An investigation of flow reversal of mixed convection in a three dimensional rectangular channel with a finite length

Wu-Shung Fu *, Yu-Chih Lai, Yun Huang, Kuan-Lan Liu

Department of Mechanical Engineering, National Chiao Tung University, Hsinchu 30010, Taiwan, ROC

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An investigation of flow reversal of mixed flow in a three dimensional channel is studied numerically. At a high Richardson number, natural convection dominates the flow and thermal fields of mixed convection. Behaviors of the residual mass flow rate produced by the difference of the strength of natural and forced convections coexisting in mixed convection are worthy to be examined deeply for industrial applications. Due to the necessity of considering the fluid compressibility, methods of the Roe scheme, preconditioning and dual time stepping are adopted to solve governing equations. The results show that at a high Richardson number situation, via the outlet in large quantities of fluids are sucked into the channel from the outside that causes the flow and thermal fields in the channel to be unsteady and vice versa.

1. Introduction

Strictly speaking, except in situations of low temperature differences between cooling fluids and heat sources, heat transfer phenomena of forced convection only in realistic situations are hardly observed; instead, the coexistence of forced and natural convections, also called mixed convection, usually appears. Heat transfer phenomena of mixed convection are strongly influenced by the ratio of natural convection to forced convection. And mixed convection is mainly divided into three parts of cross, opposite and aiding flows. Due to the same direction of the buoyancy force and a fluid flow in aiding flow mixed convection, the number of analyses of aiding flow mixed convection are relatively more than those of the other two mixing flows. From a viewpoint of enhancement of heat transfer, aiding flow mixed convection is usually regarded to be advantageous, so it attracts more investigations than those of the other two types.

Metais et al. [1] conducted an experimental work to draw a diagram indicating a flow relationship between Reynolds and Grashof numbers. In the diagram, regions of natural, forced and mixed convections divided into a laminar and a turbulent flows were clearly delimited. Behzadmehr et al. [2] investigated aiding flow mixed convection with the Boussinesq assumption under conditions of uniform heat flux and low Reynolds numbers in a vertical circular duct. Relationships of Grashof and Nusselt numbers were yielded to distinguish the laminar and turbulent regions. The results showed that the regions of Re = 1000, 8 × 105 < Gr < 5 × 107 and Re = 1500, 2 × 106 < Gr < 1010 were included in the turbulent region. Tanaka et al. [3] conducted an experimental work like [1] to draw a diagram of Reynolds via Grashof numbers, the domain of Reynolds number was between 1000 and 5000. Regions of natural, forced and mixed convections were divided into the laminar and turbulent flows. Celata et al. [4] showed the experimental results of the distribution of the buoyancy parameter of Bo in a figure of Reynolds number via Grashof number with water. In the range of Bo ≪ 1, a laminar phenomenon was observed, and the corresponding Nusselt number was slightly smaller than that of pure forced convection. Boulama and Galanis [5] studied aiding flow mixed convection in a two-dimensional vertical parallel plates with a condition of fully developed flow at the outlet, and the analytical solutions were dependent on the parameters which combined the effects of thermal and solutal buoyancy. The results revealed that buoyancy effects significantly improved heat and momentum transfer rates near the heated walls. Desrayaud and Lauriat [6] investigated aiding flow mixed convection of a two-dimensional vertical duct with a high wall temperature. The results showed that when the magnitude of Gr/Re was larger than 1, phenomena of flow reversal were observed. Under a condition of a constant Grashof number, the larger the Reynolds number was, the more difficult the flow reversal was found. Zghal et al. [7] investigated aiding flow mixed convection in a two-dimensional vertical duct with the Boussinesq assumption. Effects of parameters of length, Reynolds number and Richardson number on Nusselt number were examined. Appearance of flow reversal was mainly determined between a relationship of Peclet and Richardson numbers. Ingham et al. [8] investigated phenomena of flow reversal of mixed...
convection in a two-dimensional constant temperature vertical duct with the Boussinesq assumption. In a range of $-300 < \text{Gr} / Re < 70$, the flow reversal easily appeared at the larger magnitude of $|\text{Gr} / Re|$. Barletta [9,10] adopted an analytical method and the Boussinesq assumption to investigate mixed convection in a rectangular cross-section duct with a fully developed flow condition in the z axis. Thermal conditions of walls were composed of different combinations of high and low temperatures and constant heat flux. Phenomena of flow reversal were examined under different shapes of rectangular cross section. Barletta [11] investigated viscous dissipation effect of mixed convection in a two-dimensional vertical duct. The phenomenon of flow reversal in opposing flow mixed convection was more apparent than that in aiding flow mixed convection under the Boussinesq assumption. Yang et al. [12] adopted the Boussinesq assumption to study mixed convection in a long two-dimensional vertical duct. Heat transfer mechanisms were investigated under positive and negative magnitudes of Richardson numbers. Nguyen et al. [13] investigated transient mixed convection in a high heat flux circular duct with the Boussinesq assumption numerically. Boundary conditions at the inlet and outlet were a uniform velocity and a fully developed flow, respectively. In an opposite situation, the flow reversal appeared near the outlet region at $Gr = 3 \times 10^5$, and in an aiding situation the flow reversal appeared near the center of the axis at $Gr = 10^6$.

In retrospect, the Boussinesq assumption which is only useful for the temperature differences smaller than 30 K [14] is still conveniently used by most of the above literature. According to the limitation of the Boussinesq assumption, the analysis of mixed convection is then necessary to add an extra domain to the original domain that causes a fully developed condition to be adopted at the edge of the domain newly added. Regrettfully, by using the Boussinesq assumption, some interesting and important characteristics of mixed convection have the possibility to be omitted especially in the range of the high magnitude of Richardson number. Doubtless, at a high Richardson number situation natural convection dominates flow and heat transfer mechanisms. Then the amount of fluid which is induced by natural convection and flows through the domain is substantially larger than the amount of fluid which is provided by forced convection and flows through the domain. Behaviors of the residue of the fluid caused by the difference between the amount of fluid provided by forced convection and the amount of fluid induced by natural convection are not deeply discussed yet. As well, the inlet is usually filled with the amount of fluid provided by forced convection. As a result, a problem of consideration of behaviors of the residue of the fluid mentioned above becomes important and is worthy of deep investigation.

The aim of the study investigates flow reversal and heat transfer mechanisms of mixed convection in a three-dimensional vertical channel with a finite length under larger Richardson numbers numerically. A non-reflecting boundary condition is assigned at the outlet of the domain to adjust fluids sucked into the channel or discharged to the outside of the channel according to the magnitude of the Richardson number. The compressibility of fluid has been taken into consideration for matching the usage of the non-reflecting boundary condition which also means that the Boussinesq assumption is no longer needed. Necessary methods of the Roe scheme [16], preconditioning and dual time stepping [17] for solving a low speed compressible flow are used. The results show that in high Richardson number situations natural convection is dominant and causes part of the amount of fluid from the outside or discharged to the outside of the channel according to the magnitude of the fluid to be sucked into the channel. In a certain situation, the amount of fluid sucked into the channel is approximately equal to that flowing into the channel via the inlet provided by forced convection. That leads to drastic impingement between both the

### Nomenclature

- $a$: sound speed (m s$^{-1}$)
- $A$: area ($m^2$)
- $C_p$: constant-pressure specific heat (J kg$^{-1}$ K$^{-1}$)
- $C_v$: constant-volume specific heat (J kg$^{-1}$ K$^{-1}$)
- $d$: width of the square channel (m)
- $e$: internal energy (J kg$^{-1}$)
- $g$: acceleration of gravity (m s$^{-2}$)
- $Gr$: Grashof number defined in Eq. (12)
- $h$: enthalpy (J)
- $k$: thermal diffusivity (W m$^{-1}$ K$^{-1}$)
- $k_0$: surrounding thermal diffusivity (W m$^{-1}$ K$^{-1}$)
- $l$: length of the square channel (m)
- $M_{\text{inlet,n.c}}$: dimensionless mass flow rate of natural convection defined in Eq. (36)
- $M_{\text{inlet}}$: dimensionless mass flow rate at inlet defined in Eq. (33) (kg s$^{-1}$)
- $M_{\text{outlet}}$: dimensionless mass flow rate from outside at the outlet defined in Eq. (34) (kg s$^{-1}$)
- $Nu$: area averaged Nusselt number defined in Eq. (38)
- $Nu_l$: local Nusselt number defined in Eq. (35)
- $Nu_{th}$: time averaged local Nusselt number defined in Eq. (37)
- $P$: pressure (Pa)
- $P_0$: surrounding pressure (Pa)
- $Pr$: Prandtl number
- $R$: gas constant (J kg$^{-1}$ K$^{-1}$)
- $Ra$: Rayleigh number defined in Eq. (12)
- $Re$: Reynolds number defined in Eq. (10)
- $Ri$: Richardson number defined in Eq. (11)
- $t$: time (s)
- $t'$: dimensionless time defined in Eq. (9)
- $T$: temperature (K)
- $T^*$: dimensionless temperature defined in Eq. (9)
- $T_s$: temperature of heat surface (K)
- $\Delta T$: time difference (K)
- $u, v, w$: velocities in x, y and z directions (m/s)
- $U, V, W$: dimensionless velocities in x, y and z directions defined in Eq. (9)
- $x, y, z$: Cartesian coordinates (m)
- $X, Y, Z$: dimensionless Cartesian coordinates in Eq. (9)

### Greek symbols

- $\alpha$: Thermal diffusivity (m$^2$/s)
- $\beta$: volumetric thermal expansion coefficient (K$^{-1}$)
- $\rho$: density (kg m$^{-3}$)
- $\rho_0$: surrounding density (kg m$^{-3}$)
- $\nu$: kinematics viscosity (m$^2$/s)$^{-1}$
- $\mu$: absolute viscosity (N s/m$^2$)
- $\mu_0$: viscosity of surrounding (N s/m$^2$)
- $\gamma$: specific heat ratio
- $\Gamma$: preconditioning matrix [17]
- $\delta_x$, $\delta_y$, $\delta_z$: central-difference operators defined in Eq. (17)
amounts of fluid mentioned above to occur and the flow field to be unsteady. Oppositely, in low Richardson number situations the amount of fluid provided by forced convection is larger than the amount of fluid sucked into the channel induced by natural convection. As a result, the impingement mentioned above becomes peaceful and the flow in the channel displays a steady situation.

2. Physical model

A physical model investigated in this study is a three-dimensional vertical rectangular channel and shown in Fig. 1. The cross section of the channel is square and the width is \( d \). The length of the channel is finite and equal to \( l \). The direction of gravity \( g \) is downward and parallel to the vertical channel. The temperature of four heat wall surfaces is constant and equal to \( T_h \) which is higher than a temperature \( T_0 \) of surroundings. Boundary conditions of the temperature and velocity at the outlet of the channel EFGH are non-reflecting which was developed by Fu et al. [15].

In the situation of natural convection, via the inlet the amount of fluid flowing into the channel has difficulty to be predicted in advance. Then the non-reflecting boundary conditions of the velocity, temperature and pressure are held at the inlet to determine the amount of fluid induced by natural convection and flowing into the channel and shown in Fig. 1(a).

The same channel is also used in mixed convection, the amount of fluid via the inlet flowing into the channel is provided by forced convection and evenly distributed on the inlet and shown in Fig. 1(b). The velocity and temperature of the flowing fluid are equal to \( u_0 \) and \( T_0 \), respectively. In this situation of mixed convection, the amount of fluid induced by natural convection via the inlet flows into the channel is no longer permitted because of the complete occupancy of the amount of fluid provided by forced convection at the inlet. Therefore, in a large Richardson number situation the amount of fluid induced by natural convection is permitted only from the central region of the outlet to flow downwards into the channel and compulsionly impinges the amount of fluid which is provided by forced convection and flowing upwards into the channel from the inlet. Both the amounts of fluid then mix together and newly compose upward streams along the heat walls via the outlet to flow out of the channel. Phenomena of flow reversal are then to be observed in the channel.

For facilitating the analysis, several assumptions are made and indicated as follows.

1. A laminar flow.
2. Properties of fluids follow the equation of state of an ideal gas.
3. The pressure gradient in normal direction of surfaces is equal to zero.

The governing equation is expressed as follows

\[
\frac{\partial u}{\partial t} + \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z} = S
\]

The quantities included in \( U, F_i \) and \( S \) are separately shown in the following equations, respectively

\[
U = \begin{pmatrix} \rho \\ \rho u \\ \rho v \\ \rho w \\ \rho e \end{pmatrix}
\]

\[
F_i = \begin{pmatrix} \rho u_i \\ \rho u_i u_1 + P \delta_{i1} - \mu A_i \\ \rho u_i u_2 + P \delta_{i2} - \mu A_i \\ \rho u_i u_3 + P \delta_{i3} - \mu A_i \\ (\rho e + P) u_i - \mu A_i u_i - k \frac{\partial T}{\partial x} \end{pmatrix}, \quad \forall i = 1(x), 2(y), 3(z)
\]

\[
S = \begin{pmatrix} 0 \\ -(\rho - \rho_0)g \end{pmatrix}
\]

where \( \mu = \frac{\mu_0}{1 + \frac{T}{T_0}} \) and the ideal gas equation is written by

\[
P = \rho RT
\]

On the surfaces of ABFE, BCGF, CDHG and DAEH:

\[
T = T_h \quad \text{for both convections}
\]

On the surface of ABCD:

\[
u = u_0, \quad T = T_0 \quad \text{forced convection}
\]

On the surfaces of ABCD (for natural convection) and EFGH (for both convections):

The non-reflecting conditions are held.

The Sutherland’s law is adopted to evaluate the viscosity and the thermal conductivity as follows, respectively

\[
\mu(T) = \mu_0 \left( \frac{T}{T_0} \right)^{\frac{1}{2}} \frac{T_0 + 110}{T + 110}
\]

\[
k(T) = \frac{\mu(T) \gamma R}{(\gamma - 1)Pr}
\]

where \( \rho_0 = 1.1842 \text{ kg/m}^3 \), \( g = 9.81 \text{ m/s}^2 \), \( \mu_0 = 1.85 \times 10^{-5} \text{ N s/m}^2 \), \( T_0 = 298.0592 \text{ K} \), \( \gamma = 1.4 \), \( R = 287 \text{ J/kg/K} \) and \( Pr = 0.7 \).
To simplify the analysis, the following dimensionless variables are made
\[
X = \frac{x}{d}, \quad Y = \frac{y}{d}, \quad Z = \frac{z}{d}
\]
\[
U = \frac{u}{u_0, Re = 950}, \quad V = \frac{v}{u_0, Re = 950}, \quad W = \frac{w}{u_0, Re = 950}
\]
\[l = 3d, \quad t' = \frac{\mu_0}{\rho_0 a^2}, \quad T' = \frac{T}{T_h}
\]
(9)
where \(u_0, Re = 950\) means a uniform velocity \(u_0\) to be assigned at the inlet under the situation of \(Re = 950\).

To investigate the heat transfer in the cases of different Richardson numbers, the compressibility and viscosity of the working fluid are considered. Definitions of Reynolds number, \(Re\), Richardson number, \(Ri\), and Rayleigh number, \(Ra\), are represented as follows, respectively
\[
Re = \frac{\rho_0 u_0 d}{\mu_0}
\]
(10)
\[
Ri = \frac{Gr}{Re^2} = \frac{g \rho_0^2 \beta (T_h - T_0) d^4}{\mu_0^2} \cdot \frac{\mu_0^2}{\rho_0^2 u_0 d^2} = \frac{g \beta (T_h - T_0)}{u_0^2}
\]
(11)
\[
Ra = Pr \cdot Gr = (0.7) \cdot \frac{g \rho_0^2 \beta (T_h - T_0) d^3}{\mu_0^2}
\]
(12)

3. Numerical method

Methods of the Roe scheme [16] and preconditioning [17] are adopted to resolve the governing equations of the compressible flow shown in Eq. (1). Besides, the dual time stepping method is added to calculate transient states. And Eq. (13) can be obtained
\[
\frac{\partial U_p}{\partial t} + \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z} = S
\]
(13)
where \(t\) is an artificial time, \(\Delta t\) is a physical time, \(\Gamma\) is a preconditioning matrix proposed by Weiss and Smith [17] and \(U_p\) is a primitive form of \((p, p_r, \mu_r, \rho_w, r_e)\). By the discretization of Eqs. (13) and (14) can be obtained. Terms of the \(\frac{\partial U_p}{\partial t}, \frac{\partial F_1}{\partial x}, \frac{\partial F_2}{\partial y}\) and \(\frac{\partial F_3}{\partial z}\) are treated by a first-order forward and a second-order backward differences, respectively. Terms of the \(\frac{\partial U_p}{\partial x}, \frac{\partial F_1}{\partial y}\) and \(\frac{\partial F_3}{\partial z}\) are treated by a central difference
\[
\frac{U_{p_{i+1}} - U_{p_{i-1}}}{2\Delta x} + \frac{3U_{p_{i+1}} - 4U_{p_{i}} + U_{p_{i-1}}}{2\Delta x} + \frac{1}{\Delta x} \left( f_{i+0.5}^k - f_{i-0.5}^k \right)
\]
\[+ \frac{1}{\Delta y} \left( f_{i+0.5}^k - f_{i-0.5}^k \right) + \frac{1}{\Delta z} \left( f_{i+0.5}^k - f_{i-0.5}^k \right) = S
\]
(14)
Afterward terms of the \(U_{p_{i+1}}\) and \(f_{i+1}\) in Eq. (14) are necessary to be linearized and expressed as follows, respectively
\[
U_{p_{i+1}} = U_{p_{i}} + M \Delta U_p
\]
(15)
\[
F_{i+1}^k = F_i^k + A_p \Delta U_p
\]
(16)
where the \(A_p = \frac{\partial f^k}{\partial U_p}\) is the flux Jacobian and the same method is used for the \(B_p = \frac{\partial f^k}{\partial p_r}\) and \(C_p = \frac{\partial f^k}{\partial \rho_w}\) in linearization of the \(f_{i+1}^k\) and \(f_{i+1}^k\), respectively.

To substitute Eqs. (15) and (16) into Eq. (14), the following equation is obtained
\[
\frac{U_{p_{i+1}} - U_{p_{i}}}{\Delta t} + \frac{3U_{p_{i+1}} - 4U_{p_{i}} + U_{p_{i-1}}}{2\Delta t} + \frac{1}{\Delta t} \left( f_{i+\frac{1}{2}}^k + A_p \Delta U_p \right)
\]
\[+ \frac{1}{\Delta x} \left( f_{i+\frac{1}{2}}^k - f_{i-\frac{1}{2}}^k \right) + \frac{1}{\Delta y} \left( f_{i+\frac{1}{2}}^k - f_{i-\frac{1}{2}}^k \right) + \frac{1}{\Delta z} \left( f_{i+\frac{1}{2}}^k - f_{i-\frac{1}{2}}^k \right) = S
\]
(17)
where \(\delta_x, \delta_y, \) and \(\delta_z\) are central-difference operators.

Eq. (17) can be rearranged as the following form
\[
\left[ \frac{I}{\Delta t} + \frac{1}{\Gamma M \Delta t} \right] + \frac{3}{2\Delta t} + \frac{1}{\Gamma \Delta t} \left( \delta_x \left( A_p^k + \delta_y B_p^k + \delta_z C_p^k \right) \right) \Delta U_p = \Gamma^{-1} R^k
\]
(18)
where \(R^k = S - \left( \frac{U_{p_{i+1}} - U_{p_i}}{2\Delta t} \right) - \left( \delta_x f_i^k + \delta_y f_i^k + \delta_z f_i^k \right)\)

To solve a problem of the convergence of a low-speed compressible flow, the solver of

Eq. (19) is newly derived from the LUSGS implicit method originally proposed by Yoon and Jamesont [18]

\[
A_p = \Gamma^{-1} A_p^k
\]
(19)
\[
B_p = \Gamma^{-1} B_p^k
\]
(20)
\[
C_p = \Gamma^{-1} C_p^k
\]
(21)
where
\[
A_p^k = \frac{1}{2} (A_p \pm |\lambda|) \]
\[B_p^k = \frac{1}{2} (B_p \pm |\lambda|) \]
\[C_p^k = \frac{1}{2} (C_p \pm |\lambda|) \]
(22)

To substitute Eq. (20) into Eq. (18), the following equation is obtained
\[
\left[ \frac{I}{\Delta t} + \frac{1}{\Gamma M \Delta t} \right] + \delta_x \left( A_p^k + \delta_y B_p^k + \delta_z C_p^k \right) \Delta U_p = \Gamma^{-1} R^k
\]
(23)
\[
\delta_x \left( A_p^k + \delta_y B_p^k + \delta_z C_p^k \right) \Delta U_p = \Gamma^{-1} R^k
\]
(24)

Eq. (24) can be rearranged as follows
\[
(L + D + U) \Delta U_p = \Gamma^{-1} R^k
\]
(25)
where
\[ F_{\text{inviscid}} = \begin{pmatrix} \rho u_i \\ \rho u_i u + P \delta_{i1} \\ \rho u_i v + P \delta_{i2} \\ \rho u_i w + P \delta_{i3} \\ (\rho e + P) u_i \end{pmatrix} \]

The other is a viscous term \( F_{\text{viscous}} \)

\[ F_{\text{viscous}} = - \begin{pmatrix} 0 \\ \mu A_{i1} \\ \mu A_{i2} \\ \mu A_{i3} \\ \mu A_{i4} u_j + k \frac{\partial u_i}{\partial x_j} \end{pmatrix} \]

A Roe upwind difference scheme [16] is employed in discretization of the terms of the \( F_{\text{inviscid}} \) at the cells interface \( (i + \frac{1}{2}) \) and expressed as follows at a low Mach number situation

\[ F_{\text{inviscid},i+\frac{1}{2}} = \frac{1}{2} \left( F_R + F_I \right) - \frac{1}{2} \left( \Gamma^{-1} A_p \Delta U_p \right) \]

The MUSCL scheme with a third order proposed by Abalakin et al. [19] is used to compute the terms of the \( F_{\text{inviscid}} \). And the related derivative terms of \( A_p = \frac{\partial u_i}{\partial x_j} \) in Eq. (27) are computed by a fourth order central difference

\[ \frac{\partial u_i}{\partial x_j} = \frac{u_{i+2} - 8 u_{i-1} + 8 u_{i+1} - u_{i+2} + o(\Delta x^4)}{12 \Delta x} \]

The advantage of usage of the LUSGS implicit method is to improve efficiency.

On the heat surface, the boundary conditions are

\[ P(i, 0, k) = P(i, 1, k) \]
\[ u(i, 0, k) = -u(i, 1, k) \]
\[ v(i, 0, k) = -v(i, 1, k) \]
\[ w(i, 0, k) = -w(i, 1, k) \]
\[ T(i, 0, k) = 2T_h - T(i, 1, k) \]

Where \( T_h \) is the wall temperature.

0 indicates the ghost cell and 1 indicates the cell most near the wall.

As for the boundary conditions at the outlet, in order to avoid the flow in the channel polluted by the reflections of acoustic waves induced by the compressible flow, the non-reflecting boundary conditions are then necessarily used at the outlet of the channel.

In a high speed compressible flow condition, the method of LODI (local one-dimensional inviscid relations) proposed by Poinset and Lele [20] was suitably adopted for determining the non-reflecting boundary conditions at the outlet. However, a preconditioning matrix is not necessary in the above method that causes the method to be not adopted appropriately for determining the non-reflecting boundary conditions at the outlet under a low speed compressible flow. As a result, the method developed by Fu et al. [15] is necessary for resolving the non-reflection boundary conditions under a low speed compressible flow.
A procedure calculating the equations mentioned above is briefly described as follows.

1. Assign the inlet conditions of pressure, velocity and temperature.
2. Use the MUSCL method to calculate Eq. (18) to obtain the magnitude of $D_{UP}$.
3. Substitute the magnitude of $D_{UP}$ into Eq. (28) and use the Roe method to calculate the magnitudes of inviscid terms of the $F_{visc}$.
4. Calculate Eq. (29) to obtain the magnitudes of viscous terms and substitute in Eq. (27).
5. Solve $U_{p}^{k-1}$

\[ U_{p}^{k-1} = U_{p}^{k} + \Delta U_{p} \]  

(31)

6. Calculate Eq. (14) and examine the convergence of the iterative computation of $U_{p}^{k-1}$. Repeat (2) ~ (5) until until $U_{p}^{k+1} - U_{p}^{k} < \varepsilon, \varepsilon = 10^{-3}$.

4. Results and discussion

In this study, there are five situations of which the ratio of the height to the width is 3, tabulated in Table 1 to be performed. Rayleigh number is $6.78 \times 10^5$ in those five situations. The mass flow rate of the situation of $Re = 950$ assigned in mixed convection is the same as that obtained by the situation of natural convection. The definition of the dimensionless mass flow rate at the inlet $M_{inlet}$ and the dimensionless mass flow rate sucked from the outside $M_{outlet}$ are presented as follows, respectively

\[ \dot{m}_{Re=950} = \rho d^2 u_{0,Re=950} \]  

(32)

\[ \dot{M}_{inlet} = \rho u_{0} d^2 / \dot{m}_{Re=950} \]  

(33)

\[ M_{outlet} = \rho |u_{outlet}| d^2 / \dot{m}_{Re=950}, u_{outlet} < 0 \]  

(34)

The total mass flow rate which flows out of the channel and has advantage to heat transfer rates of heat walls is obtained by the addition of $M_{inlet}$ and $M_{outlet}$. In situations of mixed convection of $Re = 400, 200$ and $100$, the mass flow rates of $M_{outlet}$ are mainly caused by the amount of fluid to be sucked from the outside. The reason is suggested as that the strength of the driving force of natural convection is larger than those of forced convections assigned in mixed convections under situations of large magnitudes of the Richardson numbers. However, the viscous dissipation and impingement between the $M_{inlet}$ and $M_{outlet}$ occurring in situations of $Re = 400, 200$ and $100$ cannot avoid. The total mass flow rates of situations of $Re = 400, 200$ and $100$ have difficulty to be equal to that of the situation of natural convection. The dimensionless time $r^*$ are from $r^* = 0$ to the steady state for situations of $Re = 0$. 

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Fig. 4. Distributions of streamlines and thermal field for natural convection.
950, 400, and the beginning of fully unsteady state for situations of $Re = 100$ and 200, respectively.

Shown in Fig. 2, the results of local Nusselt numbers with different grid distributions along the $x$, $y$, and $z$ axes under natural convection are tested. According to the results of local Nusselt numbers, the grid distribution of $96 \times 40 \times 40$ is adopted. The definition of the local Nusselt number $N_u_x$ is expressed as follows:

$$N_u_x = \frac{d}{k(T_0 - T_0)} \kappa(T) \frac{\partial T}{\partial z}$$

(35)

Shown in Fig. 3, the variation of the mass flow rate with time for natural convection at the inlet is indicated. The mass flow rate sharply increases at an initial stage and reaches a plateau of development gradually. The negative magnitude at the initial stage indicates part of the amount of fluid originally staying in the channel to be expanded by the heat wall and extruded to the outside. The definition of the dimensionless mass flow rate $M_{inlet, n.c.}$ is presented as follows:

$$M_{inlet, n.c.} = \rho \frac{u_{inlet} d^2}{m_{Re=950}}$$

(36)

Shown in Fig. 4, the variations of the streamlines and thermal field with time for natural convection are indicated. At first, the high temperature of the heat surface causes the densities of the fluids in the channel to become small, and then the volume of the fluid is expanded that causes the fluids to flow out of the channel shown in Fig. 4(a1). Accordingly, the fluids near the inlet region gradually form flow reversal shown in Fig. 4(a2–3) in order to supplement the lack of the fluids which are discharged mentioned above. Accompanying with the increment of time, because of the influence of natural convection, the flow field becomes steady and all the fluids flow into the channel from the inlet shown in Fig. 4a(d). In addition, orderly thermal boundary layers are observed as the flow field reaches the steady state shown in Fig. 4(b).

Shown in Fig. 5, the distributions of the streamlines and thermal field for the Reynolds number of 950 are indicated, respectively. The darker the color is, the higher the temperature is shown. Because of the same mass flow rate at the inlet of both situations of natural convection and $Re = 950$, the streamlines and thermal filed for the Reynolds number of 950 are similar to the situation of natural convection shown in Fig. 4(a4) and Fig. 4(b), respectively.

Shown in Fig. 6, the distributions of streamlines and thermal field for the Reynolds number of 400 are indicated, respectively. In Fig. 6(a), a part of the fluids via the outlet are sucked into the channel from the outside, and a region of the flow reversal is observed. The Rayleigh number of this situation is the same with that of natural convection shown in Fig. 4, based upon the reason mentioned above and then the mass flow rate at the inlet of the situation of $Re = 400$ is smaller than that of natural convection. As a result, via the outlet the insufficiency of the mass flow rate is supplemented by the fluids from the outside. As well, in Fig. 6(b) the flow reversal depresses the thermal field near the outlet that somewhat increases the heat transfer rate of the region near the outlet.

In Fig. 7, the variations of streamlines with time for the Reynolds number of 200 are indicated. From Table 1, at the inlet the mass flow rate of natural convection is much larger than that of the situation of $Re = 200$ that causes the insufficient mass flow rate to be supplemented from the outside of the channel. Then some fluids via the central region of the outlet flows into the channel and impinges the amount of fluid provided by forced convection flowing upwards from the inlet. Afterward, both the amounts of fluid newly coalesce and form a new stream flowing upward along the heat wall. Due to the occurrence of impingement, an unsteady phenomenon is apparently observed in Fig. 7(d). Naturally, the phenomenon is advantageous to heat transfer mechanisms of the heat wall.
The variations of streamlines with time for the Reynolds number of Re = 100 are shown in Fig. 8. Since the mass flow rate at the inlet of this situation is also much less than that of the situation of natural convection. The location of the impingement caused by $M_{\text{inlet}}$ and $M_{\text{outlet}}$ is more close to the inlet than that shown in Fig. 7. This phenomenon leads more drastic impingement to occur that slightly decreases the mass flow rate of $M_{\text{outlet}}$. Then the mass flow rate of $M_{\text{outlet}}$ of Re = 100 is slightly smaller than that of Re = 200.

In Fig. 9, the variations of thermal field with time corresponding to streamlines shown in Fig. 8 are indicated, respectively. In an earlier stage, a mixing effect of both the amounts of fluid mentioned above just begins, and then orderly thermal boundary layers are observed on heat walls. Gradually, the mixing effect becomes complex and drastic, and then orderly thermal boundary layers are no longer observed on heat walls and high temperature fluids are irregularly distributed in the channel.

Time averaged local Nusselt numbers distributed on the central line of the heat surface of all situations are indicated in Fig. 10, respectively. The definition of the time averaged local Nusselt number $(N_u)_t$ is expressed as follows. The time interval $t$ is calculated from $t^* = 0$ to 0.15.

\[
(N_u)_t = \frac{1}{t} \int_0^t \left( \frac{d}{k_0(T_s - T_0)} \left( k(T) \frac{\partial T}{\partial z} \right) \right) dt \quad (37)
\]

In the front region ($x < 0.5$), under the situation of natural convection the amount of fluid begins to be sucked from the outside of the channel and flows into the channel. The buoyancy force driving the amount of fluid to flow upwards is gradually strengthened, and then the upward velocity is also accelerated little by little. Oppositely, under the situation of forced convection a certain quantity of mass flow rate is evenly provided at the inlet to flow into the channel. As a result, in this region time averaged local Nusselt
numbers of the situation of Re = 950 are slightly larger than those of the situation of natural convection. The other situations of forced convection, mass flow rates at the inlet are smaller than the mass flow rate of the situation of natural convection at the inlet. Naturally, time averaged local Nusselt numbers of the other three
situations of forced convection are smaller than that of the situation of natural convection. Beyond the front region, under the situation of natural convection the upward fluid driven by the buoyancy force which is gradually strengthened is mainly concentrated on the heat surface. Then time averaged local Nusselt numbers of natural convection are larger than those of situations of forced convection in which the mass flow rate is evenly distributed on the cross section. According to the statements mentioned above, in situations of lower Reynolds numbers (Re = 100 and 200) via the outlet the fluids are sucked into the channel and impinges the upward fluid provided by forced convection. This phenomenon causes the drastic impingement to occur in the channel. Consequently, time averaged local Nusselt numbers of the situations of lower Reynolds numbers (Re = 100 and 200) are larger than those of the situation of Re = 400. Near the outlet region, a part of time averaged local Nusselt numbers of the situation of Re = 100 are even larger than those of the situation of Re = 950. However, the mass flow rate of the situation of Re = 400 is medium and slightly smaller than that of the situation of natural convection. The amount of fluid sucked into the channel from the outside is small and has no ability to disturb the flow field provided by forced convection. The smallest distribution of time averaged local Nusselt numbers is then indicated.

In an earlier time region, natural convection begins to develop that causes the magnitudes of area averaged Nusselt numbers of natural convection to be smaller than those of situations of forced convection. After a certain developing time, according to the reasons mentioned above the magnitudes of the area averaged Nusselt numbers of the situation of Re = 950 situation are apparently larger than those of the other three situations of forced convection. According to the reason suggested above, the magnitudes of area averaged Nusselt numbers of the situation of Re = 400 are smaller than those of the other two situations of Re = 100 and 200. The similar phenomena of the flow reversal are found out in situations of Re = 100 and 200. The deviation of the magnitudes of situations of Re = 100 and 200 is small.

In Fig. 12, the variation of time and area averaged Nusselt numbers of all situations is indicated. The definition of the time and area averaged Nusselt number $\bar{Nu}$ is defined as follows

$$\bar{Nu} = \frac{1}{A} \int_A Nu \, dx \, dy = \frac{1}{A} \int_A \frac{d}{k(T_h - T_0)} \left[ k(T) \frac{dT}{dz} \right] dx \, dy \quad (38)$$

In an earlier time region, natural convection begins to develop that causes the magnitudes of area averaged Nusselt numbers of natural convection to be smaller than those of situations of forced convection. After a certain developing time, according to the reasons mentioned above the magnitudes of the area averaged Nusselt numbers of natural convection are gradually larger than those of situations of forced convection. Among situations of forced convection, the magnitudes of the area averaged Nusselt numbers of the situation of Re = 950 situation are apparently larger than those of the other three situations of forced convection. According to the reason suggested above, the magnitudes of area averaged Nusselt numbers of the situation of Re = 400 are smaller than those of the other two situations of Re = 100 and 200. The similar phenomena of the flow reversal are found out in situations of Re = 100 and 200. The deviation of the magnitudes of situations of Re = 100 and 200 is small.

In Fig. 14, the comparison of the average Nusselt number of the present result with that of the previous work [3].

According to the reasons mentioned above, the maximum and minimum magnitudes of the time and area averaged Nusselt numbers of natural convection and Re = 400, respectively. The difference of the magnitudes of situations of Re = 100 and 200 is slight. The magnitude of the time and area averaged Nusselt number of natural convection is slightly larger than that of the situation of Re = 950. The main reason is suggested as that the amount of

Fig. 12. Variations of time and area averaged Nusselt numbers with different Reynolds numbers.

Fig. 13. Comparisons of distributions of velocities and temperature of present results with those of the previous work [13].
fluid provided by forced convection is evenly distributed in the cross section of the channel, and the amount of fluid in the channel induced by natural convection is mainly concentrated on heat walls.

In Fig. 13, comparisons of distributions of temperature and velocity of present results with those of the previous work [13] are indicated, respectively. The physical model of the previous work was a circular cylinder, then an equivalent hydraulic diameter regarded as the width of a square duct is used to calculate distributions of temperature and velocity on the center line of the outlet cross section. The trend of both results is consistent. Since geometries of the two models are different, the existences of the slight deviations between both results are reasonable.

In Fig. 14, comparison of the present result with that of an experimental result of [3] is shown. For the Reynolds number and heat flux of both situations are 3000 and 57 $\text{W/m}^2$, respectively. The ratio of the height to the width is 10 in this work because of the limitation of computation memory, and the ratio of [3] was 25. The shorter the height is, the larger the average Nusselt number is achieved. Naturally, the result of this work ought to be larger than that of [3].

5. Conclusions
An investigation of flow reversal of mixed convection in a three dimensional rectangular channel with the consideration of the compressibility of the working fluid is studied numerically. The occurrence of flow reversal of mixed convection in the channel is mainly depended by the balance of mass flow rates induced by natural convection and provided by forced convection. The mass flow rate induced by natural convection is larger than that provided by forced convection at the inlet that causes the phenomenon of flow reversal to occur easily. The phenomenon will decay accompanying with the decrement of the difference between both mass flow rates. In the range of this work, several related features of the results are drawn as follows.

1. The magnitude of Richardson number dominates the mechanisms of the flow reversal of mixed convection.
2. Drastic phenomena of the flow reversal of mixed convection are mainly caused by the mutual impingement between the amount of downward fluid sucked from the outlet and the amount of upward fluid from the inlet provided by forced convection.
3. Under the same mass flow rate, the magnitude of time and area averaged Nusselt number of natural convection is slightly superior to that of forced convection.

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References