Modeling and experiment of three-degree-of-freedom actuators using piezoelectric buzzers

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Received 17 June 2013
Published 29 August 2013
Online at stacks.iop.org/SMS/22/105006

Abstract
This study presents innovative three-degree-of-freedom piezoelectric actuators. Under the piezoelectric force and dry friction, the piezoelectric actuators not only can move in the Z-axis direction, but also rotate around the Y-axis and Z-axis. The Z-axis displacement can reach 62 mm and the rotation angle around the Y-axis and Z-axis can reach 270° and 360°, respectively. Compared with the literature, this innovative actuator design achieves one-degree-of-freedom translation and two-degree-of-freedom rotation. Equations of motion are derived based on the piezoelectric properties and Newton’s law. Two types of actuators are created in this study. In the first type, the centers of four piezoelectric buzzers are attached to an arm while in the other type each rim of the four piezoelectric buzzers is attached to the arm. Experimental results are compared with theoretical results. According to the experimental results, the present actuator can accomplish a translational velocity of 11 mm s$^{-1}$, a Y-axis angular velocity of 8.96 rad s$^{-1}$, a Z-axis angular velocity of 2.63 rad s$^{-1}$, and a force of 2.49 mN. By using four piezoelectric buzzers, this study creates piezoelectric actuators capable of both translational and rotational motions.

1. Introduction

With the development of science and technology in various technical fields, such as aerospace, optics, electronics, and medical engineering, high-precision actuators are required [1–6]. Based on the driving principles, the types of precision actuators include electrostrictive, magnetostrictive, artificial muscle actuators, shape memory alloy, photostrictive, and mechanochemical actuators. In the electrostrictive category, piezoelectric actuators are small and possess nanoscale displacement resolution and large driving forces [7].

The piezoelectric material used in an actuator comes in various types: multilayer, unimorph, bimorph, cylindrical, ring, and disk forms [8–10]. By means of an impact drive force, moving bodies can be driven by the impulse force [11–13] or by alternate stick and slip method [14–20]. Piezoelectric actuators are employed in precision platforms, atomic force microscopes (AFM) [21], mobile phones, and digital camera lens drives. To achieve innovative and diverse actuator applications, this study is focused on actuators with three degrees of freedom (DOF), for which four piezoelectric buzzers subjected to both a piezoelectric force and dry friction are employed to undergo Z-axis displacement as well as Y-axis and Z-axis rotations. Compared with the literature, this innovative piezoelectric actuator design can achieve 1-DOF translation and 2-DOF rotation. The piezoelectric buzzers play the role as a driving source in this actuator. Under appropriate combinations of the driving voltage and the duty ratio, the piezoelectric buzzers generate different piezoelectric forces and deformation velocities, finally producing a displacement of the moving body. Actuator characteristics, including the velocity, angular velocity, and force are measured to validate the theoretical models.
Future applications of the 3-DOF piezoelectric actuators proposed in this study include mobile phone cameras, digital cameras, digital video cameras, and atomic force microscopes. The current digital products on the market carry out automatic focusing and optical zooming in only one coordinate axis. By contrast, the advantage of employing the proposed 3-DOF piezoelectric actuator is that it can directly adjust two angles of the optical lens in addition to focusing or zooming along a coordinate axis. This feature becomes desirable concerning compact products.

2. Three-DOF piezoelectric actuator design and driving process

As depicted in figure 1, this study has designed and fabricated two types of actuators: in type-A, four piezoelectric buzzers are fixed at their center points, while in type-B four piezoelectric buzzers are fixed at the buzzer rims. Hence, the only difference between both designs lies in the fixed point positions that are glued to the arms. Dealing with buzzers of disk geometry, both designs represent the largest difference in boundary conditions, one fixed at the disk center and the other at the disk rim. This is the reason why this study chooses types A and B. However, it was not known which design performed better until experiments were carried out.

Figure 2 shows that the proposed actuator moves in three DOFs, for which red arrows in figure 2 represent the directions of displacement and rotations. The actuator consists of a moving body, a rod, and a base. As depicted in figure 3, the moving body comprises a bracket, an upper part, a lower part, two driving parts, five screws, and five springs. The driving part consists of two piezoelectric buzzers and an arm. Figure 3 shows an exploded view of the 3-DOF actuator and the dimensions of the two driving part types. The diameter and length of the rod are 3 mm and 70 mm, respectively. The rod and arm are made from carbon fiber material. The bracket, upper part, lower part, and base are made from aluminum alloy. The moving body masses of types A and B are 2538 mg and 2481 mg, respectively. Screws between the moving body and the rod are adjusted in order to generate appropriate preloads and dry friction so as to expedite movement. Springs are used on the moving body to maintain a set value for preload and friction. The advantages of carbon fiber materials include light weight and a low coefficient of friction. The piezoelectric actuator selects carbon fiber as the rod material, because the coefficient of kinetic friction of carbon fiber is lower than metal and leads to a higher velocity between the rod and the moving body; thus, the moving body easily moves through a long displacement. For the arm, fiber carbon is selected due to its light weight.

The piezoelectric buzzers play the role of a driving source in this innovative piezoelectric actuator. A positive voltage causes a shrinking deformation in the piezoelectric buzzer, while a negative voltage causes an expansion. The deformation speed of the piezoelectric buzzer is adjusted by the duty ratio. Figure 4 depicts the principle of actuator translation motion along the Z-axis direction. The driving process is as follows: (a) the moving body is initially stationary. (b) When the four piezoelectric buzzers deform slowly, the piezoelectric force is smaller than the force of dry friction; thus, the moving body does not produce motion and remains in its original location. (c) When the four piezoelectric buzzers deform rapidly, the piezoelectric force becomes greater than the friction and triggers motion of the moving body, which moves along the Z-axis direction. (d) Finally, the four piezoelectric buzzers return to their undeformed state. When steps (a)–(d) are repeated, the
Figure 3. Exploded view of a 3-DOF piezoelectric actuator and the dimensions of the driving part: (a) exploded view; (b) driving part dimensions in mm of the type-A actuator; and (c) driving part dimensions in mm of the type-B actuator.

Figure 4. Principle of actuator translation: (a) the moving body is initially stationary. (b) When four piezoelectric buzzers shrink slowly, the moving body does not produce motion and remains in its original location. (c) When the piezoelectric buzzers expand rapidly, the piezoelectric force triggers motion of the moving body. (d) The four buzzers return to their undeformed state.

Conversely, if the four piezoelectric buzzers first shrink rapidly shrink before they expand slowly, the moving body moves in the opposite direction.

Figure 5 shows the principle of actuator rotation motion around the Y-axis. The axis of the piezoelectric buzzers is parallel to the rod. The principle is described as follows: (a) the moving body is initially stationary. (b) When the top buzzers shrink slowly while the bottom buzzers expand slowly, the piezoelectric force is too small to move the moving body. (c) When the top buzzers expand rapidly while the bottom buzzers shrink rapidly, the generated torque enables the moving body to rotate counterclockwise. (d) Finally, the four piezoelectric buzzers return to their undeformed state. When steps (a)–(d) are repeated, the moving body continues to rotate counterclockwise.

(c) When the top buzzers expand rapidly while the bottom buzzers shrink rapidly, the generated torque due to four piezoelectric forces in opposite directions enables the moving body to rotate clockwise. Conversely, if the top buzzers shrink rapidly while the bottom buzzers expand rapidly, the generated torque due to the four piezoelectric forces in opposite directions enables the moving body to rotate clockwise. (d) Finally, the four piezoelectric buzzers return to their undeformed state. When steps (a)–(d) are repeated, the moving body continues to rotate counterclockwise.
Figure 6. Principle of actuator rotation motion: (a) the moving body is initially stationary. (b) When the right buzzers shrink slowly while the left buzzers expand slowly, the piezoelectric force is too small to move the moving body. (c) When the right buzzers expand rapidly while the left buzzers shrink rapidly, the generated torque enables the moving body to rotate counterclockwise. (d) Finally, the four piezoelectric buzzers return to their undeformed state.

Figure 6 shows the principle of actuator rotation motion around the Z-axis. The axis of the piezoelectric buzzers is perpendicular to the rod. The principle is described as follows: (a) the moving body is initially stationary. (b) When the right buzzers shrink slowly while the left buzzers expand slowly, the piezoelectric force is too small to move the moving body, and the moving body does not produce motion or angle rotation. (c) When the right buzzers expand rapidly while the left buzzers shrink rapidly, the generated torque due to the four piezoelectric forces in opposite directions enables the moving body to rotate counterclockwise. Conversely, if the right buzzers shrink rapidly while the left buzzers expand rapidly, the generated torque due to the four piezoelectric forces in opposite directions enables the moving body to rotate clockwise. (d) Finally, the four piezoelectric buzzers return to their undeformed state. When steps (a)–(d) are repeated, the moving body continues to rotate counterclockwise.

3. Theoretical derivation

This paper presents actuators of two types: type-A, whose piezoelectric buzzers are attached to an arm at both buzzer centers, and type-B, whose piezoelectric buzzers are attached to the arm at the buzzer rims. It will be later described in another section on experiments that type-A moves faster than type-B. Therefore, this study only derives the dynamic equations of type-A. Based on the dynamic equations, this study calculates the piezoelectric forces and velocities of the moving body. These theoretical results are validated by experimental results.

Figure 7 shows the direction of both the driving voltage and the deformation of two buzzers. Figure 7(a) shows the buzzer state without the driving voltage. When two buzzers with both positive voltages travel in the same direction, two buzzers generate deformations and forces in the same direction. The principle enables the moving body to generate linear displacement, as shown in figure 7(b). However, as depicted in figure 7(c), if both buzzers with positive and negative voltages are traveling in opposite directions, the both buzzers generate deformations and forces in opposite directions, which cause the moving body shown in figure 5(c) to rotate.

Assuming that the rim of a piezoelectric buzzer is fixed but the center can vibrate freely. When the driving voltage is applied to the buzzer, the buzzer center deforms. The mechanical model of the proposed piezoelectric actuators is depicted in figure 8. As the voltage is applied to the piezoelectric buzzers, they rapidly produce deformation,
Integrating this equation three times gives
\begin{equation}
\frac{d\omega}{dr} = 0, \quad \text{when } r = 0
\end{equation}
\begin{equation}
\omega = \frac{d\omega}{dr} = 0, \quad \text{when } r = a.
\end{equation}
Substituting equation (5) into (6) yields the unknown constants
\begin{equation}
A = -\frac{1}{2} - \log a, \quad B = 0, \quad C = \frac{1}{2}a^2.
\end{equation}
Substituting equation (7) into (5) yields the buzzer deformation
\begin{equation}
\omega = \frac{P}{8\pi D} \left[ \frac{1}{2} \left( a^2 - r^2 \right) + r^2 \log \frac{r}{a} \right].
\end{equation}
The radial moment \( M_r \) and the tangential moment \( M_\theta \) are respectively written as [22]
\begin{equation}
M_r = -D \left( \frac{d^2\omega}{dr^2} + \frac{v}{r} \frac{d\omega}{dr} \right),
\end{equation}
\begin{equation}
M_\theta = -D \left( \frac{1}{r} \frac{d\omega}{dr} + \frac{v}{r^2} \frac{d^2\omega}{dr^2} \right).
\end{equation}
where \( v \) is the Poisson’s ratio. Substituting equation (8) into (9) gives the moments
\begin{equation}
M_r = \frac{P}{4\pi} \left[ (1 + \nu) \log \frac{a}{r} - 1 \right],
\end{equation}
\begin{equation}
M_\theta = \frac{P}{4\pi} \left[ (1 + \nu) \log \frac{a}{r} - \nu \right].
\end{equation}
According to Hooke’s law, the radial strain \( \varepsilon_r \) and the tangential strain \( \varepsilon_\theta \) are respectively written as
\begin{equation}
\varepsilon_r = \frac{1}{E} (\sigma_r - \nu \sigma_\theta),
\end{equation}
\begin{equation}
\varepsilon_\theta = \frac{1}{E} (\sigma_\theta - \nu \sigma_r).
\end{equation}
Substituting equations (12) into (13) gives
\begin{equation}
\varepsilon_r = \frac{-6P}{4\pi Et^2} \left[ (1 + \nu) (1 - \nu) \log \frac{a}{r} - (1 - \nu^2) \right],
\end{equation}
\begin{equation}
\varepsilon_\theta = \frac{-6P}{4\pi Et^2} \left[ (1 + \nu) (1 - \nu) \log \frac{a}{r} \right].
\end{equation}
Dealing with circular plates, the piezoelectric equation can be written as [23]
\begin{equation}
\varepsilon_r = \frac{E}{2(1-\nu)} \sigma_r + \frac{E}{2(1-\nu)} \sigma_\theta + \frac{E}{2(1-\nu)} \sigma_z + d_{31}E_z.
\end{equation}
where $s_{11}^{E}$, $s_{12}^{E}$, and $s_{13}^{E}$ denote the elasticity constants of the piezoelectric buzzer, $d_{31}$ the charge constant of the piezoelectric buzzer, $\sigma_0$ the axial stress, and $E_z$ the exerted electric field. The piezoelectric force and the electric field are respectively written as [24]

$$F_p = \sigma_0 A_1$$
$$E_z = \frac{V}{l}$$

where $A_1$ denotes the buzzer disk area and $V$ the exerted electric voltage. Substituting equations (12), (14), (16) and (17) into (15) yields the piezoelectric force

$$F_p = \frac{A_1}{S_{13}^{E}} \left( \varepsilon_t - s_{11}^{E} \sigma_t - s_{12}^{E} \sigma_0 - d_{31} \frac{V}{l} \right).$$

The friction force between the moving body and rod can be expressed by [25]

$$F_f(\dot{z}) = \gamma_1 (\tanh (\gamma_2 \dot{z}) - \tanh (\gamma_3 \dot{z})) + \gamma_4 \tanh (\gamma_5 \dot{z}) + \gamma_6 \dot{z}.$$  

Based on equation (19), the static friction coefficient can be approximated by $\gamma_1 + \gamma_4$. The term $\tanh (\gamma_2 \dot{z}) - \tanh (\gamma_3 \dot{z})$ captures the Stribeck effect, where the friction coefficient decreases from the static friction coefficient with increasing slip velocity. The third term $\gamma_4 \tanh (\gamma_5 \dot{z})$ accounts for the Coulomb friction effect. The last term $\gamma_6 \dot{z}$ accounts for the viscous dissipation effect. Finally, substituting equations (18) and (19) into (1) yields the dynamic equation. Table 1 lists the parameters used in the simulation.

4. Experimental results and discussion

4.1. System identification

System identification is conducted to investigate the piezoelectric buzzer characteristics and identify resonance frequencies before selecting a resonant frequency for driving the moving body. Exciting buzzers at the resonance frequency enables the moving body to move at high speeds. Figure 9 shows the experimental setup for system identification, for which the output is buzzer deformation and the input is a sinusoidal driving voltage of 2 V in the scanning range from 10 and 20 kHz. The driving voltage of the actuator driver is adjusted to drive the moving body on the piezoelectric actuator. The vibrometer emits laser beams to measure buzzer deformation, passing the signal value of the vibratory deformation to the vibrometer controller. Finally, vibration data stored in the vibrometer controller are converted into Bode diagrams. Corresponding to figure 1, which shows photographs of type-A and B actuators, figure 10 depicts their Bode diagrams, where the resonance frequencies of type-A include 3036, 3377, and 7445 Hz and those of type-B are 396, 536, and 6900 Hz. Accordingly, type-A has a larger bandwidth than type-B.

4.2. Performance measurement

Velocity measurement, angular velocity measurement, and force measurement experiments are carried out for comparison between actuators of types A and B. According
to the resonant peaks, the driving frequencies selected for both actuators are 3036 Hz and 6900 Hz, respectively. When the driving voltage employs square waves \cite{17, 18} to drive the piezoelectric buzzer, according to the excitation frequency, frequency response of the piezoelectric buzzer, and duty ratio of the driving voltage, the piezoelectric buzzer deforms, as depicted in figure 11. Changes in the duty ratio influence the movement direction and velocity. Figure 11 shows the measured buzzer deformation. For the actuator velocity measurement, figure 12 shows the experimental setup, which includes a waveform generator (Agilent 33210A), an actuator driver (Echo ENP-4012B), and a precise electronic scale (Precisa XS 625M) with 0.01 mN resolution. Buzers generate tiny forces due to the $t = 0.8$ mm thickness in the piezoelectric film. In order to effectively measure the actuator force resultant from the buzzers, this study uses the scale to replace force sensors whose resolution is 1 mN only. In experiments, firstly, the waveform generator is used to generate a square waveform with a duty ratio. The driving voltage of the actuator driver is adjusted so as to drive the actuator. As depicted in the upper left part of figure 13, point a on the moving body is located at the geometric center of the four buzzers. A carbon fiber rod connects point a on the moving body and the scale center. When the actuator exerts force via the carbon fiber rod to the scale center, the scale is able to measure the resultant force of the four buzzers.

Under appropriate combinations of the driving voltage and the duty ratio, the piezoelectric buzzers generate different piezoelectric forces and deformation velocities, finally producing the displacement of the moving body. Experimental and simulation results of the velocity variation with duty ratios are depicted in figure 14, in which the driving voltage is 40 V for both actuators and the driving frequencies are 3036 Hz and 6900 Hz for types A and B, respectively. When the duty ratio is prescribed as 50%, the buzzer deforms in an isosceles triangle waveform and the velocity of the moving body is difficult to control and stabilize; thus, a duty ratio of 50% is not appropriate. According to experiments, 10% and 90% duty ratios have little effect on the velocity; thus, the results of both duty ratios are not included in comparison. Figure 14 also shows that when the duty ratio is 20%–40% and 60%–80%, the moving bodies moved in opposite directions. The fastest speed is obtained at duty ratios of 40% and 60%. The type-A actuator moves faster.
Figure 12. Experimental setup for the actuator velocity measurement.

Figure 13. Experimental setup for the actuator force measurement. For the purpose of measuring forces coming from all four buzzers, the end of the carbon fiber rod is glued to touch point a, which is located at the geometric center of the moving body.

Figure 14. Comparison of the experimental and simulation results for velocity variation with the duty ratios.

than type-B actuator. Type-A simulation results are only slight variations from experimental results of type-A since the equations of motion are derived based on the type-A geometry.

Figure 15 shows that the moving velocity is proportional to the driving voltage. The type-A actuator moves significantly faster than the type-B actuator. Moreover, the moving velocity of type-A and type-B experiments under a driving voltage of 50 V can reach 11 mm s\(^{-1}\) and 8 mm s\(^{-1}\), respectively. The moving velocity of the type-A simulation under a driving voltage of 50 V is 10.7 mm s\(^{-1}\). The maximum distance traveled in the Z-axis direction is 62 mm. If there is no space constraint, the distances traveled by the moving bodies are unlimited.

Concerning the Y-axis rotational motion of the actuators, figure 16 shows that the angular velocity increases with the drive voltage. In addition, the measured angular velocity of type-A is faster than type-B. The angular
velocities of type-A and type-B experiments under a driving voltage of 50 V are 8.96 rad s\(^{-1}\) and 1.74 rad s\(^{-1}\), respectively. The angular velocity of type-A simulation under a driving voltage of 50 V is calculated as 13.54 rad s\(^{-1}\). Unbalanced weights on both sides of the rotational arm, as shown in figure 3(a), result in an angular velocity discrepancy between simulation and experimental results. The maximum rotation angle in the Y-axis direction, which is depicted in figures 1 and 2, can reach 270\(^{\circ}\) in experiments.

Concerning the Z-axis rotational motion of the actuators, figure 17 shows that the angular velocity increases with the drive voltage. In addition, the measured angular velocity of type-A is faster than type-B. The angular velocities of type-A and type-B experiments under a driving voltage of 50 V are 2.63 rad s\(^{-1}\) and 1.17 rad s\(^{-1}\), respectively. The angular velocity of the type-A simulation under a driving voltage of 50 V is 3.35 rad s\(^{-1}\). The measured angular velocities of 50 V are smaller than the calculated angular velocities in the simulation results due to unbalanced weights at both sides of the bracket and the gravitational acceleration of the bracket. The maximum rotational angle in the Z-axis direction, which is depicted in figures 1 and 2, can reach 360\(^{\circ}\) in experiments.

Figure 18 shows that the actuator force is proportional to the driving voltage. The force of the type-A and type-B experiments under a driving voltage of 50 V can reach 2.49 mN and 2.19 mN, respectively. The force of the type-A simulation under a driving voltage of 50 V is 4.01 mN. According to figure 18, actuator forces increase with the driving voltage. The measured force from the type-A actuator is larger than from the type-B actuator. The measured forces are smaller than the calculated forces in the simulation results due to the unbalanced weights and inertial force at both sides of the bracket. According to the overall comparison, type-A is superior to type-B, which is attributed to the fact that type-A is pasted at its center with the arm and hence generates larger forces and velocity while type-B is pasted at its rim.
5. Conclusion

This study has constructed a model and carried out experiments for innovative 3-DOF piezoelectric actuators, alternately subjected to a piezoelectric force and dry friction, which not only are capable of translation but also rotation. Compared with the literature, the present new piezoelectric actuator design achieves both 1-DOF translational and 2-DOF rotational motions. Experimental results depict that the actuator design achieves both mechanical and piezoelectric properties is validated by experimental results.

References