T_R = 3 fs$ [19]. Fig. 7 shows the same pulse propagation as in Fig. 6.
when Eq. (6) is used for numerical simulation. The main peak is delayed about 148 fs after propagating the distance of five soliton periods, the delay is much larger than that calculated by Eq. (5). In Fig. 8, we show the averaged frequency shifts obtained by Eqs. (5) and (6). After propagating the distance of five soliton periods, the averaged frequency shift obtained by Eq. (6) is about 37 THz and that obtained by Eq. (5) is only about 22 THz. Therefore, Eq. (6) is not suitable for describing the propagation of a second-order solitons whose pulsewidth is equal to 50 fs.

### 3.2. The propagation of the 10 fs second-order soliton

In Fig. 9, we show the power evolution of pulse shapes only with the third-order dispersion effect. One can be seen that the third-order dispersion effect not only leads to breakup of initial solitons into their constituents but stimulates a radiation within a distance of one soliton periods. The radiation is excited at and around the resonant frequency after the first constriction. Then the radiation pulse leaves the main peak in the trailing direction and gradually vanishes [10,11]. The similar behavior occurs in Fig. 3, where the effect is small and can hardly be observed. Fig. 10 shows the power evolution of pulse shapes when only the delayed Raman response effect is considered. It is seen that only small delay occurs after propagation a distance of five soliton periods. The effect of delayed Raman response is no longer important when the pulse width is short enough. It is because that the spectral width of the input pulse is much larger than the Raman gain spectral width which is about 13.2 THz. The red spectral components of the pulse can only be amplified by small parts of the blue spectral components, and the effect of delayed Raman response becomes unimportant. In Fig. 11, we show the power evolution of pulse shapes when only the self-steeping effect is considered. The two solitons gradually separate from each other at a distance of two soliton periods and are delayed together. One can see that, unlike the case of the 50 fs second-order solitons, the self-steeping effect is now larger than the delayed Raman response effect. When all the three higher-order effects are included, the power evolution of pulse shapes is shown in Fig. 2. The pulse shape of main peak broadens and the delay of the main peak become smaller. By comparing Figs. 2, 9, 10 and 11, it is seen that the third-order dispersion effect plays the most
important role, the next is the self-steeping effect, and the least is the delayed Raman response effect.

4. Conclusions

In conclusion, the propagation of the femtosecond second-order solitons in optical fiber is studied. It is shown that the higher-order nonlinear terms in the accurate equation are negligible for a second-order soliton of pulsewidth down to 10 fs, which means that the equation obtained previously is valid in those ranges. A 50 fs and a 10 fs second-order solitons of propagation in an optical fiber are numerically calculated. It is found that, for the case of 10 fs second-order soliton, the soliton decay is dominated by the third dispersion and the delayed Raman response has the least effect, in contrast to the case of 50 fs second-order soliton, where the soliton decay is dominated by the delayed Raman response. For the propagation of the 50 fs or less than 50 fs second-order soliton, we have shown that the exact delayed Raman response form must be used.

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References
