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Quality of service guarantee for real-time VBR traffic flows with different delay bound and loss probability requirements in WLANs

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The medium access control of IEEE 802.11e defines a novel coordination function, namely hybrid coordination function (HCF), which allocates transmission opportunity (TXOP) to stations taking their quality of service (QoS) requirements into account. However, the reference TXOP allocation scheme of HCF controlled channel access, a contention-free channel access function of HCF, is only suitable for constant bit rate traffic. For variable bit rate (VBR) traffic, serious packet loss may occur. In this article, we generalize the reference design with an efficient TXOP allocation algorithm, a multiplexing mechanism, and an associated admission control unit to guarantee QoS for VBR flows with different delay bound and packet loss probability requirements. We define equivalent flows and aggregate packet loss probability to take advantage of both intra-flow and inter-flow multiplexing gains so that high bandwidth efficiency can be achieved. Moreover, the concept of weighted-loss fair service scheduling is adopted to allocate the aggregate TXOP to individual flows. From numerical results obtained by computer simulations, we found that our proposed scheme meets QoS requirements and results in much higher bandwidth efficiency than previous algorithms.

Keywords: quality of service; wireless LAN; scheduling

1. Introduction

Wireless networks such as IEEE 802.11 WLANs (IEEE 2007) have recently been deployed widely with rapidly increasing users all over the world. As real-time applications such as VoIP and streaming video are getting more common in daily life, quality of service (QoS) guarantee over wireless networks is becoming an important issue. Generally speaking, QoS support includes guarantee of maximum packet delay, delay jitter, and packet loss probability. To cope with this problem, a new enhancement of WLANs, called IEEE 802.11e (IEEE 2005), was introduced to provide QoS support for real-time traffic. This amendment has been combined into WLAN standard (IEEE 2007).

The QoS-aware coordination function proposed in IEEE 802.11e is called hybrid coordination function (HCF). This function consists of two channel access mechanisms. One is contention-based enhanced distributed channel access (EDCA) and the other is contention-free HCF controlled channel access (HCCA). The contention-free nature makes HCCA a better choice for QoS support than EDCA (Mangold et al. 2003).

HCCA requires a centralized QoS-aware coordinator, called HC, which has a higher priority than normal QoS-aware stations (QSTAs) in gaining channel control. HC can gain control of the channel after sensing the medium idle for a PCF inter-frame space that is shorter than DCF inter-frame space adopted by QSTAs. After gaining control of the transmission medium, HC polls QSTAs according to its polling list. In order to be included in HC’s polling list, a QSTA needs to negotiate with HC by sending the add traffic stream frame. In this frame, the QSTA describes the traffic characteristics and the QoS requirements in the traffic specification (TSPEC) field. Based on the traffic characteristics and the QoS requirements, HC calculates the scheduled service interval (SI) and transmission opportunity (TXOP) duration for each admitted flow.

Upon receiving a poll, the polled QSTA either responds with QoS-data if it has packets to send or a QoS-null frame otherwise. When the TXOP duration of some QSTA ends, HC gains the control of channel again and either sends a QoS-poll to the next station on its polling list or releases the medium if there is no more QSTA to be polled. Polling, together with TXOP allocation, will be referred to as a scheduling scheme of HCCA in this article.

Scheduling schemes can be classified into two categories, namely static and dynamic. In a static...
scheduling scheme, HC allocates the same TXOP duration to a QSTA every time it is polled. Moreover, the SI is often selected as the minimum of delay bound requirements of all traffic flows. The sample scheduler provided in IEEE 802.11 standard (IEEE 2007) is a typical example of a static scheduling scheme. The HC of the sample scheduler allocates TXOP duration based on mean data rate and nominal medium access control (MAC) service data unit (MSDU) size. It performs well for constant bit rate traffic. For variable bit rate (VBR) traffic, severe packet loss may occur. Some other static scheduling schemes have been proposed previously (Fan et al. 2004, Huang et al. 2007, Gao et al. 2008, Lee and Huang 2008). These schemes generalized the sample scheduler with modified TXOP allocation algorithm and admission control unit so that both delay bound and packet loss probability requirements of admitted traffic flows can be fulfilled. The fact that many real-time VBR applications can tolerate packet loss to a certain degree was taken into consideration in those schemes to improve bandwidth efficiency. The sample scheduler does not take advantage of bandwidth efficiency. The sample scheduler because they considered inter-flow multiplexing gain because the TXOP duration allocated to a QSTA is the sum of the TXOP durations allocated to individual flows attached to it. The schemes (Fan et al. 2004, Huang et al. 2007, Gao et al. 2008, Lee and Huang 2008) result in higher bandwidth efficiency than the sample scheduler because they considered inter-flow multiplexing effect. In references Fan et al. (2004) and Huang et al. (2007), it was assumed that all traffic flows have the same delay bound of one SI and the same packet loss probability requirement. In the work of Lee and Huang (2008), traffic flows are allowed to have different delay bounds but identical packet loss probability requirements. A finite buffer is provided for packets with delay bounds greater than one SI. With such a buffer, packets that arrived in different previous SIs (and have not violated their delay bound requirement) can share the current TXOP to achieve intra-flow multiplexing gain. The rate-variance envelope-based admission control (RVAC) algorithm (Gao et al. 2008) uses token buckets for traffic shaping. With the token buckets, the envelope of traffic arrival can be determined. Using the traffic envelope and the given delay bound requirement, one can compute the packet loss probability for an allocated bandwidth. This scheme only considers identical delay bound and packet loss probability requirements.

In contrast to static ones, a dynamic scheduling scheme allocates TXOP duration to a QSTA dynamically, according to system status, to provide delay bound guarantee and/or fairness. Some dynamic scheduling schemes can be found in Ansel et al. (2003), Cicconetti et al. (2007), Higuchi et al. (2007), Rashid et al. (2008), Bourawy et al. (2009), Huang et al. (2009), Luo and Shyu (2009), Huang et al. (2010), and Hantrakoon and Phonphoem (2010). To achieve delay bound guarantees, a dynamic scheduling scheme requires QSTAs to report their queue statuses to HC in a timely manner. As an example, in the prediction and optimization-based HCCA (PRO-HCCA) scheme (Rashid et al. 2008) that was proposed recently, the SI is set to be smaller than or equal to half of the minimum of delay bounds requested by all traffic flows. As a consequence, compared with static scheduling schemes, QSTAs are polled more frequently, which implies higher overhead for poll frames. Furthermore, static and periodic polling allows QSTAs to easily eliminate overhearing to save energy. Therefore, although dynamic scheduling has the potential to achieve high bandwidth efficiency, it is worth studying static scheduling schemes.

The purpose of this article is to present an efficient static scheduling scheme to provide QoS guarantee for VBR traffic flows with different delay bounds and packet loss probability requirements. The proposed scheme achieves both intra-flow and inter-flow multiplexing gains. In this scheme, HC calculates TXOP duration and performs admission control while every QSTA implements a weighted-loss fair service scheduler to determine how the allocated TXOP is shared by traffic flows attached to it. Numerical results obtained by computer simulations show that our proposed TXOP allocation algorithm results in much better performance than previous work. Moreover, the proposed weighted-loss fair service scheduler successfully manages the TXOP so that different delay bounds and packet loss probability requirements of all traffic flows can be fulfilled.

The remainder of this article is organized as follows. In Section 2, we describe the system model. In Section 3, we review related previous works. Section 4 contains our proposed TXOP allocation algorithm, the weighted-loss fair service scheduler, and the associated admission control unit. Simulation results are provided and discussed in Section 5. Finally, we draw conclusion in Section 6.

2. System model

The studied system consists of $K$ QSTAs, called $QSTA_1$, $QSTA_2$, . . . , and $QSTA_K$ such that $QSTA_i$ has $n_i$ existing VBR flows. Transmission over the wireless medium is divided into SIs and the duration of each SI, denoted by $S_i$, is a sub-multiple of the length of a beacon interval $T_b$. Moreover, an SI is further
divided into a contention period and a contention-free period. The HCCA protocol is adopted during contention-free periods.

It is assumed that every QSTA has the capability to measure channel quality to determine a feasible transmission rate which yields a frame error rate sufficiently smaller than the packet loss probability requirements requested by all traffic flows attached to the QSTA. The relationship between measured channel quality and frame error rate can be found in the article presented by Kim et al. (2005).

The QoS requirements of traffic flows are specified with delay bound and packet loss probability. Every QSTA is equipped with sufficiently large buffer so that a packet is dropped if and only if (iff) it violates the delay bound. It is assumed that there are \( I \) different packet loss probabilities, represented by \( P_1, P_2, \ldots, \) and \( P_J \) with \( P_1 > P_2 > \cdots > P_J \), and \( J \) possible delay bounds, denoted by \( D_1, D_2, \ldots, \) and \( D_J \) with \( D_1 < D_2 < \cdots < D_J \). We assume that \( D_1 = SI \) and \( D_J = \beta_j SI \) for some integer \( \beta_j > 1 \).

HC allocates TXOPs to QSTAs based on a static and periodic schedule. As illustrated in Figure 1, the TXOP for \( QSTA_k \), denoted by \( TXOP_k \), is allocated every SI and is of fixed length. The length of scheduled SI is chosen to be the minimum of all requested delay bounds. Note that SI has to be updated if a new flow with delay bound smaller than those of existing ones is admitted or the only flow with the smallest delay bound is disconnected. In this case, the TXOPs allocated to QSTAs have to be recalculated accordingly.

Consider the existing flows of a specific QSTA, say \( QSTA_w \). The \( n_w \) flows attached to \( QSTA_w \) are classified into groups according to their QoS requirements. Let \( F_{ij} \) represent the set which contains all traffic flows with packet loss probability \( P_i \) and delay bound \( D_j \). Furthermore, let \( F_i = \cup_{1 \leq j \leq J} F_{ij} \) and \( F = \cup_{1 \leq i \leq I} F_i \). To reduce computational complexity, we assume that the traffic arrivals of different flows are independent Gaussian processes. Since the sum of independent Gaussian random variables remains Gaussian, the aggregated flow of all the flows in set \( F_{ij} \) is Gaussian and will be represented by \( f_{ij} \). For convenience, we shall consider \( f_{ij} \) as a single flow. A separate queue, called \( Queue_{ij} \), is maintained for flow \( f_{ij} \), \( 1 \leq i \leq I \) and \( 1 \leq j \leq J \). Let \( N(\mu_{ij}, \sigma_{ij}^2) \) denote the distribution of traffic arrival for flow \( f_{ij} \) in one SI. Note that the values of \( \mu_{ij} \) and \( \sigma_{ij}^2 \) can be calculated by \( \mu_{ij} = E(N_{ij}) \cdot E(X_{ij}) \) and \( \sigma_{ij}^2 = E(N_{ij}) \cdot VAR(X_{ij}) + E(X_{ij})^2 \cdot VAR(N_{ij}) \), where \( N_{ij} \) and \( X_{ij} \) represent, respectively, the number of packets belonging to flow \( f_{ij} \) that arrive in one SI and the packet size. Interested readers can find the derivations in the paper (Huang et al. 2007).

3. Previous works

3.1. The sample scheduler (IEEE 2007)

Consider \( QSTA_w \) which has \( n_w \) flows. Let \( \rho_i, L_i \) denote, respectively, the mean data rate and the nominal MSDU size of the \( i \)th flow attached to \( QSTA_w \). HC calculates TXOPs as follows. Firstly, it decides, for flow \( l \), the average number of packets \( N_l \) that arrive at the mean data rate during one SI

\[
N_l = \left\lceil \frac{\rho_l \times SI}{L_l} \right\rceil.
\]

Secondly, the TXOP duration for this flow is obtained by

\[
TD_l = \max\left\{ N_l \times \left( \frac{L_l}{R_{\min}} + O \right), \frac{L_{\max}}{R_{\min}} + O \right\},
\]

where \( R_{\min} \) is the minimum physical transmission rate of \( QSTA_w \), and \( L_{\max} \) and \( O \) denote, respectively, the maximum allowable size of MSDU and per-packet overhead in time units. The overhead \( O \) includes the transmission time for an ACK frame, inter-frame space, MAC header, CRC field, and PHY PLCP preamble and header.

Finally, the total TXOP duration allocated to \( QSTA_w \) is given by

\[
TXOP_w = \left( \sum_{i=1}^{n_w} TD_l \right) + SIFS + t_{POLL}.
\]
where SIFS and \( t_{\text{poll}} \) are, respectively, the short inter-frame space and the transmission time of a CF-poll frame.

Admission control is performed as follows. Assume that \( QSTA_n \) negotiates with HC for admission of a new traffic flow, i.e., the \((n_0 + 1)\)th flow of \( QSTA_n \). For simplicity, we further assume that the delay bound of the new flow is not smaller than \( SI \). The process is similar if this assumption is not true. HC updates \( TXOP_a \) as \( TXOP_a = TXOP_{a'} + TD_{n_0 + 1} \). The new flow is admitted iff the following inequality is satisfied

\[
\frac{TXOP_a}{SI} + \sum_{k=1,k\neq a}^{K} \frac{TXOP_k}{SI} \leq T_b - T_{\text{cp}},
\]

where \( T_{\text{cp}} \) is the time used for EDCA traffic during one beacon interval.

It is clear that the TXOP allocation algorithm of the sample scheduler does not consider delay bound and packet loss probability requirements. Moreover, it does not take advantage of inter-flow multiplexing gain.

3.2. Scheme for traffic flows with identical packet loss probability requirements

In Huang et al. (2007), it was assumed that all traffic flows request the same packet loss probability and the same delay bound of one \( SI \). The assumption was relaxed in Lee and Huang (2008) to allow flows requesting different delay bounds but identical packet loss probabilities. We only describe the scheme proposed by Lee and Huang (2008) because it is a generalization of that presented by Huang et al. (2007). Without loss of generality, assume that the packet loss probability requested by all flows is \( P_1 \). As a result, we have \( F = F_1 \). Further, for ease of description, we assume that there is at least one traffic flow with delay bound \( D_1 \).

Again, consider \( QSTA_n \) which has \( n_0 \) flows. The \( n_0 \) flows are classified into \( J \) disjoint sets \( F_{1,1}, F_{1,2}, \ldots, \) and \( F_{1,J} \) such that a flow belongs to \( F_{1,j} \) iff its delay bound is \( \beta_j SI \). Let \( f_{1,j} \), \( 1 \leq j \leq J \), with traffic arrival distribution \( N(\mu_{1,j}, \sigma_{1,j}^2) \) denote the aggregated flow of all the flows in set \( F_{1,j} \). The first come first serve service discipline was adopted for packet transmission. The effective bandwidth \( c_{1,j} \) of flow \( f_{1,j} \) is defined as the minimum TXOP which is sufficient to guarantee a packet loss probability smaller than or equal to \( P_1 \) for flow \( f_{1,j} \). Calculation of \( c_{1,j} \) takes advantage of intra-flow multiplexing gain. Since the delay bound of flow \( f_{1,j} \) is \( \beta_j SI \), \( c_{1,j} \) can be determined with a finite-buffer queuing model where the buffer size is \( \beta_j c_{1,j} \), the server transmission capability is \( c_{1,j} \), and the desired packet loss probability is \( P_1 \). Given the traffic arrival distribution \( N(\mu_{1,j}, \sigma_{1,j}^2) \), \( c_{1,j} \) can be written as \( c_{1,j} = \mu_{1,j} + \alpha_{1,j} \sigma_{1,j} \), where \( \alpha_{1,j} \) was called the QoS parameter of flow \( f_{1,j} \). Derivation of packet loss probability for a finite-buffer system is complicated. Kim and Shroff (2001) provided a good approximation based on the tail probability of an infinite-buffer system and the loss probability of a buffer-less system, as shown in Equation (5).

\[
P_L(x) \approx \frac{P_L(0)}{P(X > 0)} \cdot P(X > x).
\]

In the above equation, \( P_L(x) \) represents the packet loss probability of a finite-buffer system with buffer size \( x \) and \( P(X > x) \) denotes the tail probability above level \( x \) of an infinite-buffer system. The equation for \( P_L(0) \) can be found in the paper presented by Kim and Shroff (2001). It is pretty complicated and thus is omitted due to space limitation. The equation for \( P_L(0) \) is given by

\[
P_L(0) = Q(\alpha_{1,j}) + \left[ \frac{\sigma_{1,j}}{\mu_{1,j} \sqrt{2\pi}} e^{-\left(\frac{\sigma_{1,j}^2}{2}\right)} - \left(1 + \frac{\alpha_{1,j} \sigma_{1,j}}{\mu_{1,j}}\right) Q(\alpha_{1,j}) \right],
\]

where \( Q(\alpha_{1,j}) = \int_{\alpha_{1,j}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\left(\frac{x^2}{2}\right)} dx \). Having \( P_L(x), P(X > x), \) and \( P_L(0) \), one can obtain the (approximate) packet loss probability of a finite-buffer system with server transmission capability \( c_{1,j} \) and buffer size \( \beta_j c_{1,j} \) as

\[
P_L(\beta_j c_{1,j}) \approx \frac{\sigma_{1,j}}{\mu_{1,j} \sqrt{2\pi}} e^{-\left(\frac{\sigma_{1,j}^2}{2}\right)} - \left(1 + \frac{\alpha_{1,j} \sigma_{1,j}}{\mu_{1,j}}\right) Q(\alpha_{1,j}).
\]

Consequently, given mean \( \mu_{1,j} \), variance \( \sigma_{1,j}^2 \), delay bound \( \beta_j SI \), and the desired packet loss probability \( P_1 = P_L(\beta_j c_{1,j}) \), the QoS parameter \( \alpha_{1,j} \) can be computed with Equation (7) which in turn can be used to derive the effective bandwidth \( c_{1,j} = \mu_{1,j} + \alpha_{1,j} \sigma_{1,j} \).

Let \( L_{1,j} \) represent the nominal packet size of flow \( f_{1,j} \). The average number of packets which can be transmitted in one \( SI \), denoted by \( N_{1,j} \), can be estimated as

\[
N_{1,j} = \left\lfloor \frac{c_{1,j}}{L_{1,j}} \right\rfloor.
\]

The allocated TXOP duration for flow \( f_{1,j} \) is given by

\[
TD_{1,j} = \max \left\{ \frac{c_{1,j}}{R_n} + N_{1,j} \times O, \frac{L_{\text{max}}}{R_n} + O \right\}.
\]
where $R_a$ represents the feasible physical transmission rate of $QSTA_a$.

As mentioned before, using buffer to store packets achieves intra-flow multiplexing gain. To further achieve inter-flow multiplexing gain, an equivalent flow of delay bound $D_t$, denoted by $f_{1,t}$, is defined for flow $f_{1,j}$, $1 \leq j \leq J$. Let $N(\hat{\mu}_{1,j}, \hat{\sigma}_{1,j}^2)$ be the traffic arrival distribution of $f_{1,j}$. We have $f_{1,1} = f_{1,1}$. The equivalent flow $f_{1,2}$ for $2 \leq j \leq J$ is obtained by letting its mean and effective bandwidth equal those of flow $f_{1,j}$, i.e., $\hat{\mu}_{1,j} = \mu_{1,j}$ and $\hat{\sigma}_{1,j} = \sigma_{1,j}$, where $\hat{\sigma}_{1,j}$ is the QoS parameter of the equivalent flow. Since the delay bound of the equivalent flow $f_{1,j}$ is equal to $D_t = SI$, a packet of $f_{1,j}$ which arrives in the $n$th SI will violate its delay bound and be dropped if it is not served in the $(n+1)$th SI. As a consequence, the effective bandwidth for $f_{1,2}$ can be derived based on a buffer-less system. That is, the QoS parameter $\hat{\sigma}_{1,j}$ can be computed according to Equation (6) for $P(0) = P_t$. Lee and Huang (2008) showed that $\hat{\sigma}_{1}$ can be well approximated by $Q^{-1}(P_t)$. With the approximation, we have $\hat{\sigma}_{1,j} = \frac{\sigma_{1,j}}{Q^{-1}(P_t)}$. After obtaining the equivalent flows $f_{1,j}$, $1 \leq j \leq J$, one can determine the aggregate equivalent flow $f_{1}$. Let $N(\hat{\mu}_{1}, \hat{\sigma}_{1}^2)$ denote the distribution of traffic arrival in one SI for the aggregate equivalent flow $f_{1}$. Since the sum of independent Gaussian random variables remains Gaussian, we have $\hat{\mu}_{1} = \mu_{1,1} + \sum_{j=2}^{J} \hat{\mu}_{1,j}$ and $\hat{\sigma}_{1}^2 = \sigma_{1,1}^2 + \sum_{j=2}^{J} \hat{\sigma}_{1,j}^2$. Again, given $\hat{\mu}_{1}$ and $\hat{\sigma}_{1}^2$, the QoS parameter $\hat{\sigma}_{1}$ of flow $f_{1}$ can be derived according to Equation (6) for $P(0) = P_t$. Having $\hat{\sigma}_{1}$, one can compute the effective bandwidth $\hat{c}_1$ for flow $f_{1}$. The TXOP duration allocated to $QSTA_a$ is then determined as follows

$$TXOP_a = \max \left\{ \frac{\hat{c}_1}{R_a} + \frac{\sum_{i=1}^{I} L_{1,i}}{L_{1,i}} \times \left( N_{1} + O + SIFS + t_{POLL} \right) \right\},$$

where

$$\hat{c}_1 = \hat{\mu}_1 + \hat{\sigma}_1 \hat{\sigma}_{1},$$

$$N_{1} = \left\lfloor \frac{\hat{c}_1}{L_{1,i}} \right\rfloor.$$

In Equation (12), $L_{1,i}$ denotes the weighted average nominal packet size of all the flows in $F_1$, and is calculated by

$$L_{1,i} = \frac{\sum_{j=1}^{J} N_{i,j} \times L_{1,i}}{\sum_{j=1}^{J} N_{i,j}}.$$

The criterion shown in Equation (4) was used for admission control.

Clearly, assuming all traffic flows have identical packet loss probabilities is a big constraint of the above scheme. A straightforward solution to handle flows with different packet loss probabilities is to assume that all flows have the most stringent requirement. Unfortunately, such a solution increases the effective bandwidths of flows which allow packet loss probabilities greater than the smallest one. Another possible solution is to compute separately the effective bandwidth $\hat{c}_1$ for aggregated equivalent flow $f_{1}$, $1 \leq i \leq I$, and allocate $TXOP_a = \sum_{i=1}^{I} \hat{c}_1$. Such a solution, however, does not take advantage of inter-flow multiplexing gain. In the following section, we present our proposed scheme which considers different packet loss probabilities and takes advantage of inter-flow multiplexing gain.

4. Our proposed scheme

Our proposed scheme consists of an aggregate TXOP allocation algorithm, the weighted-loss fair service scheduler, and the associated admission control unit. As mentioned before, TXOP allocation and admission control are performed in HC and weighted-loss fair service scheduler is implemented in QSTAs. An overview of our proposed scheme is shown in Figure 2. Once again, let us consider $QSTA_a$ with $n_a$ traffic flows, which are classified into $I \times J$ groups according to their QoS requirements.

4.1. Aggregate TXOP allocation algorithm

First of all, an aggregate equivalent flow, denoted by $\hat{f}_i$, is determined using the technique described in the last section for flows $f_{1,1}, f_{2,2}, \ldots,$ and $f_{J,J}$, for all $1 \leq i \leq I$. Note that the packet loss probability requirement of $f_i$ is $P_t$. Let $N(\hat{\mu}_i, \hat{\sigma}_i^2)$ represent the traffic arrival distribution for flow $f_i$. Define $\hat{f}$ as the ultimate equivalent flow with traffic arrival distribution $N(\sum_{i=1}^{I} \hat{\mu}_i, \sum_{i=1}^{I} \hat{\sigma}_i^2)$. The desired packet loss probability of flow $\hat{f}$, denoted by $P_{\text{ultimate}}$, is given by

$$P_{\text{ultimate}} = \frac{\sum_{i=1}^{I} P_t \cdot \hat{\mu}_i}{\sum_{i=1}^{I} \hat{\mu}_i}.$$

Note that the delay bounds of the aggregate equivalent flows $f_i$, $1 \leq i \leq I$, and the ultimate equivalent flow $\hat{f}$ are equal to $SI$. Consequently, the QoS parameter $\hat{\sigma}_i$ of flow $f_i$ is equal to $\hat{\sigma}_1$ as the ultimate equivalent $SI$ can be derived based on a buffer-less system.
\[ P_{\text{ultimate}} \text{. The aggregate TXOP allocated to } QSTAn \text{ can be calculated using Equation (10), except that the aggregate effective bandwidth and the average number of packets which can be served in one SI are obtained by} \]

\[ \hat{c} = \frac{I}{\sum_{i=1}^{I} \frac{\mu_i}{\sigma_i^2}}, \]

\[ N = \left\lceil \frac{\hat{c}}{F} \right\rceil. \]  

In Equation (16), \( T \) denotes the weighted average nominal packet size of all the flows in \( F \) and is calculated by \( T = (\sum_{i=1}^{I} \sum_{j=1}^{J} N_{i,j} \cdot T_{i,j}) / (\sum_{i=1}^{I} \sum_{j=1}^{J} N_{i,j}) \), where \( N_{i,j} \) and \( T_{i,j} \) can be obtained using Equations (12) and (13), respectively. The aggregate TXOP allocation procedure for \( QSTAn \) is summarized below.

**Step 1:** For \( 1 \leq i \leq I \), determine the aggregate equivalent flow \( \hat{f}_i \) with packet loss probability requirement \( P_i \), for flows \( f_{i,1}, f_{i,2}, \ldots, f_{i,I} \).

**Step 2:** Calculate the packet loss probability \( P_{\text{ultimate}} \) using Equation (14).

**Step 3:** Compute the QoS parameter of the ultimate equivalent flow using Equation (6) with \( P_{\text{ultimate}} \) as the desired packet loss probability.

**Step 4:** Determine the effective bandwidth and average number of packets served in one SI using Equations (15) and (16), respectively. Compute the aggregate transmission duration \( TXOP_a \) according to Equation (10).

**4.2. Weighted-loss fair service scheduler**

When polled, \( QSTAn \) needs to determine how the corresponding flows share the allocated TXOP. Let \( Queue_{i,j} \) denote the queue maintained in \( QSTAn \) for storing packets of flow \( f_{i,j} \). As shown in Figure 3, \( Queue_{i,j} \) is divided into \( \beta_j \) virtual sub-queues such that the \( p \)th sub-queue, represented by \( Queue_{i,j}^p \), \( 1 \leq p \leq \beta_j \), contains packets which can be kept for up to \( p \) SIs before violating the delay bound. How the allocated TXOP is shared is controlled by our proposed weighted-loss fair service scheduler.

Consider the \( n \)th SI. The proposed weighted-loss fair service scheduler is similar to the earliest deadline first (EDF) scheduler (Georgiades et al. 1997). Let \( Q_n^{p,j}[n] \), \( 1 \leq p \leq \beta_j \), represent the buffer occupancy in terms of transmission time for \( Queue_{i,j}^p \) and \( Q_n^{j}[n] = \sum_{p=1}^{\beta_j} Q_n^{p,j}[n] \). If the aggregate TXOP allocated to \( QSTAn \) satisfies \( TXOP_n \geq \sum_{i,j} Q_n^{j}[n] \), then all packets in \( Queue_{i,j} \) can be served and, therefore, no traffic is lost in the \( n \)th SI. In this case, our proposed weighted-loss fair service scheduler is the same as the EDF scheduler.

Assume that \( TXOP_n < \sum_{i,j} Q_n^{j}[n] \). Under this assumption, there exists a minimum \( m \) such that \( \sum_{i,j} Q_n^{j}[n] > TXOP_n \). Packets with deadlines smaller than \( m \cdot SI \) are served in this SI according to the EDF scheduler. Any packet which can be kept for longer than \( m \cdot SI \) stays in queue. Packets in \( Queue_{i,j}^p \), \( 1 \leq i \leq I \), \( \beta_j \geq m \), are handled differently by our proposed weighted-loss fair service scheduler and the EDF scheduler. In the proposed weighted-loss fair service scheduler, which packets should stay in queue (if \( m > 1 \)) or be dropped (if \( m = 1 \)) is
decided based on running packet loss probabilities. Once the decision is made, the service order of those packets to be transmitted is determined by the EDF scheduler.

Define $\text{Loss}[n] = \sum_{i,j} \sum_{m=1}^{m_{\text{max}}} Q^m_{ij}[n] - TXOP_a$. For $\text{Queue}_{ij}$, let $A_{ij}[n]$ and $L_{ij}[n]$ denote, respectively, the accumulated amount of traffic arrived and lost up to the $n$th SI. Define $l_{ij}[n]$ as the amount of lost traffic (if $m = 1$) or the amount of traffic with deadline $m \cdot SI$ that stays in $\text{Queue}_{ij}$ (if $m > 1$). Also, define $TD_{ij}[n]$ as the TXOP duration shared by $\text{Queue}_{ij}$. It holds that $\sum_{ij} TD_{ij}[n] = TXOP_a$. Finally, let $P_{ij}[n] = (L_{ij}[n] - 1) + l_{ij}[n]) / A_{ij}[n]$. We call $P_{ij}[n]$ the running packet loss probability for $\text{Queue}_{ij}$ up to the $n$th SI if $m = 1$, or a pseudo one if $m > 1$.

Our proposed weighted-loss fair service scheduler tries to minimize the total amount of packet loss while maintaining a kind of fairness in the sense that the (pseudo) running packet loss probabilities of traffic flows are proportional to their packet loss probability requirements. To achieve the goal, we let $l_{ij}[n] = 0$ if $\beta_j < m$ or $\beta_j \geq m$ and $Q^m_{ij}[n] = 0$. For $\text{Queue}_{ij}$ with $\beta_j \geq m$ and $Q^m_{ij}[n] > 0$, the following equations are solved for $l_{ij}[n]$.

$$\frac{P_{ij}[n]}{P_j} = \frac{P_{r,i}[n]}{P_r} \forall (i,j), (r,s) \in U_{\text{active}},$$

$$\text{Loss}[n] = \sum_{(i,j) \in U_{\text{active}}} l_{ij}[n].$$

In Equations (17) and (18), $U_{\text{active}}$ is a set which contains $(i,j)$ such that $Q^m_{ij}[n] > 0$. For ease of description, we assume that every $\text{Queue}_{ij}$ is in $U_{\text{active}}$ if $\beta_j \geq m$. After some derivations (shown in Appendix 1), we get

$$l_{ij}[n] = \sum_{(r,s) \in U_{\text{active}}} P_r \cdot A_{ij}[n] \left[ \frac{1}{P_i \cdot A_{ij}[n] \cdot \left( \text{Loss}[n] + \sum_{(r,s) \in U_{\text{active}}} L_{rs}[n] \cdot (n - 1) \right)} - \frac{l_{ij}[n] - 1}{P_i \cdot A_{ij}[n]} \right]$$

$$= \sum_{(r,s) \in U_{\text{active}}} \frac{P_r \cdot A_{ij}[n]}{P_i} \cdot \frac{\text{Loss}[n] + \sum_{(r,s) \in U_{\text{active}}} L_{rs}[n] \cdot (n - 1)}{P_i \cdot A_{ij}[n]} - \frac{l_{ij}[n] - 1}{P_i \cdot A_{ij}[n]}$$

(19)

If the solution satisfies $0 \leq l_{ij}[n] \leq Q^m_{ij}[n]$ for all $(i,j) \in U_{\text{active}}$, then a feasible solution is obtained. The TXOP duration for $\text{Queue}_{ij}$, i.e., $TD_{ij}[n]$, is given by

$$TD_{ij}[n] = \left( \sum_{p=1}^{m-1} Q^p_{ij}[n] \right) + Q^m_{ij}[n] - l_{ij}[n].$$

Unfortunately, the solution obtained by Equation (19) may be infeasible, i.e., it is possible to have $l_{ij}[n] > Q^m_{ij}[n]$ or $l_{ij}[n] < 0$ for some $(i,j) \in U_{\text{active}}$. If it happens, adjustment is necessary to make the solution feasible. The adjustment is accomplished by the loss computation algorithm shown in Appendix 2. Its basic idea is described below. There are four possible cases for the solution obtained by Equation (19).

Case 1: $0 \leq l_{ij}[n] \leq Q^m_{ij}[n]$ for all $(i,j) \in U_{\text{active}}$.

If $0 \leq l_{ij}[n] \leq Q^m_{ij}[n]$ for all $(i,j) \in U_{\text{active}}$, then a feasible solution is found.

Case 2: $l_{ij}[n] \geq 0$ for all $(i,j) \in U_{\text{active}}$ and $l_{rs}[n] > Q^m_{rs}[n]$ for some $(r,s)$.

In this case, let $\text{Loss}'[n] = \text{Loss}[n]$. For every $(i,j)$ such that $l_{ij}[n] \geq Q^m_{ij}[n]$, assign $l_{ij}[n] = Q^m_{ij}[n]$, remove $(i,j)$ from $U_{\text{active}}$, and set $\text{Loss}'[n] = \text{Loss}[n] - Q^m_{ij}[n]$. Use Equation (19) again to compute $l_{ij}[n]$ for the updated $U_{\text{active}}$ and $\text{Loss}'[n]$. Note that, as proved in Theorem 1, the updated solution should fall in either Case 1 or Case 2. If it falls in Case 1, then a feasible solution is obtained. Otherwise, the same process is repeated. Eventually, a feasible solution will be obtained because it holds that $\sum_{ij} Q^m_{ij}[n] > \text{Loss}[n]$.

**Theorem 1:** Given $U_{\text{active}}$ and $\text{Loss}[n]$. Assume that the solution shown in Equation (19) satisfies $l_{ij}[n] \geq 0$ for all $(i,j) \in U_{\text{active}}$ and $l_{rs}[n] > Q^m_{rs}[n]$ for some $(r,s)$. Let $U = U_{\text{active}} - \{(r,s)\}$ and $\text{Loss}'[n] = \text{Loss}[n] - Q^m_{rs}[n]$. Further, let $l_{ij}'[n]$. $(i,j) \in U$, be the solution of Equations (17) and (18) for $U$ and $\text{Loss}'[n]$. It holds that $l_{ij}'[n] > l_{ij}[n] > 0$.

**Proof:** Assume that $l_{ij}'[n] \leq l_{ij}[n]$ for some $(i,j)$. According to Equation (17), we have $l_{ab}[n] \leq l_{ab}[n]$ for any $(a,b) \in U$. As a result, it holds that

$$\sum_{(a,b) \in U} l_{ab}[n] \leq \sum_{(a,b) \in U} l_{ab}[n] = \sum_{(a,b) \in U_{\text{active}}} l_{ab}[n] - l_{rs}[n]$$

$$< \sum_{(a,b) \in U_{\text{active}}} l_{ab}[n] - Q^m_{rs}[n] = \text{Loss}[n].$$

This contradicts Equation (18). Therefore, Theorem 1 is true.

**Theorem 1** says that if we set $l_{ij}[n] = Q^m_{ij}[n]$ when $l_{rs}[n] > Q^m_{rs}[n]$, then $l_{ij}[n]$ has to be increased for all $(i,j) \in U$ in order to satisfy Equation (17) for queues in...
$U$ and Equation (18). In fact, the amount $l_{r,s}[n] - Q_{r,s}^n[n]$ is proportionally shared by queues in $U$, i.e., it holds that \( (l_{r,s}[n] - l_{a,b}[n])/A_{a,b}[n]P_a = (l_{r,s}[n] - l_{c,d}[n])/A_{c,d}[n]P_c \) for all \((a,b),(c,d)\) ∈ \(U\). It is worth pointing out that although Theorem 1 is stated for one \((r,s)\) which satisfies $l_{r,s}[n] > Q_{r,s}^n[n]$, it actually implies the same conclusion if multiple queues satisfy the condition.

**Case 3:** $l_i[n] < Q_{i}^n[n]$ for all \((i,j)\) ∈ \(U_{\text{active}}\) and $l_{r,s}[n] < 0$ for some \((r,s)\).

In this case, we assign $l_i[n] = 0$ for every \((i,j)\) such that $l_i[n] \leq 0$, remove \((i,j)\) from \(U_{\text{active}}\), and solve for new $l_i[n]$ with Equation (19) for the updated $U_{\text{active}}$ and $\text{Loss}[n]$. The updated solution will fall in either Case 1 or Case 3. This is implied by Theorem 2 stated below. Similarly, a feasible solution is found if the updated solution falls in Case 1. Otherwise, the same process is repeated until a feasible solution appears. The proof for Theorem 2 is similar to that for Theorem 1 and is omitted.

**Theorem 2:** Given $U_{\text{active}}$ and $\text{Loss}[n]$. Assume that the solution shown in Equation (19) satisfies $l_i[n] \leq Q_i^n[n]$ for all \((i,j)\) ∈ \(U_{\text{active}}\) and \(l_{r,s}[n] < 0\) for some \((r,s)\). Let $U = U_{\text{active}} - \{(r,s)\}$ and \(l_i[n], (i,j) \in U\), be the solution of Equations (17) and (18) for \(U\) and $\text{Loss}[n]$. It holds that $l_i[n] < l_{i}[n] < Q_{i}^n[n]$.

Theorem 2 states that if we set $l_{r,s}[n] = 0$ when $l_{r,s}[n] \leq 0$, then $l_{i}[n]$ has to be decreased for all \((i,j)\) ∈ \(U\) in order to satisfy Equation (17) for queues in \(U\) and Equation (18). Again, although we state Theorem 2 for one \((r,s)\) which satisfies $l_{r,s}[n] < 0$, it implies the same conclusion if multiple queues satisfy the condition. Therefore, for Case 3, we can repeatedly set $l_i[n] = 0$ for all \((i,j)\) such that $l_i[n] \leq 0$ and solve Equations (17) and (18) for the updated $U_{\text{active}}$ and $\text{Loss}[n]$ until a feasible solution is found.

**Case 4:** $l_{r,s}[n] > Q_{r,s}^n[n]$ for some \((r,s)\) and $l_{r,s}[n] < 0$ for some \((r',s')\).

Let $U$ be the set which contains all \((i,j)\) ∈ \(U_{\text{active}}\), such that $l_i[n] \geq 0$. Case 4 is further divided into two sub-cases.

Sub-case 1. $\sum_{(i,j) \in U} Q_{i}^n[n] < \text{Loss}[n]$

For this sub-case, define

$$V_1 = \{(i,j) \in U_{\text{active}} : l_i[n] \geq Q_{i}^n[n]\}$$

and $V_2 = U_{\text{active}} - V_1$. We set $l_i[n] = Q_{i}^n[n]$ for all \((i,j)\) ∈ \(V_1\) and $\text{Loss}[n] = \text{Loss}[n] - \sum_{(i,j) \in V_1} l_i[n]$. Then, solve Equations (17) and (18) for $V_2$ and $\text{Loss}[n]$. Let $l_i[n], (i,j) \in V_2$, be the solution. No further adjustment is necessary if the solution falls in Case 1. If the solution falls in Case 2, then Case 2 is performed repeatedly until a feasible solution is found. Similarly, if the solution falls in Case 3, then Case 3 will be repeatedly executed until a feasible solution is obtained. Finally, if the solution falls in Case 4, then either Sub-case 1 or Sub-case 2 is performed again.

Sub-case 2. $\sum_{(i,j) \in U} Q_{i}^n[n] \geq \text{Loss}[n]$

For this sub-case, let $V_1 = U$ and $V_2 = U_{\text{active}} - V_1$. Equations (17) and (18) are solved for $V_1$ and $\text{Loss}[n]$. If the solution falls in Case 1, then no further processing is required. Assume that the solution fails in Case 2. Let $l_i[n], (i,j) \in V_1$, be the solutions and $W_1$ and $W_2$ be two sub-sets of $V_1$ such that $W_1 = \{(i,j) \in V_1 : l_i[n] < Q_{i}^n[n]\}$ and $W_2 = V_1 - W_1$. We set $l_i[n] = Q_{i}^n[n]$ for all \((i,j)\) ∈ \(W_2\). Let $V_2 = V_2 \cup W_1$ and $\text{Loss}[n] = \text{Loss}[n] - \sum_{(i,j) \in W_1} l_i[n]$. Equations (17) and (18) are solved for $V_2$ and $\text{Loss}[n]$. Note that this step is necessary to achieve the equality described in Equation (17) for queues in the updated $V_2$. If the solution falls in Case 3, then Case 3 will be repeatedly executed until a feasible solution is obtained. Finally, if the solution fails in Case 4, then either Sub-case 1 or Sub-case 2 is performed again.

The computational complexity of the loss computation algorithm is stated in the following Theorem 3.

**Theorem 3:** The loss computation algorithm takes at most $2(N-1)$ iterations to find the feasible solution, where $N = |U_{\text{active}}|$, the size of $U_{\text{active}}$.

**Proof:** It is clear that the solution of the last iteration falls in Case 1. Let $M$ denote the size of $U$ in that iteration. We shall prove that the loss computation algorithm takes at most $2(N - M)$ iterations to find the feasible solution if $M < N$ or one iteration if $M = N$. The case of $M = N$ is obviously true. We prove the case of $M < N$ by mathematical induction. For simplicity, we use Sub-case $i$ ($i = 1, 2$) to represent Sub-case $i$ of Case 4 in this proof.

For $N = 2$, we have $M = 1$. Since $M < N$, we know that the solution of the first iteration cannot fall in Case 1. By tracing the algorithm, one can see that the number of iterations required to find the feasible solution is equal to $2 = (N - M)$. Assume that the statement is true for $N = H$ and $M = 1, 2, \ldots, H - 1$ (Hypothesis I). Consider the case of $N = H + 1$. If Sub-case 2 is never visited, then the number of iterations required is at most $N - M + 1 \leq 2(N - M)$ because at least one queue is removed from $U_{\text{active}}$ in each iteration before the last one. Assume that Sub-case 2 was visited before the feasible solution was found. If the solution of the first iteration does not fall in Sub-case 2, then the size of $U$ in the second iteration is at most $H$. According to Hypothesis I, the maximum
number of iterations required to find the feasible solution, starting from iteration 2, is equal to $2(H - M)$. As a result, the total number of iterations is upper bounded by $2(H - M) + 1 < 2(N - M)$.

Assume that the solution of the first iteration falls in Sub-case 2. Let $|V_1| = i$ and $|V_2| = j$ with $i + j = N$. Further, let $k$ represent the number of queues added to $V_2$ when iteration 1 resumes its execution. The total number of iterations required is at most $1 + B(i, k) + 2(j + k - M)$, where $B(i, k)$ represents the maximum number of iterations required before iteration 1 resumes its execution and $2(j + k - M)$ denotes the upper bound of the number of iterations required to find the feasible solution for the updated $V_2$, according to Hypothesis I. Theorem 3 is true if $B(i, k) \leq 2(i - k) - 1$. We shall prove this by mathematical induction.

By tracing the algorithm, one can see that it is true for $i = 2$ and $k = 0$ or 1. Assume that it is true for $i = p$ and $k = 0, 1, \ldots, p - 1$ (Hypothesis II). Consider the case of $i = p + 1$. If Sub-case 2 is not visited again before iteration 1 resumes its execution, then we have $B(i, k) \leq i - k$. Note that if $k = 0$, then Case 2 is not visited. If $k > 0$, then there are $0$ to $(i - k - 1)$ times of Sub-case 1 followed by a Case 2. Since $k \leq i - 1$, we have $B(i, k) \leq 2(i - k) - 1$. Assume that, before Sub-case 1 resumes its execution, Sub-case 2 is visited for the second time in iteration $r$. This implies the solutions of iterations 2, ..., and $r - 1$ all fall in Sub-case 1 and, therefore, at least $r - 2$ queues are removed from $U_{active}$. Let $x$, $y$, and $z$ represent, respectively, the size of $V_1$, the size of $V_2$, and the number of queues added to $V_2$ when iteration $r$ resumes its execution. It is clear that $x + y \leq i - r + 2$. After iteration $r$ resumes its execution, the situation is the same as iteration 1 except that the size of $V_1$ (of iteration 1) is changed from $i$ to $y + z$. As a result, we have $B(i, k) \leq (r - 1) + B(x, z) + B(y + z, k)$. According to Hypothesis II, it holds that $B(i, k) \leq (r - 1) + 2(x - z + 1 - 1 + 2(y + z - k) - z) \leq 2(i - k) - 1$. This completes the proof of Theorem 3.

After the feasible solution is found, $TD_{ij}[n]$ can be obtained according to Equation (20). If data are dropped (i.e., $m = 1$), $L_{ij}[n]$ is updated as follows:

$$L_{ij}[n] = L_{ij}[n - 1] + l_{ij}[n].$$

Since the number of real-time flows attached to each QSTA is normally small, the complexity of the loss computation algorithm should be acceptable. Furthermore, because of static and periodic TXOP allocation, each QSTA has one SI worth of time to compute the solution. Therefore, the proposed weighted-loss fair service scheduler should be feasible for real systems.

4.3. The associated admission control unit

Assume that $QSTA_a$ is negotiating with HC for its $(n_a + 1)$th flow that requires packet loss probability $P_i$ and delay bound $D_i$. Define available bandwidth $BW_{av}$ as

$$BW_{av} = SI \left(1 - \frac{T_p}{T_b}\right) - \sum_{i=1}^{K} TXOP_i.$$  (22)

Let $\theta$ and $\rho$ denote, respectively, the mean and variance of traffic arrival in one SI for the new traffic flow. The new flow, if admitted, will become part of flow $f_{ij}$. As a result, we need only update the parameters related to flows $f_{ij}$, $\hat{f}$, and $\hat{f}$. Let $N(\mu_i, \sigma_i^2)$, $N(\hat{\mu}_i, \hat{\sigma}_i^2)$, and $N(\hat{\mu}_i, \hat{\sigma}_i^2)$ denote, respectively, the traffic arrival distributions for flows $f_{ij}$, $\hat{f}$, and $f$ before the new flow is admitted. Assume that this new flow is admitted. The parameters of $f_{ij}$ are updated as $\mu_i = \mu_i + \theta$ and $\sigma_i^2 = \sigma_i^2 + \rho^2$. Moreover, the traffic arrival distribution of the aggregate equivalent flow $\hat{f}$ is updated as $N(\hat{\mu}_i, \hat{\sigma}_i^2)$, where $\hat{\mu}_i = \mu_i + \sum_{j \neq i} \hat{\mu}_j$. The traffic arrival distribution of the ultimate equivalent flow $f$ is updated as $N(\hat{\mu}_i, \hat{\sigma}_i^2)$, where $\hat{\mu}_i = \mu_i + \sum_{j \neq i} \hat{\mu}_j$. The ultimate packet loss probability has to be recalculated using Equation (14) with the above-updated parameters as input. Finally, the effective bandwidth and the required TXOP, denoted by $TXOP_a$, can be computed, respectively, by Equations (6) and (10). Define $\Delta TXOP = TXOP_a - TXOP_a$. The new flow is admitted iff the following inequality is satisfied:

$$BW_{av} \geq \Delta TXOP.$$  (23)

If the new flow is admitted, we update $BW_{av}$ as $BW_{av} = BW_{av} - \Delta TXOP$.

Note that, if an existing flow of $QSTA_a$ is disconnected, a process similar to that shown above is conducted to obtain $\Delta TXOP = TXOP_a - TXOP_a$ and $BW_{av}$ is updated by $BW_{av} = BW_{av} + \Delta TXOP$. Note that if admission or disconnection of a flow leads to change of SI, then the TXOPs for all QSTAs should be recalculated.

5. Simulation results

The PHY and MAC parameters and all related information in simulations are shown in Table 1.

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Table 1. Related parameters used in simulations.

<table>
<thead>
<tr>
<th>PHY and MAC parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>SIFS</td>
<td>10μs</td>
</tr>
<tr>
<td>MAC header size</td>
<td>32 bytes</td>
</tr>
<tr>
<td>CRC size</td>
<td>4 bytes</td>
</tr>
<tr>
<td>QoS-ACK frame size</td>
<td>16 bytes</td>
</tr>
<tr>
<td>QoS CF-poll frame size</td>
<td>36 bytes</td>
</tr>
<tr>
<td>PLCP preamble length</td>
<td>4 bytes</td>
</tr>
<tr>
<td>PLCP header length</td>
<td>20 bytes</td>
</tr>
<tr>
<td>PHY rate (R)</td>
<td>11 Mbps</td>
</tr>
<tr>
<td>Minimum PHY rate (R_{min})</td>
<td>2 Mbps</td>
</tr>
</tbody>
</table>

Transmission time for different header and per-packet overhead (μs)

<table>
<thead>
<tr>
<th>Header Type</th>
<th>Transmission Time (μs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PLCP preamble and header (t_{PLCP})</td>
<td>96</td>
</tr>
<tr>
<td>Data MAC header (t_{HDR})</td>
<td>23.27</td>
</tr>
<tr>
<td>Data CRC (t_{CRC})</td>
<td>2.90909</td>
</tr>
<tr>
<td>ACK frame (t_{ACK})</td>
<td>107.63636</td>
</tr>
<tr>
<td>QoS-CF-poll (t_{POLL})</td>
<td>122.1818</td>
</tr>
<tr>
<td>Per-packet overhead (O)</td>
<td>249.81818</td>
</tr>
</tbody>
</table>

Note that the sizes of QoS-ACK and QoS-poll in the table only include the sizes of MAC header and CRC overhead. The simulations are performed using Matlab on a PC with an Intel (R) Core (TM) 2 Quad CPU Q9550 operated at 2.83 GHz with 3072 MB of RAM.

Traffic is delivered from QSTAs to AP and the contention-free period occupies the whole SI. We investigate three types of QSTA in the simulations. Each type of QSTA is assumed to be attached to two real-time traffic flows. Real traffic traces, developed by Fitzek and Reisslein (2006), are used for Types I and II QSTAs in our simulations. A Type III QSTA is attached with two flows, one with constant packet size and the other with variable packet size. The arrival processes are assumed to be Poisson. For flows which generate variable-size packets, the packet size varies according to exponential distribution. The length of each traffic flow lasts for one hour. The detailed information of traffic flows, including QoS requirements and traffic parameters, are described in Table 2.

For each flow, the mean μ and the variance σ² of traffic arrivals in one SI can be calculated from the mean data rate μ and the variance of frame size σ² provided in the trace file or derived using the technique described in Section 2. The calculated μ and σ² of each flow are shown in the last two rows of Table 2. Note that Type III QSTA is included to study the effect of aggregating flows with identical QoS requirements.

Simulations are divided into two parts. In the first part, we compare the loss probabilities of our proposed scheme with those of several other static scheduling algorithms. Results show that our proposed scheme outperforms the other ones. The second part contains a comparison of our proposed scheme with PRO-HCCA (Rashid et al. 2008), a recently proposed dynamic scheduling algorithm which has been shown to possess good capability of QoS support. Note that in the first part, the behavior of a QSTA is independent of other QSTAs for all the investigated static schemes, since loss probability is adopted as the performance metric. As a result, it suffices to study a system which consists of one HC and three QSTAs, one for each type. The performance metrics used in the second part include average transmission delay and packet loss probability. It is assumed that there are one HC and 10 Type I QSTAs in the system. For both parts, a packet is dropped if it violates the delay bound.

In the first part, the aggregate TXOP duration allocated by the sample scheduler is calculated by plugging the simulation parameters into Equations (1) and (2). The aggregate TXOP duration for RVAC (Gao et al. 2008) is obtained assuming that all flows request the most stringent packet loss probability and delay bound because it only considers traffic flows with identical QoS requirements. The RVAC algorithm requires peak data rate and maximum frame size, which are infinite under the adopted mathematical model. In our simulations, we use the 99 percentiles for these values. For the scheme presented by Lee and Huang (2008), the aggregate TXOP allocation is calculated assuming all flows request the most stringent packet loss probability. For the scheme proposed in this article, the aggregate TXOP duration is derived according to the procedure described in Section 4. In the comparison, we adopt the proposed weighted-loss fair service scheduler for all the investigated schemes.

In Table 3, we compare packet loss probabilities after all data are delivered. Since there is only one trace for each video, we conducted simulations with 1000 different starting positions to collect the 99% confidence intervals. The symbol $a \pm b$ in Table 3 means the 99% confidence interval is given by $(a-b, a+b)$. Transmission error is also considered for our proposed scheme. The frame transmission error probability is set to be $0.5 \times 10^{-3}$. The packet loss probability considering transmission error is marked with $^*$ and shown in the last row of Table 3. According to the results, our proposed scheme can meet the individual QoS requirements requested by traffic flows whether or not there is aggregation of flows with identical QoS requirements. Moreover, no matter which TXOP allocation scheme is adopted, our proposed weighted-loss fair service scheduler can achieve the goal of maintaining the ratio of actual packet loss probabilities at the same rate as requested values. For example, the ratios of the actual packet loss probabilities of Jurassic Park I and Lecture Camera for the sample scheduler, the RVAC
Table 2. TSPECs of traffic flows attached to Types I, II, and III QSTAs.

<table>
<thead>
<tr>
<th>Type of QSTA</th>
<th>Type I</th>
<th>Type II</th>
<th>Type III</th>
</tr>
</thead>
<tbody>
<tr>
<td>Attached traffic model</td>
<td>Jurassic Park I</td>
<td>Lecture Camera</td>
<td>Mr. Bean</td>
</tr>
<tr>
<td>Packet loss rate requirement ($P_L$)</td>
<td>0.01</td>
<td>0.001</td>
<td>0.01</td>
</tr>
<tr>
<td>Maximum service interval ($SI_{\max}$) (ms)</td>
<td>80</td>
<td>160</td>
<td>80</td>
</tr>
<tr>
<td>Mean data rate ($\rho$) (bps)</td>
<td>268 k</td>
<td>210 k</td>
<td>184 k</td>
</tr>
<tr>
<td>Nominal MSDU size ($L$) (bytes)</td>
<td>1339</td>
<td>1048</td>
<td>920</td>
</tr>
<tr>
<td>Variance of frame size ($\sigma^2$)</td>
<td>1,273,237</td>
<td>828,990</td>
<td>801,216</td>
</tr>
<tr>
<td>Frame inter-arrival time</td>
<td>40 ms</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Scheduled $SI$</td>
<td>80 ms</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Calculated mean per $SI$ ($\mu$) (bytes)</td>
<td>2680</td>
<td>2100</td>
<td>1840</td>
</tr>
<tr>
<td>Calculated variance per $SI$ ($\sigma^2$)</td>
<td>2,546,474</td>
<td>1,657,980</td>
<td>1,602,432</td>
</tr>
</tbody>
</table>

scheme, the scheme proposed by Lee and Huang (2008), our proposed scheme, and our proposed scheme with transmission error are, respectively, 0.1857:0.0018, 0.0008:0.0001, 0.0052:0.0005, 0.0099:0.0010, and 0.0100:0.0010, which are all very close to the ratio of the requested packet loss probabilities, i.e., 0.01:0.001. Another important observation is that the results of our proposed scheme are satisfactory even for a frame error rate of $0.5 \times 10^{-3}$. This implies that, to cope with transmission errors, one need only select an appropriate feasible physical transmission rate so that the probability of transmission error is sufficiently smaller than the requested packet loss probability. The average execution times of the proposed weighted-loss fair service scheduler are 0.31, 0.32, and 0.52 ms for Types I, II, and III QSTA, respectively. These numbers are much smaller than $SI$ (80 ms) and, therefore, the scheduler is feasible for real systems.

We also record the running packet loss probabilities of traffic flows attached to Type I QSTA for all investigated schemes. Here, the running packet loss probability for flow $f_{ij}$ up to the $n$th $SI$ is given by $L_{ij}[n]/A_{ij}[n]$. For the sample scheduler, as shown in Figure 4, the running packet loss probabilities of all simulated traffic flows are more than 10 times larger than their requested levels for most of the time. For TXOP allocation schemes which consider packet loss probability, we compare the sample paths of each traffic flow attached to Type I QSTA. The results are illustrated in Figures 5 and 6. It can be seen that the long-term packet loss probability meets the requirement for all the investigated schemes. However, our proposed scheme is the most efficient one because it allocates the smallest TXOP durations to QSTAs. To compare the bandwidth efficiency of the investigated schemes, we list the over-allocation ratios in Table 4. Here, the over-allocation ratio is defined as the ratio of unused TXOP duration to the allocated TXOP duration. As one can see, our proposed scheme has the smallest over-allocation ratio among the investigated schemes which meet QoS requirements. In other words, compared with other static TXOP allocation algorithms, our proposed scheme reduces over-allocation ratio and hence improves bandwidth utilization without sacrificing QoS guarantee.

Figure 7 compares the admissible regions of the investigated TXOP allocation schemes. For a particular scheme, the system can accommodate $x$ Type I QSTAs and $y$ Type II QSTAs with QoS guarantee if $(x, y)$ falls in the triangle formed by the $x$-axis, $y$-axis, and the curve labeled for the scheme. Our proposed scheme allows 8% and 18% more QSTAs to be admitted than the scheme proposed by Lee and Huang (2008) and RVAC, respectively.

In the second part of simulations, the circular round robin is adopted as the polling scheme so that all QSTAs are treated equally. In other words, the polling order in the $i$th $SI$ is QSTA $i$ (mod 10), QSTA $i + 1$ (mod 10) . . . , and QSTA $i + 9$ (mod 10). As a consequence, it suffices to consider the performance of one specific QSTA. The results are shown in Table 5. Note that, being a dynamic scheme, the PRO-HCCA has to calculate TXOP allocations at the beginning of each $SI$ which is an overhead to the HC. According to our simulation results, the PRO-HCCA achieves smaller average transmission delay than our proposed scheme because it allocates TXOPs to QSTAs dynamically based on the queue status. However, compared with PRO-HCCA, our proposed scheme has smaller delay jitter, which is defined as the difference between maximum and minimum delays in this article. The reason is that the TXOP duration allocated by our proposed scheme is a constant which equals 7.6 ms while that allocated by PRO-HCCA is
Table 3. The 99% confidence intervals of packet loss probability of flows attached to Types I, II, and III QSTAs.

<table>
<thead>
<tr>
<th>Packet loss probability ($P_L$)</th>
<th>Type I QSTA</th>
<th>Type II QSTA</th>
<th>Type III QSTA</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Jurassic Park I</td>
<td>Lecture Camera</td>
<td>Mr. Bean</td>
</tr>
<tr>
<td>Sample scheduler</td>
<td>0.1857 ± 4 × 10^{-3}</td>
<td>0.0186 ± 3 × 10^{-3}</td>
<td>0.2323 ± 3 × 10^{-5}</td>
</tr>
<tr>
<td>RVAC</td>
<td>0.0008 ± 4 × 10^{-6}</td>
<td>0.0001 ± 4 × 10^{-6}</td>
<td>0.0025 ± 9 × 10^{-6}</td>
</tr>
<tr>
<td>Scheme of Lee and Huang (2008)</td>
<td>0.0052 ± 2 × 10^{-5}</td>
<td>0.0005 ± 2 × 10^{-5}</td>
<td>0.0032 ± 1 × 10^{-5}</td>
</tr>
<tr>
<td>Our proposed scheme</td>
<td>0.0099 ± 3 × 10^{-5}</td>
<td>0.0010 ± 3 × 10^{-5}</td>
<td>0.0072 ± 2 × 10^{-5}</td>
</tr>
<tr>
<td>Our proposed scheme</td>
<td>0.0100 ± 1 × 10^{-4}</td>
<td>0.0010 ± 3 × 10^{-5}</td>
<td>0.0073 ± 1 × 10^{-4}</td>
</tr>
</tbody>
</table>
Figure 4. Running packet loss probabilities of flows attached to Type I QSTA for the sample scheduler.

Figure 5. Running packet loss probabilities of Jurassic Park I attached to Type I QSTA.

Figure 6. Running packet loss probabilities of Lecture Camera attached to Type I QSTA.
dynamic and can be larger than 7.6 ms. As a result, the maximum delay of PRO-HCCA is larger than that of our proposed scheme, which implies the delay jitter of our proposed scheme is smaller because the minimum delays are roughly the same. Moreover, our proposed scheme guarantees packet loss probability requirements while PRO-HCCA does not. For PRO-HCCA, the packet loss probability of Lecture Camera is equal to 0.0028, which is greater than its requirement 0.001. If our proposed weighted-loss fair service scheduler is combined with PRO-HCCA, then packet loss probabilities become 0.0075 and 0.0007 for Jurassic Park I and Lecture Camera, respectively. The average delay and delay jitter change slightly. The average delays are 0.0261 and 0.0289 s and the delay jitters are 0.0785 and 0.1591 s for Jurassic Park I and Lecture Camera, respectively.

6. Conclusion

In this article, we have presented an efficient static TXOP allocation algorithm, a weighted-loss fair service scheduler, and the associated admission control
unit to provide QoS guarantee for VBR traffic flows with different packet loss probability and delay bound requirements in WLANs. Computer simulations were conducted to evaluate the performance of our proposed scheme. Results show that our proposed scheme is effective in QoS guarantee and, moreover, performs much better than previous work. Our proposed weighted-loss fair service scheduler can also be combined with dynamic TXOP allocation algorithms to provide better QoS support. In real systems, it is likely that there are only a limited number of possible applications. Therefore, one can pre-compute the QoS parameter of each type of application so that admission control can be performed in real time. An interesting further research topic is to extend the results to different traffic models and other types of wireless networks.

Acknowledgments
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Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{ij}[n]$</td>
<td>the accumulated amount of $f_{ij}$ arrived up to the end of the $n$th SI</td>
</tr>
<tr>
<td>$a_{ij}$</td>
<td>QoS parameter of $f_{ij}$</td>
</tr>
<tr>
<td>$\hat{a}_{ij}$</td>
<td>QoS parameter of $f_{ij}$</td>
</tr>
<tr>
<td>$c_{ij}$</td>
<td>effective bandwidth of $f_{ij}$ in one SI (i.e., $c_{ij} = \mu_{ij} + \alpha_i \sigma_{ij}$)</td>
</tr>
<tr>
<td>$\hat{c}_{ij}$</td>
<td>the effective bandwidth of $\hat{f}<em>{ij}$ (i.e., $\hat{c}</em>{ij} = \hat{\mu}<em>{ij} + \hat{\alpha}</em>{ij}$) in one SI</td>
</tr>
<tr>
<td>$D_j$</td>
<td>the $j$th delay bound requirement ($D_j = \beta_j \cdot SI$)</td>
</tr>
<tr>
<td>$F_{ij}$</td>
<td>the set which contains traffic flows with packet loss probability and delay bound requirement equal to $P_i$ and $D_j$, respectively</td>
</tr>
<tr>
<td>$F_i$</td>
<td>the set containing traffic flows with packet loss probability equal to $P_i$ (i.e., $F_i = \cup_{j=1}^{n} F_{ij}$)</td>
</tr>
<tr>
<td>$F$</td>
<td>the set which contains all traffic flows</td>
</tr>
<tr>
<td>$f_{ij}$</td>
<td>the aggregate traffic flow for all flows in $F_{ij}$ with distribution assumed to be $N(\mu_{ij}, \sigma_{ij}^2)$</td>
</tr>
<tr>
<td>$\hat{f}_{ij}$</td>
<td>the equivalent flow of $f_{ij}$ with distribution denoted by $N(\hat{\mu}<em>{ij}, \hat{\sigma}</em>{ij}^2)$</td>
</tr>
<tr>
<td>$f_j$</td>
<td>the aggregate traffic flow for all flows in $F_j$</td>
</tr>
<tr>
<td>$\hat{f}_j$</td>
<td>the equivalent flow of $f_j$ with distribution denoted by $N(\hat{\mu}_j, \hat{\sigma}_j^2)$</td>
</tr>
<tr>
<td>$F$</td>
<td>the aggregate traffic flow for all flows in $F$</td>
</tr>
<tr>
<td>$\hat{f}$</td>
<td>the equivalent flow of $f$ with distribution denoted by $N(\hat{\mu}, \hat{\sigma}^2)$</td>
</tr>
<tr>
<td>$L_{ij}$</td>
<td>nominal MSDU size of $f_{ij}$</td>
</tr>
<tr>
<td>$\bar{L}_i$</td>
<td>the weighted average packet size of $\hat{f}_i$</td>
</tr>
<tr>
<td>$L_{ij}[n]$</td>
<td>the accumulated amount of traffic lost up to the end of the $n$th SI</td>
</tr>
<tr>
<td>$N_{ij}$</td>
<td>the number of packet arrivals belonging to $f_{ij}$ in one SI</td>
</tr>
<tr>
<td>$\hat{N}_{ij}$</td>
<td>the estimated number of packet arrivals belonging to $f_{ij}$ in one SI</td>
</tr>
<tr>
<td>$N_i$</td>
<td>the estimated number of packet arrivals belonging to $f_j$ in one SI</td>
</tr>
<tr>
<td>$P_i$</td>
<td>the $i$th packet loss probability requirement</td>
</tr>
<tr>
<td>$P_{ij}[n]$</td>
<td>the running packet loss probability of $f_{ij}$</td>
</tr>
<tr>
<td>Queue$_{ij}$</td>
<td>the queue for buffering data of flow $f_{ij}$</td>
</tr>
<tr>
<td>Queue$_{ij}$</td>
<td>the queue for buffering data of flow $f_{ij}$ which can be buffered up to $p \cdot SI$</td>
</tr>
<tr>
<td>$Q_i[t]$</td>
<td>buffer occupancy in terms of transmission time for Queue$_{ij}$ at the beginning of the $n$th SI</td>
</tr>
<tr>
<td>$Q^p_i[t]$</td>
<td>buffer occupancy in terms of transmission time for Queue$_{ij}$ at the beginning of the $n$th SI</td>
</tr>
<tr>
<td>$X_{ij}$</td>
<td>packet size distribution of $f_{ij}$</td>
</tr>
</tbody>
</table>

References


conference on communications and mobile computing, 12–14 April 2010, Shenzhen, China, 485–489.


After substituting the above equation into Equation (17), we get

\[ L_{ij}[n] + l_{ij}[n] = L_{ij}[n-1] + l_{ij}[n] \]

which implies

\[ l_{ij}[n] = -L_{ij}[n-1] + \left( \frac{P_{i} \cdot A_{ij}[n]}{P_{i} \cdot A_{ij}[n]} \right)(L_{ij}[n-1] + l_{ij}[n]). \]

Summing over all \((r,s) \in U_{active}\) except for \((r,s) = (i,j)\), we have

\[ \sum_{(r,s) \neq (i,j), (r,s) \in U_{active}} l_{rs}[n] = \sum_{(r,s) \neq (i,j), (r,s) \in U_{active}} \left[ -L_{rs}[n-1] + \left( \frac{P_{r} \cdot A_{rs}[n]}{P_{r} \cdot A_{rs}[n]} \right)(L_{rs}[n-1] + l_{rs}[n]) \right]. \]

According to Equation (18), it holds that

\[ \text{Loss}[n] - l_{ij}[n] = \sum_{(r,s) \neq (i,j), (r,s) \in U_{active}} \left[ -L_{rs}[n-1] + \left( \frac{P_{r} \cdot A_{rs}[n]}{P_{r} \cdot A_{rs}[n]} \right)(L_{rs}[n-1] + l_{rs}[n]) \right]. \]

After some manipulations, we get

\[ l_{ij}[n] = \frac{1}{\sum_{(r,s) \neq (i,j), (r,s) \in U_{active}} P_{r} \cdot A_{rs}[n]} \left[ \sum_{(r,s) \neq (i,j), (r,s) \in U_{active}} P_{r} \cdot A_{rs}[n] \cdot \left( \text{Loss}[n] - \sum_{(r,s) \neq (i,j), (r,s) \in U_{active}} L_{rs}[n-1] \right) \right]. \]

**Appendix 2: pseudocode for computing feasible \( l_{ij}[n] \)**

**Algorithm: Loss computation**

**Initialization**

\( U_{temp} = U_{active} \), \( Loss_{temp} = \text{Loss}[n] \), \( Flag = 0 \)

**Begin**

\[ l_{ij}[n] \forall (i,j) \in U_{active} | \text{Loss}[n] = \text{LossComputation}(Loss_{temp}, U_{temp}) \]

**End**

1. **LossComputation(Loss, U)** /*Loss computation module*/

2. WeightedLossCalculation(Loss, U) /*Compute \( l_{ij}[n] \) with Equation. (19)*/

3. if \( 0 \leq l_{ij}[n] \leq Q_{ij}^U[n] \forall (i,j) \in U / \text{Case 1} */

4. exit

5. elseif \( 0 \leq l_{ij}[n] \forall (i,j) \in U \) and \( \exists (i,j) \in U, \)

6. \( l_{ij}[n] > Q_{ij}^U[n] / \text{Case 2} */

7. for all \( (i,j) \in U \)

8. if \( l_{ij}[n] \geq Q_{ij}^U[n] \)
8. $l_{i,j}[n] = \mathcal{Q}_m[i,j]$
9. $U = U - \{(i,j)\}$
10. $\text{Loss} = \text{Loss} - l_{i,j}[n]$  
11. end if
12. end for
13. if Flag = 1
14. Flag = 0
15. exit
16. else
17. LossComputation(Loss, U)
18. end if
19. elseif $l_{i,j}[n] \leq \mathcal{Q}_m[i,j] \forall (i,j) \in U$ and $\exists (i,j) \in U, s.t. l_{i,j}[n] < 0$ /*Case 3*/
20. for all $(i,j) \in U$
21. if $l_{i,j}[n] \leq 0$
22. $l_{i,j}[n] = 0$
23. $U = U - \{(i,j)\}$
24. end if
25. end for
26. LossComputation(Loss, U)
27. elseif $\exists (i,j)$ and $(r,s) \in U, s.t. l_{i,j}[n] > \mathcal{Q}_m[i,j]$ and $l_{r,s}[n] < 0$ /*Case 4*/
28. $V_1 = \{(i,j) \in U : l_{i,j}[n] \geq 0\}$
29. $V_2 = U - V_1$
30. if $\sum_{(i,j) \in V_1} \mathcal{Q}_m[i,j] < \text{Loss}[n]$ /*Sub-case 1*/
31. for all $(i,j) \in U$
32. if $l_{i,j}[n] \geq \mathcal{Q}_m[i,j]$
33. $l_{i,j}[n] = \mathcal{Q}_m[i,j]$
34. $U = U - \{(i,j)\}$
35. $\text{Loss} = \text{Loss} - l_{i,j}[n]$
36. end if
37. end for
38. elseif $\sum_{(i,j) \in V_1} \mathcal{Q}_m[i,j] \geq \text{Loss}[n]$ /*Sub-case 2*/
39. $\text{Flag} = 1$
40. LossComputation(Loss, V_1)
41. if $\text{Flag} = 0$ and $\exists (i,j) \in V_1, s.t. l_{i,j}[n] < \mathcal{Q}_m[i,j]$
42. for all $(i,j) \in V_1$
43. if $l_{i,j}[n] < \mathcal{Q}_m[i,j]$
44. $V_2 = V_2 \cup \{(i,j)\}$
45. else
46. $\text{Loss} = \text{Loss} - \mathcal{Q}_m[i,j]$
47. end if
48. end for
49. LossComputation(Loss, V_2)
50. else
51. for all $(i,j) \in V_2$
52. $l_{i,j}[n] = 0$
53. end for
54. exit
55. end if
56. end if
57. end if