Reliability analysis of network design for a hub-and-spoke air cargo network

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This study develops a series of models for designing an air cargo network, including flight frequencies, aircraft types and routes, that minimises the total logistics cost. The time value of cargo, economies of scale, and route distance are considered in the hub-and-spoke network design. We calculate the boundaries of the type of aircraft to be used and the route selection, thereby providing airlines with efficient decision-making tools to respond to variations in both the cargo flow and the distance of a route. In addition, this study analyses how fluctuations in demand affect the optimal network design, and evaluates its reliability. The evaluation results show that due to flow consolidation of a hub-and-spoke system, the impact of demand fluctuations can be intensified. Airlines adopting a hub-and-spoke system ought to plan their routes and flights with sufficient flexibility in order to respond to sudden events.

Keywords: hub-and-spoke; reliability analysis; network design; fluctuation in demand; economies of scale

1. Introduction

Since the deregulation of the USAs’ domestic market in 1978, airlines have widely adopted the hub-and-spoke system in order to reduce operation costs. The hub-and-spoke system is commonly employed in the cargo sector because unlike passengers, a cargo would not mind or complain about time-consuming transfers. Most international integrated air cargo carriers, such as FedEx and UPS, have already established their hubs in major cities of the Asia Pacific. For those airlines using the hub-and-spoke network, factors such as the time value of cargo, cargo flow, and route distance are usually considered in order to select the type of aircraft and to decide whether or not to use the hub for cargo transport.

In practice, the network design of airlines, including flight frequencies, aircraft types and routes, is planned for either the summer or the winter season according to a variety of demands. Due consideration must be given to the predicted demands for the coming months and the average demand on each route is used in the planning. However, a cyclical boom in business, regional economic growth or decline and other unexpected events, such as wars or strikes, may cause inevitable disturbances in the flow of all considered routes. The empirical analysis performed by Rimmer (2000) and Dobruszkes and Hamme (2011) even showed that economic crisis has different impacts on the geography of air traffic volumes. When fluctuations in demand occur as

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a result of such disturbances, the flight network designed according to the average demand may become inappropriate, and thus adjustments must be made.

The hub-and-spoke network is an overall plan designed, in principle, for all routes to achieve system optimum. Even though the demand of cargo traffic on a specific route remains unchanged, the decision-making process on that route may be affected by disturbing factors on other routes.

Besides traffic flow, the operating procedures of airports may also necessitate adjustments to be made. For example, the emphasis on airport security checks after the 9/11 terrorist attack may prolong the operation time at airports and cause additional delays to airlines. In addition, airlines servicing the fresh product delivery markets may have to adjust flight frequencies, aircraft types or even routes not only according to load factors, but also punctuality.

Once the network for an airline is determined, decisions on aircraft maintenance and crew assignment will then be made to ensure harmonised operation and smooth coordination. In other words, once the allocation of resources is determined, airlines have little room for adjustments in their flight network despite of disturbances. Therefore, when designing a flight network, the possibility that a disturbance may impact the reliability of the network should be well taken into account.

Numerous related studies on air transport have been conducted, but most of them concern passenger transportation. Considering the discrepancy in characteristics and complexities between passenger and cargo flights, Kasilingam (1996) addressed the differences from the perspective of air cargo revenue management. Among the studies examining air cargo routes, many, such as the research done by O’Kelly (1986, 1992), Aykin (1988), O’Kelly and Miller (1994) and Martin and Roman (2004), tackled the issue of selecting and locating hubs. In addition, Chan and Ponder (1979) and Chestler (1985) described and analysed the various characteristics of air cargo and the application of the hub-and-spoke system in the design of the air cargo network. Kuby and Gray (1993), on the other hand, introduced the situation of stopovers and feeders to analyse the design issue of a hub-and-spoke air cargo network, and expressed it as an integer programming problem for seeking a solution. Yang, Shiau, and Lee (2004) took into account direct flights and transfers via hubs as well as the number of hubs used and capacity limitation to establish a mixed-integer programming problem model for planning the optimal network design. However, the economy of scale of the hub-and-spoke system was treated only as an exogenous parameter in their study and the relationship between network design and economy of scale was not explicitly expressed. In addition, they did not consider the uncertainty of demands and other factors. Sanso and Milot (1999) assessed the performability measure of an urban transportation network given the possibility of an accident. Du and Nicholson (1997) considered a transportation system as degradable when its system state and performance can be degraded by a variety of events, such as earthquakes, floods, traffic accidents and industrial actions. They proposed a framework for the sensitivity analysis of a degradable transportation system on the basis of an integrated equilibrium model with elastic travel demand. Furthermore, taking into account stochastic link capacity degradation and stochastic demand, Siu and Lo (2008) developed a habitual travel time budget equilibrium model to capture the commuters’ considerations in route selection. Similarly, when the equilibrium of a hub-and-spoke air cargo network is affected by any disturbing factor, the network design has to be adjusted accordingly. Regarding logistics and physical distribution issues, Haughton and Stenger (1999) proposed a model dealing with delivery shortages when the demands of customers in a logistic network are stochastic. The research compared different strategies on the basis of how these strategies affect inventory and transportation, and explored the information cost thresholds for accepting/rejecting route reoptimisation. With respect to the airline passenger network, Hsu and Wen (2002) evaluated the reliability of the network design under fluctuations in demand. Wu (2005) simulated the impact of delay propagation in airline networks caused by operation delays and pointed out that when designing their networks, airlines should consider stochastic disruptions in operations to enhance schedule reliability.
In general, past research on hub-and-spoke network design focused on transportation cost, with the economies of scale of the hub-and-spoke network not treated as an endogenous variable or clearly presented. In addition, neither the impact of disturbances on the network design nor how these disturbances affected the reliability of the design were addressed or formulated as a model in the literature. To make up for such deficiencies, this paper attempts to explore these issues in depth using an analytical approach.

The remainder of this paper is organised as follows. Section 2 develops the proposed logistics cost function and designs a network for airlines to minimise total logistics costs. Section 3 analyses the impact of various demand disturbances on the network design. In Section 4, a case study is presented to demonstrate the application of the model developed and the results are discussed. Finally, in Section 5, conclusions are drawn and suggestions are made.

2. Design model for an air cargo network operation

Hall (1987) developed a model to analyse the direct versus terminal freight routing in a network with concave costs. The routing choice was decided by considering the transportation and inventory costs simultaneously. This paper takes the characteristics of air freight into consideration and discusses the hub-and-spoke network design.

The pattern of the hub-and-spoke system can vary according to practical requirements. The hub-and-spoke system proposed in this paper is shown in Figure 1. The serving markets of an airline are divided into several regions including Asia Pacific, North America and Western Europe, each with a hub established. Cargo transported into the respective region must be redistributed via the regional hub. In a hypothetical situation where there is no limitation on traffic rights and flight frequencies, the cargo from airport \( m \) to a regional hub \( n \) can be transported directly on route \( m-n \), or via its regional hub \( h \), where route \( h-n \) is the line haul of the flight network.

Generally speaking, air cargo can be transported by general airlines and integrated air cargo carriers. General airlines offer purely cargo transport service by air with the ground transport undertaken by forwarders; while integrated air cargo carriers provide door-to-door cargo transport service both on land and in the air. In this study, the network concerns mainly air cargo transport and does not involve ground delivery. In actual practice, it is mostly the integrated air cargo carriers that adopt the complicated hub-and-spoke network; however, general airlines have also incorporated the hub-and-spoke network in its simpler form to reduce operation costs. For example, both China Airlines and EVA Airways often use Taipei as their hub for cargo transport from Mainland China and Southeast Asia to the USA. This study aims to develop models that are applicable to both integrated air cargo carriers and general airlines.

Route selections are different for cargo and passenger flights mainly because goods have no preferences or feelings. Airlines design their network with the objective of minimising the total logistics costs. If a particular route can achieve lower operation costs and speed up the delivery of goods, it will be considered as the better choice.

The total logistics costs examined in this study comprise transportation costs, inventory costs, warehouse rent and cargo handling costs. As mentioned above, the proposed network design concerns mainly air cargo transport and hence focuses on airport-to-airport transportation costs. Whether the ground delivery is handled by forwarders or integrated air cargo carriers, the costs incurred will have no impact on the air cargo network design, and are thus neglected in our analysis. Both inventory costs and warehouse rent fluctuate with the transport cycle. In view of the fact that all cargoes, whether in packages or not, require pallet or container consolidation, and for the completeness of the model developed, the cargo handling costs are incorporated into the logistics cost function in our analysis.
Transportation costs and cargo handling costs are undertaken by air carriers, and these costs affect directly the strategies for flight frequency, aircraft type and route selection. Inventory costs and warehouse rent are borne by shippers or forwarders, and while they have little influence on the airlines’ operation cost, they may affect their level of service. However, since air carriers operate in an increasingly competitive and market-driven environment, they aim not only to lower their transportation and handling costs, but also to improve their services and thus enhance their competitiveness. Air carriers usually better their services by providing high-frequency service and well-planned routes so as to shorten logistics time. Therefore, Daganzo (1991) suggested that all costs incurred by cargo from its origin to its destination should be taken into account regardless of who pays for it.

In this study, the definition of the logistics cost function followed that of Hsu and Wang (1997). The following notations will be used in the proposed model.

- $q_{mn}$ weekly cargo flow between OD pair m–n (lb)
- $Q_{mh}$ total weekly cargo flow on route m–h (lb)
- $T_i$ airport user charge per flight for aircraft type $i$ (US$)
- $f_i$ average operation cost per mile for aircraft type $i$ (US$/mile)
- $w$ average ratio between chargeable weight and gross weight of cargo
- $r$ warehouse rent (US$/lb per week)
- $V_i$ optimal shipment volume of aircraft type $i$ (lb)
- $U_i$ maximum payload at the specific route flown by aircraft type $i$ (lb)
- $S_i$ optimal shipment volume without payload constraint for aircraft type $i$ (lb)
- $d_{mn}$ direct transport distance on route m–n (miles)
In general, the unit for aircraft flying time, aircraft takeoff/landing time, and cargo waiting time at the hub are all in hours. However, in actual practice, airlines schedule their flights on a weekly basis in the network design. Hence, in this study, the time for the related parameters in the proposed model is converted from hours to weeks to facilitate calculation.

Transportation costs comprise airport user charge and aircraft operation costs. Airport user charges differ from one airport to another, and include landing fees, parking fees, ground services (excluding cargo handling) and fees for using facilities. Generally speaking, airport user charges, $T_i$, are calculated according to the aircraft weight. Aircraft operating costs, $f_i$, include fuel costs, personnel expenses, aircraft repair and maintenance costs, aircraft depreciation and leasing costs. Each type of aircraft has its own specific flight range. Aircraft operating costs, apart from aircraft size, are also proportional to the distance flown. The weekly transportation costs for delivering cargo on route m–n by an aircraft of type $i$ are the product of the transportation cost per flight, $T_i + f_i d_{mn}$, and the weekly flight frequency, $q_{mn}/V_i$. In other words, $(T_i + f_i d_{mn})q_{mn}/V_i$, where $V_i$ denotes the optimal shipment volume per flight, which is dependent on the type of aircraft used. The optimal shipment volume is not necessarily the maximum capacity of the aircraft. It is because to minimise the total logistics costs, airlines can opt for more frequent but smaller shipments so as to reduce the waiting time incurred. This phenomenon will be further discussed later.

Cargo handling costs can be divided into operation costs of pallet or container consolidation and loading/unloading of cargo. To avoid damaging the cargo and to facilitate handling, most cargo is stored in containers or on pallets. The cargo must be assembled at the departure points and then taken apart upon arrival at their destination. As shown in Figure 1, there are not many destination points in this study; therefore, we assume that the cargo has been sorted and packaged at the points of origin according to their destinations, thus eliminating the need for packing and unpacking at hubs. Therefore, the handling costs at the departing airports include the consolidation and the loading/unloading costs. On the other hand, the handling costs at a hub consist of loading/unloading costs only. Generally speaking, handling costs are directly proportional to cargo volume. The total weekly handling costs of the cargo volume on route m–n are $q_{mn}(g_1 + g_2)$.

Inventory costs represent the loss of opportunity cost and a loss of value because cargo cannot be used or sold when being transported. These costs are positively correlated with cargo volume, the time value of cargo and the duration of transport. In this study, inventory costs include cargo waiting time cost and flying time cost. The waiting time cost is the cost related to the flight frequency due to schedule delays. The higher the flight frequency, the lower the waiting time cost will be.

Air cargo shippers try as much as possible to deliver the shipment to the airport right before aircraft takeoff. Even before being delivered to the airport, the shipment already incurs inventory costs. There is also a depreciation cost involved for perishable goods such as food and flowers. Even for industrial products, it is impossible to time their production right before shipment. The waiting time incurred by such goods should be appropriately incorporated. When designing an air cargo network, airlines should take into account the demand of the entire air cargo market, rather than that of a single product or consignee. In addition, there are ongoing economic productions and business activities, and hence continuous demands for air cargo transport. Assuming that the arrival process of the cargo follows a uniform distribution, the average waiting time at the

\[
P \quad \text{cargo value per pound (US$/lb)} \\
R \quad \text{loss value of cargo per dollar inventory per week} \\
\tau \quad \text{aircraft takeoff and landing time per flight (week)} \\
\tau' \quad \text{aircraft flying time per mile for the cruising stage (week/mile)} \\
g_1 \quad \text{operating cost of pallet or container consolidation (US$/lb)} \\
g_2 \quad \text{operating cost of pallet or container loading/unloading (US$/lb)} \\
e_h \quad \text{additional waiting time incurred at hub h (week)}
departure point is counted as half of the transportation cycle and is denoted as $V_i/2q_{mn}$. The time value of cargo is shown as $PR$. Thus, the waiting time cost of the cargo, $q_{mn}$, at the point of departure can be denoted as $PRV_i/2$.

Aircraft flying time can be divided into two parts. One refers to the time during takeoff and landing when the speed of the aircraft is usually slower; the other is the flight time after the aircraft reaches a certain altitude and when it is flying at a cruise speed on the route. At present, the speed for medium- and long-range aircraft are similar, and the time required for takeoff and landing is almost the same for all aircraft. In this study, we assume that the takeoff/landing time at the airport per flight is $\tau$, and the flying time per unit distance at the cruising stage is $\tau'$ for all aircraft types. The flying time cost is expressed as $PRq_{mn}(\tau + \tau'd_{mn})$. The total inventory costs of cargo on route m–n by an aircraft of type $i$ can, therefore, be expressed as the sum of the waiting time cost and the flying time cost. That is, $PRV_i/2 + PRq_{mn}(\tau + \tau'd_{mn})$.

Before being transported, the cargo must be stored at an airport warehouse, and the rent costs are directly proportional to the cargo volume and storage time. Warehouse rent is dependent on the volume of the cargo, which determines the space required to be rented. However, in actual operation, air cargo terminals calculated the warehouse rent according to either the gross weight or volume weight of the cargo, whichever is higher. In other words, the chargeable weight $= \text{MAX} \ (\text{gross weight}, \text{volume weight})$. As stated in ‘The Air Cargo Tariff Manual’ issued by the International Air Transport Association (IATA, 2012), volume weight can be calculated as 1 pound $= \text{cubic inches}/166$. Cargo for air shipment is usually of specific types and there may exist a relationship between the chargeable weight and gross weight that can be obtained through observation. Hence, to better reflect the actual situation, the proposed model incorporates a parameter $w$, which denotes the ratio between average chargeable weight and average gross weight. Assuming that the rent per unit chargeable weight per week for the airport warehouse is $r$, and $wr$ would be the weekly warehouse rent per unit gross weight of cargo. Then, $wrV/2$, which denotes the rent during the waiting period, is similar to the inventory cost of the cargo at the airport waiting to be transported.

The main purpose of developing a cost function is to compare the direct transportation costs to the costs of transfer-route transportation via air hub. Consequently, we only need to take into account the differences between two transportation routes and omit any equivalent costs. Thus, there is no need to include the inventory cost at the destination point since there is no difference between direct and transfer-route transportation. In addition, airport user charges differ from airport to airport. Whether it is direct or transfer-route transportation, the transportation costs will include airport user charges both at the departure and the destination points. To simplify the expressions as well as the analysis, we express the airport user charges for aircraft type $i$, $T_i$.

To sum up the analysis, the total logistics costs of the cargo flow between OD pair m–n per week transported on direct route m–n, $C_{mn}(q_{mn}, d_{mn})$, can be formulated as follows:

$$C_{mn}(q_{mn}, d_{mn}) = (T_i + f_d d_{mn})q_{mn}/V_i + q_{mn}(g_1 + g_2) + PRq_{mn}(\tau + \tau'd_{mn}) + (PR + wr)V_i/2.$$

(1)

This study aims to explore the network design including aircraft type, flight schedule and routes adopted according to the location of the hub and the cargo flows for airlines implementing the hub-and-spoke system, and to examine its reliability under demand disturbances. Indeed, the issues examined bear some resemblance to those problems, such as optimal shipment volume and frequency, handled by the economic order quantity (EOQ) model. Moreover, the EOQ can fully reflect the characteristic of high time value of cargo in terms of optimal shipment volume and flight frequency and is thus applicable in our analysis. In addition, the proposed model also takes into account the suitable type of aircraft to be employed and their constraints on the optimal shipment volume per flight.
According to the standard EOQ model, the optimal shipment volume can be derived by minimising the total logistics costs as follows:

$$V_i = \sqrt{\frac{2(T_i + f_id_{mn})q_{mn}}{(PR + wr)}}. \quad (2)$$

Equation (2) shows that the optimal shipment volume, $V_i$, depends on airport user charges, $T_i$, and on the average operating cost per mile, $f_i$, which is related to aircraft type $i$, and also on the distance flown on route m–n per week. There exists an inverse relationship between $V_i$ and $PR$; that is, the higher the time value of cargo, the smaller the volume per shipment and the higher the flight frequency will be, so as to reduce the waiting time for the cargo to be transported. The optimal shipment volume per flight is also constrained by the payload of the used aircraft type $i$.

In this study, the following steps are developed to determine the optimal shipment volume.

Step 1: Find the payload for a specific distance (i.e. $d_{mn}$) for all aircraft types. Horonjeff and McKelvey (2002) pointed out a relationship between aircraft payload and flight range, as illustrated in Figure 2. As can be seen, between points A and B, the flight range increases from $R_a$ to $R_b$, while its payload decreases from $P_a$ to $P_b$ owing to the fuel required for the extra distance. Although the flight range of the aircraft can reach $R_c$, the additional fuel that has to be carried will reduce the payload to zero, meaning that it cannot carry any cargo. As a matter of fact, all aircraft types are designed for a specific most suitable range. This study assumes that only the DAB segment as shown in Figure 2 is considered, while the BC segment is not because it is extremely uneconomical. The BC segment involves a long flight distance, implying that the aircraft has to carry large quantity of fuel, which in turn decreases the payload. If $U_i$ denotes the maximum payload for aircraft type $i$ for a specific distance, then

$$U_i = \begin{cases} P_a & d_{mn} \leq R_a, \\ P_a - (d_{mn} - R_a) \times (P_a - P_b)/(R_b - R_a) & R_a < d_{mn} < R_b. \end{cases} \quad (3)$$

![Figure 2. Relationship between payload and range of an aircraft.](image-url)
Therefore, the total logistics cost function of the cargo flow between OD pair m–n per week can be shown as:

\[ C_{mn}(q_{mn}, d_{mn}) = \begin{cases} \sqrt{2(T_i + f_i d_{mn})q_{mn}(PR + r) + q_{mn}(g_1 + g_2) + PR q_{mn}(\tau + \tau'd_{mn})} & V_i < U_i, \\ (T_i + f_i d_{mn})q_{mn}/U_i + q_{mn}(g_1 + g_2) + PR q_{mn}(\tau + \tau'd_{mn}) + (PR + wr) U_i/2q_{mn} & V_i = U_i. \end{cases} \] (4)

Step 4: Calculate and compare the total logistics costs for all aircraft types.

Step 5: Select the aircraft type with the lowest costs to be the optimal aircraft. Furthermore, the average logistics costs per pound, \( AC_{mn}(q_{mn}, d_{mn}) \), and the marginal logistics costs, \( MC_{mn}(q_{mn}, d_{mn}) \), can be derived from the total logistics costs of Equation (4) and are shown as Equations (5) and (6), respectively.

\[ AC_{mn}(q_{mn}, d_{mn}) = \begin{cases} \sqrt{2(T_i + f_i d_{mn})(PR + wr)/q_{mn}} + g_1 + g_2 + PR(\tau + \tau'd_{mn}) & V_i < U_i, \\ (T_i + f_i d_{mn})/U_i + g_1 + g_2 + PR(\tau + \tau'd_{mn}) + (PR + wr) U_i/2q_{mn} & V_i = U_i. \end{cases} \] (5)

\[ MC_{mn}(q_{mn}, d_{mn}) = \begin{cases} \sqrt{(T_i + f_i d_{mn})(PR + wr)/2q_{mn}} + g_1 + g_2 + PR(\tau + \tau'd_{mn}) & V_i < U_i, \\ (T_i + f_i d_{mn})/U_i + g_1 + g_2 + PR(\tau + \tau'd_{mn}) & V_i = U_i. \end{cases} \] (6)

Equation (5) shows that the average logistics costs per pound decrease with increase in cargo flow, \( q_{mn} \), due to the economy of scale. Equation (6) shows that the marginal cost declines with the flow when shipment volume, \( V_i \), is less than the payload of aircraft \( i \) at range \( d_{mn} \), \( U_i \), and approaches a constant, equaling \( (T_i + f_i d_{mn})/U_i + g_1 + g_2 + PR(\tau + \tau'd_{mn}) \) once the load factor is 1 (i.e. \( V_i = U_i \)).

In general, using a hub increases the distance flown and the flying time; hence, both transportation costs and inventory costs will be increased. Moreover, the additional landing required will incur airport user charges. Most hubs are well established with the consolidation function. Increased transportation and inventory costs are thus reduced by realising economies of scale and by shortening the waiting time, thereby encouraging more cargoes to be transferred via hubs. In addition, the decision of whether or not to use a hub depends strongly on the efficiency of hub operations. As a result, when analysing logistics costs, the efficiency of hub operations should also be taken into account.

If the cargo flow between OD pair m–n is transported through hub h, then the entire route can be divided into route m–h and route h–n. The total cargo flow transported on route m–h per week can be shown as \( Q_{mn} \) and includes the cargo destined for hub h and also those transferred via hub h. Similar to the direct transport method, the total logistics cost function of cargo \( Q_{mn} \) on route...
m–h can be denoted as $C_{mh}(Q_{mh}, d_{mh})$ and expressed as follows:

$$C_{mh}(Q_{mh}, d_{mh}) = (T_i + f_jd_{mh})Q_{mh}/V_i + Q_{mh}(g_1 + g_2) + PRQ_{mh}(\tau + \tau'd_{mh}) + (PR + wr)V_i/2.$$  (7)

The actual cargo flow between OD pair m–n is $q_{mn}$, $Q_{mh} - q_{mn}$ refers to the route m–h but not to be transported to destination n. As a result, the logistics costs caused by $q_{mn}$ on route m–h can be shown as $C_{mh}(Q_{mh}, d_{mh}) - C_{mh}(Q_{mh} - q_{mn}, d_{mh})$.

On the other hand, busy hubs or hubs with low operation efficiency will hinder the aircraft operations and prolong aircraft parking time at the apron. Additional waiting time is often incurred for cargo at the hub owing to inefficient flight scheduling or operations at the hub. Too much additional waiting time incurred will affect the decision of airlines in choosing a certain airport as the hub. In the proposed model, this factor is taken into consideration and is denoted by $e_h$. The inventory costs and warehouse rent for the cargo at hubs can be expressed as $(PR + wr)V_i/2 + (PR + wr)q_{mn}e_h$. Therefore, the weekly logistics cost of the cargo on route h–n is denoted as $C_{hn}(Q_{hn}, d_{hn})$, and

$$C_{hn}(Q_{hn}, d_{hn}) = (T_i + f_jd_{hn})Q_{hn}/V_i + Q_{hn}g_2 + PRQ_{hn}(\tau + \tau'd_{hn}) + (PR + wr)V_i/2 + (PR + wr)Q_{hn}e_h.$$  (8)

The logistics costs as a result of $q_{mn}$ on route h–n can be shown as $C_{hn}(Q_{hn}, d_{hn}) - C_{hn}(Q_{hn} - q_{mn}, d_{hn})$. By combining the logistics costs caused by $q_{mn}$ on route m–h and route h–n, the total logistics costs of $q_{mn}$ per week for transfer route via hub can be shown as follows:

$$C^{n}_{mn}(q_{mn}, Q_{mh}, Q_{hn}, d_{mh}, d_{hn}) = C_{mh}(Q_{mh}, d_{mh}) - C_{mh}(Q_{mh} - q_{mn}, d_{mh}) + C_{hn}(Q_{hn}, d_{hn}) - C_{hn}(Q_{hn} - q_{mn}, d_{hn}).$$  (9)

By comparing the logistics costs of the direct routes and the transfer routes, the optimal route with the minimum logistics cost can be selected.

Equation (6) shows that marginal cost declines with increase in weekly cargo flow on route. When the cargo flow is large enough to make the shipment volume equal the payload of aircraft, then the marginal cost will become a constant. The destination points in this study are gateways of regions as shown in Figure 1, route h–n denotes the route between two inter-continental hubs, which handle most of the cargo flow between the two continents. Hence, it is reasonable to assume that the flow on route h–n is large enough to make the marginal cost a constant. The total logistics costs of $q_{mn}$ on route h–n can then be expressed as $C_{hn}(Q_{hn}, d_{hn}) - C_{hn}(Q_{hn} - q_{mn}, d_{hn}) = \alpha q_{mn}$, and the constant, $\alpha$, is then equal to $(T_i + f_jd_{hn})/U_i + g_2 + PR(\tau + \tau'd_{hn}) + (PR + wr)e_h$. Consequently, the weekly logistics costs of $q_{mn}$ for the transfer route in Equation (9) can be revised as follows:

$$C^{n}_{mn}(q_{mn}, Q_{mh}, Q_{hn}, d_{mh}, d_{hn}) = C_{mh}(Q_{mh}, d_{mh}) - C_{mh}(Q_{mh} - q_{mn}, d_{mh}) + \alpha q_{mn}.\quad (10)$$

Furthermore, when $C_{mh}(Q_{mh}, d_{mh}) - C_{mh}(Q_{mh} - q_{mn}, d_{mh}) < C_{mn}(q_{mn}, d_{mn}) - \alpha q_{mn}$, then the transfer route for shipping the cargo flow between OD pair m–n, $q_{mn}$, is selected, where $C_{mh}(Q_{mh}, d_{mh})$ is concave with respect to $Q_{mh}$. For a given $q_{mn}$, $C_{mh}(Q_{mh}, d_{mh}) - C_{mh}(Q_{mh} - q_{mn}, d_{mh})$ decreases with an increase of $Q_{mh}$, as shown in Figure 3. As a result, the optimal route depends on whether $q_{mn}$ is greater than or smaller than a ‘critical flow’, denoted by $Q_{mh}$. The critical flow is the value of $Q_{mh}$, which satisfies the following equation:

$$C_{mh}(Q_{mh}, d_{mh}) - C_{mh}(Q_{mh} - q_{mn}, d_{mh}) = C_{mn}(q_{mn}, d_{mn}) - \alpha q_{mn}\quad (11)$$
When \( q_{mh} \geq \bar{q}_{mh} \), then the transfer cost is less than or equal to the direct cost and Equation (12) is sustainable.

\[
C_{mn}^h(q_{mn}, Q_{mh}, Q_{hn}, d_{mh}, d_{hn}) \leq C_{mn}(q_{mn}, d_{mn}). \tag{12}
\]

Under this circumstance, the cargo flow between OD pair m–n, \( q_{mn} \), is better transported through the hub; otherwise, a direct route is the better choice when \( Q_{mh} \) is smaller than the critical flow, \( \bar{Q}_{mh} \).

\( \bar{Q}_{mh} - q_{mn} \) denotes the critical flow on route m–h before the addition of the transferred cargo \( q_{mn} \). If we obtain the critical flow \( \bar{Q}_{mh} - q_{mn} \) in relation to every \( q_{mn} \), and plot all the loci of \( (q_{mn}, \bar{Q}_{mh} - q_{mn}) \), we can then draw a critical trajectory, as shown in Figure 4. When \( (q_{mn}, \bar{Q}_{mh} - q_{mn}) \) is located above the critical line, then \( q_{mh} - q_{mn} \) is larger than the critical flow \( \bar{Q}_{mh} - q_{mn} \).

As a result, the transfer cost is smaller, and the transfer route will be selected for \( q_{mn} \). On the other hand, if \( (q_{mn}, \bar{Q}_{mh} - q_{mn}) \) is located in the area under the critical line, then a direct route will be selected.

The above inference shows that the larger the flow on route m–h prior to the addition of the transfer flow, the more likely a transfer route is selected for \( q_{mn} \). In some cases, no value of \( Q_{mh} - q_{mn} \) with respect to \( q_{mn} \) will satisfy Equation (11). In that case, the cargo flow between OD pair m–n, \( q_{mn} \), will always be routed directly or through a hub, independent of \( Q_{mh} \). For instance, if \( q_{mn} \) is small and the hub is located near the origin m, then the transfer route will always be preferred. Or, if \( q_{mn} \) is large enough to realise an economy of scale, and if the transfer route is circuitous, a direct route is preferred. As a result, not all \( q_{mn} \) has a corresponding \( \bar{Q}_{mh} - q_{mn} \). The above analysis implies that the upper bound \( \bar{q}_{mn} \) and the lower bound \( q_{mn}^l \) of \( q_{mn} \) are possible. The values of \( \bar{q}_{mn}^l, q_{mn}^l \) and \( \bar{Q}_{mh} - q_{mn} \) affect significantly the decision for selecting the routes for the cargo flow between OD pair m–n, \( q_{mn} \), such that

- When \( q_{mn} > \bar{q}_{mn} \), the direct route should be selected;
- When \( q_{mn} < q_{mn}^l \), the transfer route should be selected; and
When \( \bar{q}_{mn} < q_{mn} < \bar{q}^{u}_{mn} \), the optimal route depends on whether \( \bar{Q}_{mh} - q_{mn} \) is larger than or smaller than \( \bar{Q}_{mh} - q_{mn} \).

The relationship between \( \bar{q}_{mn} \), \( \bar{q}^{l}_{mn} \), and \( \bar{Q}_{mh} - q_{mn} \) is shown in Figure 4. The left-hand side of the curve corresponds to transfer routing, while the right-hand side corresponds to direct routing. The larger \( \bar{q}_{mn} \) and \( \bar{q}^{l}_{mn} \) are, the more likely it is that the cargo flow, \( q_{mn} \), should be transferred through hub \( h \). It is evident from Figure 4 that the larger the values of \( \bar{q}_{mn} \) and \( \bar{q}^{l}_{mn} \), the larger the left-hand side of the curve is. The values of \( \bar{q}_{mn} \) and \( \bar{q}^{l}_{mn} \) are mainly affected by the relative geographic positions between points of origin/destination and hubs.

The above analysis did not constrain the flight frequency as an integer in order to monitor the connection between the cost function and flow. If the actual flight frequencies are taken into account and if all cargoes are to be transported completely, the flight frequency can be replaced by \( \lceil q_{mn}/V_i \rceil \), which is the smallest integer exceeding \( q_{mn}/V_i \). As a result, the total logistics cost function of the cargo flow between OD pair \( m-n \) per week transported on the direct route \( m-n \) can be rewritten from Equations (4) into (13).

\[
C_{mn}(q_{mn}, d_{mn}) = (T_i + f_id_{mn})[q_{mn}/V_i] + q_{mn}(g_1 + g_2) + PRq_{mn} \left( \tau + \tau d_{mn} \right) + (PR + wr)q_{mn} \left[ V_i/q_{mn} \right]/2. \tag{13}
\]

3. Reliability analysis of demand fluctuations

The above cost function is analysed according to the assumption of using the average cargo flow for every OD pair. However, the cargo flow in fact differs from week to week and from month to month. Let us assume that the cargo flow between OD pair \( m-n \) at week \( t \) is a random variable, \( q'_{mn} \). When the cargo flow decreases, the load factor of the original network design becomes smaller. This in turn will result in higher transportation costs per pound (with the same inventory cost and warehouse rent). Airlines may reduce the impact of the increased transportation costs by
reducing the flight frequencies. However, this will increase the inventory costs and warehouse rent correspondingly. On the other hand, using smaller aircraft to maintain the same flight frequency might be an alternative. At the same time, when the cargo flow increases, increasing the frequency or using larger aircraft may obtain an economy of scale and lower the logistics cost per pound. Regardless whether the cargo flow is going up or down, it is only when the changes in the flows are large enough that adjustment in the flight network are justified; if not, it will only affect the load factors.

Let us assume that $SC^0_{mn}$ is the network design for OD pair m–n to accommodate the specific weekly flow, $q^0_{mn}$. Then, this study further derives $q^0_{mn}$ and $\overline{q}_{mn}$. When $q^0_{mn} \leq q'_{mn} \leq \overline{q}_{mn}$, this means that the variations in the cargo flow are not large enough; and therefore, there is no need to adjust the network design, $SC^0_{mn}$. In other words, the original flight network design is appropriate with the minimum costs incurred. When $q^0_{mn} < q'_{mn}$ or $q^0_{mn} > \overline{q}_{mn}$, then this means that the change in demand is such that the average logistics costs per pound with the original network design, $SC^0_{mn}$, is no longer at its optimal lowest. In other words, there should exist a network design, $SC'_{mn}$, that achieves lower average logistics costs than the present one.

In this paper, we defined the reliability of the network design as the probability that pre-planned frequencies, aircraft type and routes can maintain relatively low logistics costs under various future short-run fluctuations.

The above definition allows the reliability of the network planned according to flow, $q^0_{mn}$, at week $t$ to be expressed as a conditional probability, $R_{mn}(q'_{mn}, q^0_{mn})$, in Equation (14) and is shown in Figure 5.

$$R_{mn}(q'_{mn}, q^0_{mn}) = \Pr(q^0_{mn} \leq q'_{mn} \leq \overline{q}_{mn}). \quad (14)$$

The network design of an airline comprises flight frequencies, aircraft types and routes. For the network that is designed according to $q^0_{mn}$, all traffic flows fall within the range of the upper boundary, $\overline{q}_{mn}$, and the lower boundary, $q^0_{mn}$, and can use the same network design with regard to aircraft type, frequencies and routes.

In the network design stage, airlines should take into consideration short-run fluctuations in cargo flow while keeping other parameters constant. In general, when the cargo flow is changed, the frequency may be the first factor to be altered, followed by an adjustment of the aircraft type, and finally a change in routes. However, this also depends on the conditions of every route. In addition, the upper or lower boundary for changing a shipping route in Figure 5 can be obtained from the line of the critical flow in Figure 4.

However, large fluctuations in parameters would necessitate changes in the original network design even during the implementation stage. Depending on the actual situation and resources.
available, airlines can consider rescheduling aircraft and crew. Well-considered formulation stage and realistic parameters adopted can avoid large adjustment of network design during the implementation stage.

Fluctuations in cargo flow are common. However, except for normal seasonal shifts, there may also be some unexpected events that would cause a change in demands such as a large international exhibition, war or labour strikes, thus leading to a change in the cargo flow and influencing the original network design. The extent of these events in terms of significance and duration will affect airlines’ decision-making on whether to adjust their flight network or not.

Let $S_{mn}$ represent the set of all distinct states that occur during a planned period of time, and let $S_{mn} = \{s_0, s_1, s_2, \ldots, s_x\}$, where $s_1, s_2, \ldots, s_x$, represent distinct abnormal states; $s_0$ represents a normal state, and subscript $x$ gives the number of distinct abnormal states. Let $Pr(s_i)$ be the probability that occurs in state $i$, where $Pr(s_i) \geq 0$ and $\sum_{i=0}^{x} Pr(s_i) = 1$.

Let $q_{mn}^{t}|s_i$ denote the cargo flow between OD pair $m$–$n$ at week $t$ when state $i$ occurs. The mean of the cargo flow between OD pair $m$–$n$ at week $t$ can be shown as $q_{mn}^t = \sum_{i=0}^{x} q_{mn}^{t}|s_i Pr(s_i)$ and the conditional reliability that varies with the network design according to the original demands, $q_{mn}^0$, is

\[ R_{mn}(q_{mn}^t, q_{mn}^0) = \sum_{i=0}^{x} R_{mn}(q_{mn}^t|s_i, q_{mn}^0) Pr(s_i). \]  

(15)

An unreliable network design means that the average logistics costs per pound are not as low as originally planned. Although the network design adjustment will reduce costs, it involves rescheduling of aircraft and crews, which would incur extra adjustment costs. Therefore, the decision on whether to adjust the network design or not would depend on the benefits and the costs of the adjustment when the cargo flow changes.

Let the benefit of the adjustment be the cost saved by adjusting the network design in accordance with the cargo flow, then

\[ ADR(q_{mn}^t, q_{mn}^0, d_{mn}) = C_{mn}(q_{mn}^t, d_{mn}, SC_{mn}^0) - C_{mn}(q_{mn}^t, d_{mn}, SC_{mn}^t). \]  

(16)

Generally speaking, when the cargo flow starts to decline, airlines may consider cutting back flight frequencies or using smaller aircraft for shipping. If the situation will persist over a long time, then they may rent out some of the aircraft or sell them. On the other hand, when the cargo flow starts to rise, airlines should increase the flight frequencies or use larger aircraft. However, existing aircraft fleet, size of crew, aircraft maintenance plans and whether the subsequent decision impacts upon other routes are all factors that should be considered when making an adjustment on the network design. If the fleet of an airline is not sufficient, changing the aircraft type or increasing the number of flights on one route may affect some or all other routes. In the short run, airlines may choose to dispatch more aircraft, whereas in the long run, aircraft leasing, purchasing or strategic cooperation with other airlines can be adopted.

Let $\theta_{mnj}$ denote the adjustment cost per flight caused by substituting aircraft type $j$ with aircraft type $i$ on route $m$–$n$, and that the adjustment costs can be shown as $ADC(q_{mn}^t, q_{mn}^0, d_{mn}) = \theta_{mnj}([q_{mn}^t/V_j] - [q_{mn}^0/V_j])$.

When $ADR(q_{mn}^t, q_{mn}^0, d_{mn}) > ADC(q_{mn}^t, q_{mn}^0, d_{mn})$ is tenable, then the benefit of adjustment is greater than the adjustment cost; otherwise, no adjustment should be made since it would not be cost effective. If $q_{mn}^t > q_{mn}^0$ and no adjustment is made, any cargo that cannot be transported may be diverted to other airlines. This will affect the airlines’ market share, revealing the importance of a network design and the prediction of traffic demand. Once a network design is completed, relevant resources such as size of the fleet and the crew as well as maintenance plans will be arranged accordingly. If the flight network needs to be adjusted, then considerable costs will be incurred.
In Section 2, it is assumed that the total cargo flow transported from hub h to destination n is large enough to realise economies of scale, and that the marginal cost is close to a constant, so that the impact of demand fluctuations on logistics costs per pound on that route is small.

However, the change of cargo flow on route m–h and route m–n may be large enough to influence the route selection. When $Q_{mh}'$ and $q_{mn}'$ are disturbed by normal or abnormal factors, the flight frequencies and aircraft types on route m–h and route m–n may be altered. This may cause changes in decisions regarding the direct and transfer costs of the flow between OD pair m–n, $q_{mn}$, as well as the shipping routes. In particular, when $Q_{mh}'$ is close to $Q_{mh}$, or when $q_{mn}'$ is close to $q_{mn}$ or $q_{mn}^t$, a fluctuation in the cargo flow may require an adjustment to be made in the shipping routes. Such impact may extend to more than just one route. For example, let us assume that the original cargo flow on route m–h at week $t$, $Q_{mh}'$, is smaller than $Q_{mh}$. The direct shipping route is then adopted for the flow between the OD pair m–n at week $t$, $q_{mn}'$. However, if $Q_{mh}'$ changes and becomes larger than $Q_{mh}$, then the flow between the OD pair m–n at week $t$, $q_{mn}'$, may have to be transferred through hub h. When the increase of $Q_{mh}'$ is caused by cargo transferred via hub h, then the logistics costs per pound on route m–h will change and affect the cargo flow on other OD pairs transported via hub h, which were originally to be transported directly.

In addition, upsets do not always happen to a single route but may take place on several routes simultaneously. In the hub-and-spoke system, any cargo flow transported on route m–h including the flow between OD pair m–h and those not destined for hub h but are transferred via hub h. When the cargo flow between all OD pairs fluctuates due to an upset, the range of the fluctuation will be significantly widened because of the consolidation of cargo. Let us assume that the cargo flow of k OD pairs merges on route m–h, then the variances of the flow on this route can be expressed as Equation (17).

$$
\sigma^2(Q_{mh}) = \sigma^2 \left( \sum_{i=1}^{k} q_{mi} \right) = \sum_{i=1}^{k} \sum_{j=1}^{k} \sigma(q_{mi}, q_{mj}) = \sum_{i=1}^{k} \sigma^2(q_{mi}) + \sum_{i=1}^{k} \sum_{j=1}^{k} \sigma(q_{mi}, q_{mj}) \quad (i \neq j).
$$

The variances in the traffic flow transported on route m–h, plus the integrating variances in flow on k OD pairs, as well as the covariance of the flow between k OD pairs must be added. For example, generally speaking, all OD pairs are affected when the upset is caused by economic reasons and the direction of the fluctuation is the same. The range of fluctuations of the flow on route m–h may be greater than the sum of the fluctuations of the flow in k OD pairs.

4. Case studies

To demonstrate the operation of the proposed model, this study uses an international airline in Taiwan as an example. This airline uses Taipei (Taiwan Taoyuan International Airport) as its hub for the East Asia region and uses Anchorage, Amsterdam and Sydney as their hubs for the North American, Western European and Australian markets, respectively. There are six different aircraft types available for that airline. The range and payload for all aircraft types are given in Table 1. The operating networks and the cargo flow on all OD pairs are given in Table 2. The mileages of every route are given in Table 3. In addition, the parameters in the model are collected and estimated as follows.

- $P = 20$ US$/lb
- $R = 0.02$
- $w = 1.2$
- $r = 0.25$ US$/lb per week
Table 1. Characteristics of all aircraft types.

<table>
<thead>
<tr>
<th>i</th>
<th>Aircraft type</th>
<th>$P_a$ (lb)</th>
<th>$R_a$ (miles)</th>
<th>$P_b$ (lb)</th>
<th>$R_b$ (miles)</th>
<th>$T_i$ (US$)</th>
<th>$f_i$ (US$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>B-727-200F</td>
<td>65015</td>
<td>1830</td>
<td>58406</td>
<td>2071</td>
<td>868</td>
<td>14.10</td>
</tr>
<tr>
<td>2</td>
<td>B-757-200F</td>
<td>72210</td>
<td>3340</td>
<td>60000</td>
<td>4260</td>
<td>1008</td>
<td>14.46</td>
</tr>
<tr>
<td>3</td>
<td>DC-10-10F</td>
<td>135300</td>
<td>2014</td>
<td>45973</td>
<td>5639</td>
<td>1781</td>
<td>28.15</td>
</tr>
<tr>
<td>4</td>
<td>DC-10-30F</td>
<td>177400</td>
<td>3913</td>
<td>97835</td>
<td>6421</td>
<td>2545</td>
<td>32.11</td>
</tr>
<tr>
<td>5</td>
<td>MD-11F</td>
<td>190753</td>
<td>3758</td>
<td>81000</td>
<td>7710</td>
<td>2660</td>
<td>24.83</td>
</tr>
<tr>
<td>6</td>
<td>B-747-400F</td>
<td>248300</td>
<td>5118</td>
<td>152090</td>
<td>7715</td>
<td>3658</td>
<td>36.34</td>
</tr>
</tbody>
</table>

Notes: These six aircraft types are widely adopted by cargo airlines. The meanings of $P_a$, $P_b$, $R_a$ and $R_b$ of all aircraft types are shown in Figure 2. Source: Boeing Commercial Airplane Group. $T_i$ is the airport user charge for all aircraft types at Taiwan Taoyuan International Airport. $f_i$ in the table is estimated from aircraft operating costs according to the work report of ICAO ALLPIRG/Advisory Group in 2000.

Table 2. Traffic flow between all OD pairs.

<table>
<thead>
<tr>
<th>From/to</th>
<th>Taipei</th>
<th>Hong Kong</th>
<th>Seoul</th>
<th>Shanghai</th>
<th>Tokyo</th>
<th>Jakarta</th>
<th>Kuala Lumpur</th>
<th>Manila</th>
<th>Singapore</th>
<th>Bangkok</th>
</tr>
</thead>
<tbody>
<tr>
<td>Taipei</td>
<td>212,232</td>
<td>216,654</td>
<td>107,388</td>
<td>696,840</td>
<td>74,455</td>
<td>70,488</td>
<td>77,879</td>
<td>58,041</td>
<td>155,902</td>
<td></td>
</tr>
<tr>
<td>Anchorage</td>
<td>977,864</td>
<td>193,076</td>
<td>873,860</td>
<td>1,567,816</td>
<td>399,542</td>
<td>686,795</td>
<td>281,877</td>
<td>426,147</td>
<td>733,593</td>
<td></td>
</tr>
<tr>
<td>Amsterdam</td>
<td>1,228,736</td>
<td>866,567</td>
<td>1,602,504</td>
<td>1,839,540</td>
<td>318,632</td>
<td>510,327</td>
<td>177,644</td>
<td>362,662</td>
<td>955,952</td>
<td></td>
</tr>
<tr>
<td>Sydney</td>
<td>54,708</td>
<td>6707</td>
<td>67,858</td>
<td>140,468</td>
<td>71,314</td>
<td>60,441</td>
<td>173,830</td>
<td>22,207</td>
<td>123,959</td>
<td></td>
</tr>
</tbody>
</table>

Notes: The data of cargo flow between each OD pairs of major Asia Pacific cities in 2009 are made with reference to the customs statistics in 2009 on exports of major Asia Pacific cities to the USA, Europe and Australia. The above data are compiled with the assumption that the airline is to have 30% market share of each route.

Table 3. Mileage of all routes.

<table>
<thead>
<tr>
<th>From/to</th>
<th>Taipei</th>
<th>Hong Kong</th>
<th>Seoul</th>
<th>Shanghai</th>
<th>Tokyo</th>
<th>Jakarta</th>
<th>Kuala Lumpur</th>
<th>Manila</th>
<th>Singapore</th>
<th>Bangkok</th>
</tr>
</thead>
<tbody>
<tr>
<td>Taipei</td>
<td>501</td>
<td>904</td>
<td>419</td>
<td>1355</td>
<td>2367</td>
<td>2017</td>
<td>730</td>
<td>2000</td>
<td>1544</td>
<td></td>
</tr>
<tr>
<td>Anchorage</td>
<td>4672</td>
<td>5073</td>
<td>3794</td>
<td>4300</td>
<td>3426</td>
<td>7031</td>
<td>6651</td>
<td>5307</td>
<td>6661</td>
<td>5998</td>
</tr>
<tr>
<td>Amsterdam</td>
<td>5865</td>
<td>5760</td>
<td>5313</td>
<td>5555</td>
<td>5788</td>
<td>7051</td>
<td>6358</td>
<td>6468</td>
<td>6529</td>
<td>5701</td>
</tr>
<tr>
<td>Sydney</td>
<td>4523</td>
<td>4591</td>
<td>5178</td>
<td>4883</td>
<td>4863</td>
<td>3427</td>
<td>4089</td>
<td>3888</td>
<td>3909</td>
<td>4683</td>
</tr>
</tbody>
</table>

$$g_1 = 0.02 \text{ US$}/\text{lb}$$
$$g_2 = 0.1 \text{ US$}/\text{lb}$$
$$e^h = 0.024 \text{ week (4 h)}$$
$$\tau = 0.0033452 \text{ week (0.562 h)}$$
$$\tau' = 0.000010629 \text{ week (speed = 560 miles per hour)}$$

The aircraft type used on each route depends on the cargo flow and the distance of that route. The optimal aircraft type helps to achieve the lowest logistics costs. By following the calculation steps of aircraft presented in Section 2, the optimal aircraft types for the routes with different distances and cargo flows are determined and their relationships are shown in Figure 6. In practice, by inserting the distance and the cargo flow of any route, the optimal aircraft type can be obtained from Figure 6. In essence, the greater the cargo flow, the larger the aircraft should be adopted. This is known as the economies of plane size. Furthermore, every aircraft is designed for a specific range; therefore, the distance flown should not exceed its $R_b$ (Figure 2). When the flying distance exceeds $R_b$, the payload will be reduced and substantial costs will be incurred. When the route distance reaches a certain level, the type of aircraft used should be replaced by aircraft of longer range. Airlines can use Figure 6 as a tool to find the optimal aircraft types for their planned routes for various flows and distances, and it can also be used as a reference in fleet planning.
Let us find out whether or not the cargo flow from major airports in the East Asia region to Anchorage should be transferred via Taipei. The critical line for the route selection of every OD pair is determined by plotting \((q_{mn}, Q_{mh} - q_{mn})\) as shown in Figure 7. Because the corresponding geographic positions for every origin/destination airports to Taipei are different, the route selection will be different even though the cargo flow \((q_{mn}, Q_{mh} - q_{mn})\) is the same. In other words, even with the same \(q_{mn}\), the different geographical positions of hub and spoke airports will correspond to different \(Q_{mh} - q_{mn}\). The critical line for the route selection of every OD pair reflects the corresponding position between its origin/destination points and the hub.

When \(Q_{mh} - q_{mn}\) is larger than \(Q_{mh} - q_{mn}\), the cargo flow between OD pair m–n will be transferred via the hub. The area on the left-hand side of the critical line means that a transfer route should be selected, whereas the area on the right-hand side of the critical line indicates that a direct route is preferred. Figure 7 shows the clear distribution and variation of the critical lines for the OD pairs of Tokyo, Seoul, Shanghai, Hong Kong and Manila to Anchorage. The critical lines of other OD pairs are located on the x-coordinate because of the long distance of the flights serving those pairs that affect the aircraft’s payload and increase the total logistics costs of direct routes. For such OD pairs, transfer routes are selected. Airlines employing long-range aircraft,
such as A380s or B787s, are capable of lowering the total logistics costs of the direct routes, thus resulting in a shift of the critical lines. Figure 7 clearly illustrates the route choice of every OD pair. In addition, if the flow on a specific OD pair changes with the unchanged flow of other OD pairs, the impact of the flow changes on the route selection can be analysed using the critical line shown in Figure 7.

The results of the cargo flow assignment are given in Table 4. Blanks in the table reflect those OD pairs that chose transfer routes, and their flow is merged into the routes from their origin to the hub and from the hub to their destination. Take the Hong Kong–Anchorage pair for example. The cargo flow of 193,076 lb per week for the pair is strategically better transferred via Taipei to achieve the least cost. Because the flow on the Hong Kong–Taipei route consists of cargo transported from Hong Kong to Taipei, Hong Kong to Anchorage and Hong Kong to Sydney, the cargo flow is therefore increased to 412,015 lb per week and a B-757-200F type aircraft can be used on that route. There will be six flights per week with a load factor of 0.95.

As mentioned in Section 2, an assumption when developing the model is made. That is, the cargo flow on route h–n is large enough to make the marginal cost a constant. The cargo flow assigned on routes Taipei–Anchorage, Taipei–Amsterdam and Taipei–Sydney all met the assumption of shipment volume equal to the payload of aircraft and achieved the economy of scale; hence, the marginal cost will be a constant.

Airlines may use these results of cargo flow assignment in their network design and then adjust it in response to any flow fluctuations. Generally speaking, the sequence of adjustment is as follows. First, the flight frequencies are modified, followed by a change in the type of aircraft. When there is a very high variation in the flow, the optimal routes will be adjusted last. According to the analysis in Section 3, the upper and lower boundaries of the flows on all routes for maintaining planned flight frequencies and aircraft types are calculated and are given in Tables 5 and 6, respectively.

It should be noted that in reality, the cargo flow on every route is usually not fixed, and the upset will cause all kinds of fluctuations. Assuming that the flows are distributed in a normal manner on every route, the value in Table 4 is further used as the mean flow with 10% of the mean as its standard deviation. This study then employs the data in Tables 5 and 6 to determine the probability of maintaining the original planned flight frequencies and aircraft type under flow fluctuations. The results are given in Tables 7 and 8, respectively.

Take the Hong Kong–Taipei route for instance. The original network design remains optimal for the cargo flow between [361,049 and 433,261]. However, if the variation of the flow exceeds that range, the optimal network design will have to be adjusted. Under the hypothesis of the above fluctuation, the probability that the original flight frequencies remain optimal on that route is 0.38. Moreover, the reliability of the flight frequencies on the Taipei–Anchorage route is only 0.04, indicating that for routes with larger flows, the reliability of the flight frequency tends to be low. This is because routes with larger flows are often accompanied with larger fluctuations for the same standard deviation, thus increasing the possibility of a necessary adjustment in the flight frequency.

In addition, the hub-and-spoke system has the characteristic of a consolidated flow. Intuitively, when the flow of all O–D pairs show coincident fluctuation trends, then the fluctuations of the
Table 5. Critical flow for maintaining the original flight frequencies.

<table>
<thead>
<tr>
<th>From/to</th>
<th>Taipei</th>
<th>Hong Kong</th>
<th>Seoul</th>
<th>Shanghai</th>
<th>Tokyo</th>
<th>Jakarta</th>
<th>Kuala Lumpur</th>
<th>Manila</th>
<th>Singapore</th>
<th>Bangkok</th>
</tr>
</thead>
<tbody>
<tr>
<td>Taipei</td>
<td>Up</td>
<td>433,261</td>
<td>288,842</td>
<td>288,842</td>
<td>953,766</td>
<td>953,766</td>
<td>763,013</td>
<td>577,681</td>
<td>572,260</td>
<td>1,144,519</td>
</tr>
<tr>
<td></td>
<td>Low</td>
<td>361,049</td>
<td>216,630</td>
<td>181,716</td>
<td>763,012</td>
<td>763,012</td>
<td>572,259</td>
<td>505,469</td>
<td>433,260</td>
<td>953,765</td>
</tr>
<tr>
<td>Anchorage</td>
<td>Up</td>
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<td>3,638,136</td>
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Table 6. Critical flow for maintaining the original aircraft types.

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<th>Seoul</th>
<th>Shanghai</th>
<th>Tokyo</th>
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<td>433,261</td>
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<tr>
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<td>–</td>
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<td>124,538</td>
<td>87,677</td>
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Table 7. Reliability of flight frequencies of the original schedule.

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<th>Kuala Lumpur</th>
<th>Manila</th>
<th>Singapore</th>
<th>Bangkok</th>
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</thead>
<tbody>
<tr>
<td>Taipei</td>
<td>0.38</td>
<td>0.50</td>
<td>0.85</td>
<td>0.48</td>
<td>0.42</td>
<td>0.44</td>
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<td>0.58</td>
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<tr>
<td>Amsterdam</td>
<td>0.28</td>
<td>0.38</td>
<td>0.49</td>
<td>0.42</td>
<td>0.44</td>
<td>0.73</td>
<td>0.51</td>
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</table>

Table 8. Reliability of aircraft types of the original schedule.

<table>
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<th>Hong Kong</th>
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<th>Shanghai</th>
<th>Tokyo</th>
<th>Jakarta</th>
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<th>Manila</th>
<th>Singapore</th>
<th>Bangkok</th>
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</thead>
<tbody>
<tr>
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<td>0.27</td>
<td>0.64</td>
<td>0.96</td>
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<td>0.20</td>
<td>0.73</td>
<td>0.99</td>
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</tr>
<tr>
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<td>0.97</td>
<td>1.00</td>
<td></td>
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<td></td>
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</tr>
<tr>
<td>Amsterdam</td>
<td>0.74</td>
<td>0.79</td>
<td>0.99</td>
<td>1.00</td>
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<td>0.76</td>
<td>0.72</td>
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</tr>
<tr>
<td>Sydney</td>
<td>0.67</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.67</td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

consolidated flow would be amplified. On the contrary, when the O–D pairs show different fluctuation trends, with some increasing and some decreasing, the fluctuations will be cancelled out. Even when the fluctuation of each OD pair is independent of each other, variances of the consolidated flow would be the sum of variances in flow on all OD pairs, which would definitely exceed the variance of individual OD pair. Therefore, the extent of flow fluctuations becomes amplified. In this case, the standard deviation of the flow on the Hong Kong–Taipei route is changed to 1.73 times its original value, and the reliability of the flight frequency on this route is reduced from 0.59 to 0.38. Moreover, in reality, the flow fluctuation on every route of a hub-and-spoke network may not be independent. Under the current trend of globalisation, the market for air cargo transport is for certain affected by economic conditions around the world. Fluctuations are usually caused by economic impact and tend to be coincident; hence, the extent of fluctuations for consolidated flows on the network is certain to be large.

5. Conclusions

This study analysed the logistics costs and network design of an airline. The findings can serve as useful references for airlines when planning a hub-and-spoke system. The reliability analysis of our study showed how to analyse the impact of a flow fluctuation on the reliability of a flight network in advance and how to assess what is required to make the corresponding adjustments.

The total logistics costs in this study took into account the time value of cargo, aircraft type, cargo flow and the relative geographic locations of the origin and destination airports. When using the figures from our study results, no complicated calculations are needed to determine which aircraft type and which route should be selected to serve the various cargo flows of OD pairs with different geographic locations and distances. Furthermore, when the cargo flow of a specific OD pair changes, there is no need to recalculate and compare the costs of two different routes. Merely examining whether the resulting flow is within the range of the cargo flows would indicate the reliability of the original network design. As a result, this study provides airlines with efficient tools to determine if the impact of a flow fluctuation is large enough to justify an adjustment of the network design, including changes in flight frequencies, aircraft types and routes.

The results of the case studies show that the variations in flows have different influences on the reliability of flight frequencies, aircraft types and routes. In addition, because of the consolidation
effect of the hub-and-spoke system on route flows, the impact of fluctuations of cargo flows can be quite severe. In other words, the probability of having to adjust the optimal flight frequencies and aircraft types is higher. Consequently, airlines adopting the hub-and-spoke system should plan their flights and their fleet with greater flexibility so as to enable them to respond to all sorts of sudden events.

The cost of adjusting a network design when an airline faces an upset in their operation is largely affected by the size of the fleet and the crew. This issue is not investigated in-depth in this paper. Nevertheless, even if the model proposed by this research is used, in practice, the optimal aircraft type for every route may not always be used owing to constraints on the operating fleet. When the optimal aircraft type is not available under the condition of a flight network adjustment, the second most suitable aircraft type is suggested to be adopted for lowering costs. In that case, the benefit of adjustment will be decreased due to fleet limitation.

References