Switchable tunneling mode for cylindrical photonic quantum well consisting of photonic crystals containing liquid crystal

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We propose a cylindrical photonic quantum well made of photonic crystals containing liquid crystals, the properties of which are theoretically calculated and investigated by the transfer matrix method in the cylindrical symmetry system. Liquid crystals are introduced into the photonic quantum well structure as tunable defect layers. When the liquid crystals are pseudo-isotropic state and the azimuthal mode order of incident waves are $m = 0$, there were two pass-bands around certain wavelength. When the liquid crystals are homeotropic state, the reflectance of pass-band at shorter wavelength decreases from 0.75 to 0.05 in the TM mode, but the reflectance does not change in the TE mode. When mode order $m = 1$ and the liquid crystals are pseudo-isotropic state, the reflectance of defect mode stayed the same as $m = 0$. However, the result is reversed while the phase of liquid crystals change from pseudo-isotropic to homeotropic state. The reflectance is the same as in the TM mode, but that in the TE mode decreases substantially from 0.75 to 0.05. The application of our structure to switching device is highly potential.

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1. Introduction
Photonic crystal (PC) is a composite material with permittivity and permeability in one, two or three dimensions periodic structure [1–3]. The studies of PC have attracted a large number of interests for their capability in prohibiting certain frequency-range of electromagnetic (EM) waves from propagating in such structure over the past decade. These frequency regions are called photonic band gaps (PBGs), which is the most prominent characteristic of PCs as tunable defect layers. When the liquid crystals are pseudo-isotropic state and the azimuthal mode order of incident waves are $m = 0$, there were two pass-bands around certain wavelength. When the liquid crystals are homeotropic state, the reflectance of pass-band at shorter wavelength decreases from 0.75 to 0.05 in the TM mode, but the reflectance does not change in the TE mode. When mode order $m = 1$ and the liquid crystals are pseudo-isotropic state, the reflectance of defect mode stayed the same as $m = 0$. However, the result is reversed while the phase of liquid crystals change from pseudo-isotropic to homeotropic state. The reflectance is the same as in the TM mode, but that in the TE mode decreases substantially from 0.75 to 0.05. The application of our structure to switching device is highly potential.

Some tunabilities which will change the position of PBG during operation. This kind of tunable capacity depends on adjustment of the permittivity or permeability of constituent materials by some means.
external parameters such as temperature, external magnetic fields or electric fields. Due to tunable features, PC can be designed and utilized extensively in optoelectronic devices and microwave communication system. One of the prospective materials for fabricating tunable PC is liquid crystals (LCs) due to their characteristics of optical anisotropy and dielectric anisotropy based on the molecular orientation order. Thermal forces are also employed for the reorientation of LC molecules in PC structures to possess the tuning property. The variation of the refractive index due to electro–optical effect can reach up to $\Delta n = 0.2$ which is much larger than the variation cause by thermal forces. However, most works paid attention on thermal tuning effect rather than electrical tuning effect.

In this paper, we focus on the electro–optical effect of LCs and introduce LCs into a cylindrical photonic crystal as defect layers. First, we take advantage of the concept of photonic quantum well (PQW) structure [15–17] to broaden the PBGs for the azimuthal mode number $m = 0$ and $m = 1$. By introducing LCs into the structure, a tunable photonic defect mode will be obtained. Owing to the variation of refractive index of LCs via an applied electric field, a tunable defect mode is observed for the azimuthal mode number $m = 0$ and $m = 1$. However, the feature of defect mode will be reversed for the azimuthal mode number $m = 1$. This interesting phenomenon has not been exploited up to the present in our knowledge.

The rest of this paper is organized as follows: in Section 2, the TMM for cylindrical waves in an annular cylindrical structure was illustrated to calculate the reflectance spectrum for different azimuthal mode. In Section 3, the effect of starting radius is discussed. Then, the concept of PQW structure was used to broaden the PBGs. Besides, LCs were then used to create a tunable feature. Finally, the results of calculation were discussed and then summarized.

2. Theory

The TMM for calculating the reflectance spectrum in cylindrical symmetry structures [8] is briefly described. The method is applied to investigate the propagation of cylindrical EM waves in different polarization and for different azimuthal mode through each cylindrical interface. As illustrated in Fig. 1(a), there is an annular cylindrical structure consists of multilayer media. The boundary of inner core is formed the first layer, where the core radius $r_0$ is also called the starting radius. For the next layer, the label $r_1$ is the radius from the center of structure to the interface between layer 1 and layer 2 and so forth. The refractive index of the core, material 1, material 2 and the surrounding media are represented by $n_0, n_1, n_2, n_3$. We assume that the cylindrical structure has an infinite length from $z = 0$ to $z = \infty$. A linear source is inserted inside the core of starting radius to generate cylindrical EM waves, hence cylindrical EM waves propagate along $\rho$ axis from $\rho = 0$ to $\rho = \infty$, which incident on each interface normally. The reflectance at the interface can be obtained by using TMM in cylindrical symmetry system. Fig. 1(b) and (c) are TE and TM directions on top of the cross-section geometry of cylindrical structure. Generally speaking, Maxwell equation can be rewritten in cylindrical coordinate form as following:

$$\nabla \cdot (\rho \nabla V) - \mu \frac{\partial^2 E_z}{\partial \phi^2} + \epsilon \frac{\partial^2 E_z}{\partial \phi^2} + \epsilon \frac{\partial^2 E_z}{\partial \phi^2} - \mu \nabla \times \nabla \times E_z = 0$$

(1)

This equation is the governing differential equation in the TE mode. To solve Eq. (1), we used the method of separation of variables and let $E_z = V(\rho)\Phi(\phi)$, where $\Phi(\phi) = \exp(\ii m \phi)$. The radial part $V$ satisfies

$$\rho \frac{\partial}{\partial \rho} \left( \rho \frac{\partial V}{\partial \rho} \right) + \left( k^2 \rho^2 - m^2 \right) V = 0$$

(2)

This is Bessel’s equation with the solution

$$V(\rho) = A_m(k\rho) + B_m(k\rho)$$

(3)

where $A$ and $B$ are constant, $J_m$ is a Bessel function, $Y_m$ is a Neumann function, $k = \omega \sqrt{\epsilon_0 \mu_0} \rho / c$ and $m$ is an integer which is referred to as the azimuthal mode number. In the similar way, magnetic field $H_0 = U(\rho)e^{\ii \phi}$ is assumed. We have

$$U(\rho) = \frac{1}{J_m(k\rho)} \frac{\partial V}{\partial \rho} = \frac{k}{J_m}(A'_m(k\rho) + B'_m(k\rho))$$

(4)

Let us rewrite Eq. (4) as

$$U(\rho) = -j\rho(A'_m(k\rho) + B'_m(k\rho))$$

(5)

where $p$ is defined as the intrinsic admittance of the medium.

$$k = \frac{\omega \sqrt{\epsilon_0 \mu_0}}{c} = -j \frac{\sqrt{\epsilon_0}}{\mu} = -jp$$

(6)

Combine Eqs. (3) and (5) to establish the relationship of transfer matrix $M_1$ is expressed as

$$\begin{bmatrix} V(\rho_1) \\ U(\rho_1) \end{bmatrix} = M_1 \begin{bmatrix} V(\rho_0) \\ U(\rho_0) \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \begin{bmatrix} V(\rho_0) \\ U(\rho_0) \end{bmatrix}$$

(7)

each matrix elements of $M_1$ is written as

$$m_{11} = \frac{\pi}{2} k_1 \rho_0 \left[ Y_m(k_1 \rho_0) J_m(k_1 \rho_1) - J_m(k_1 \rho_0) Y_m(k_1 \rho_1) \right]$$

$$m_{12} = j \frac{\pi}{2} k_1 \rho_0 \left[ J_m(k_1 \rho_0) Y_m(k_1 \rho_1) - Y_m(k_1 \rho_0) J_m(k_1 \rho_1) \right]$$

$$m_{21} = -j \frac{\pi}{2} k_1 \rho_1 \left[ Y_m(k_1 \rho_0) J_m(k_1 \rho_1) - J_m(k_1 \rho_0) Y_m(k_1 \rho_1) \right]$$

$$m_{22} = \frac{\pi}{2} k_1 \rho_0 \left[ J_m(k_1 \rho_0) Y_m(k_1 \rho_1) - Y_m(k_1 \rho_0) J_m(k_1 \rho_1) \right]$$

(8)

where $p_1 = \sqrt{\epsilon_1} / \mu_1$. Similarly, we can obtain the transfer matrix $M_i$ for each layer and the total system matrix $M$ can be found by

$$\begin{bmatrix} V(\rho_i) \\ U(\rho_i) \end{bmatrix} = M_i \begin{bmatrix} V(\rho_{i-1}) \\ U(\rho_{i-1}) \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \begin{bmatrix} V(\rho_{i-1}) \\ U(\rho_{i-1}) \end{bmatrix} = M_1 \cdots M_3 M_2 M_1$$

(9)

According to the matrix elements in Eq. (9), we can define the elements of the inverse matrix of $M$ by $M_{11}, M_{12}, M_{21}$ and $M_{22}$. Therefore, we will have the reflection and transmission coefficients as follows for E-polarization:

$$r_d = \frac{M_{12} + \ii p_0 C_{m1}^{C_m} M_{31} - \ii p_0 C_{m2}^{C_m} M_{12} + \ii p_0 C_{m0}^{C_m} M_{22}}{M_{12} + \ii p_0 C_{m1}^{C_m} M_{31} + \ii p_0 C_{m2}^{C_m} M_{12} + \ii p_0 C_{m0}^{C_m} M_{22}}$$

(10)

$$t_d = \frac{4 \epsilon_0 / \mu_0}{\pi k_0 p_0 H_0^{C_m} (k_0 \rho_0) H_0^{C_m} (k_0 \rho_0) - \ii p_0 C_{m0}^{C_m} M_{22}}$$

(11)

where $H_0^{C_m}$ and $H_0^{C_{m2}}$ are the Hankel function of the first and second kind, $K = \omega \sqrt{\epsilon_0 \mu_0}$, $p_0 = \sqrt{\epsilon_0} / \mu_0$ and $C_{m1}^{C_m}$

(12)

Similarly, we exchanged $c$ with $\mu$ and $j$ with $-j$ in the formulations of reflection and transmission coefficient, the corresponding results for H-polarization can be acquired.
In our works, TM method is different from solving partial differential Eq. (1) to include all set of azimuthal mode terms. We investigate transmission properties of each azimuthal mode in TE (TM) mode propagating outward through multilayers. In our work, there are only two azimuthal modes \( m = 0 \) and \( m = 1 \) to be considered, other higher order modes neglected.

3. Results and discussion

To present the numerical results, we first investigate the reflection properties in a single cylindrical PC (CPC) which is comprised of silicon for A and air for B whose filling factor is \( \text{ff} = 0.55 \). To construct the concept of a PQW structure in the cylindrical symmetric system is utilized. At first, two kinds of material constitute cylindrical photonic quantum well (CPQW) structure and the photonic band gaps of type 1 and type 2 from \( \text{ff} = 0.10 \) to \( \text{ff} = 0.95 \) are calculated by the plane wave method (PWM). From band dispersion diagrams, we can set the position of the PBGs in the reflectance spectrum by tuning the filling factor.

Two type of CPQW structures are chosen: type 1 (AB), is constructed of silicon for A and air for B whose filling factor is \( \text{ff} = 0.55 \); type 2 (AC), is constructed of silicon for A and LC for C whose filling factor is \( \text{ff} = 0.69 \) where \( p \) is the number of unit cell, respectively. We choose \( \text{ff} = 0.55 \) for type 1 and \( \text{ff} = 0.69 \) for type 2 because of their PBGs are adjacent. The diagrams that represent type 1 and type 2 PC structure are shown in Fig. 3(a) and (b). Fig. 4(a) and (b) shows the band diagrams of type 1 and type 2 calculated by PWM where the refractive indices are \( n_3 = 3.6, n_{air} = 1 \) and \( n_{LC} = 1.56 \). As indicated in figures, there are many small PBGs in type 1 and type 2. All of PBGs are marked by shaded areas. If we could connect these PBGs together, we will obtain a broaden PBG between normalized frequency 0.15–0.95. The reflectance spectra of the corresponding band diagrams of type 1 and type 2 in the wavelength range 300 nm–1100 nm when the incident wave is E (dashed line) and H polarized (solid line) are shown in Fig. 5(a) and (b) where the starting radius and the number of unit cell are assume to be \( \rho_0 = 80 \) nm and \( p = 8 \). In reflectance spectra, the first PBG of type 1 locates at wavelength \( \lambda = 325 \) nm to \( \lambda = 375 \) nm and the second PBG locates at \( \lambda = 470 \) nm to \( \lambda = 620 \) nm; the first PBG of type 2 locates at \( \lambda < 320 \) nm; the second PBG locates at \( \lambda = 360 \) nm to \( \lambda = 440 \) nm and the third PBG locates at \( \lambda = 540 \) nm to \( \lambda = 690 \) nm. The second PBG of type 2 is between the two PBGs of type 1. Thus, we take advantage of the concept of PQW structure in order to broaden the PBG. We use type 1 as barrier PCs to sandwich type 2 which is treated as a well PC as shown in Fig. 3(c). Fig. 6(a) and (b) shows that the reflectance spectrum of the CPQW (AB)\(_d\) (AC)\(_d\) (AB)\(_d\) in the TE (dashed line) and TM mode (solid line) for azimuthal mode \( m = 0 \) and \( m = 1 \). The CPQW structure which is composed of type 1 with filling factor \( \text{ff} = 0.55 \) and type 2 with filling factor \( \text{ff} = 0.69 \), respectively. The PBG of the CPQW structure is located at the range from \( \lambda = 420 \) nm to \( \lambda = 920 \) nm. It is demonstrated that the PBG is widened successfully not only for mode order \( m = 0 \), but also for \( m = 1 \). However, the PBG in
the TM mode for azimuthal mode \( m = 1 \) was widened larger toward longer wavelength than that in the TE mode as shown in Fig. 6(b). As the starting radius \( r_0 \) is increased gradually, the phenomenon of the PBG in the TM mode which is wider than that in TE mode will gradually vanish. Therefore, the PBG of CPQW structure for \( H \)-polarization wave for azimuthal mode \( m = 1 \) is sensitive to the starting radius but the PBG for \( E \)-polarization wave is insensitive. According to the difference of reflectance spectra between \( TE \) and TM mode cause by small starting radius effect, we set the starting radius \( r_0 = 80 \text{ nm} \) in the following calculations.

Further, we used the property of LCs to create a tunable defect mode. First, we adjusted the filling factor of type 2 from 0.69 to 0.72 in order to match a partial of PBG of type 1 and pass-band of type 2 to create a narrow defect mode. An external electric field \( E = 2.78 \times 10^6 (V/m) \) is applied to the CPQW structure which is used to control the refractive index of the LCs in the simulation. The electric field which is radially outward from the center of the CPQW structure in all direction, the state of the LCs will transform from pseudo-isotropic to homeotropic when the electric field is turned on. At first, the LC is pseudo-isotropic state and the

**Fig. 2.** The schematic represents (a) type 1 PC structure, (b) type 2 PC structure and (c) photonic quantum well structure.

**Fig. 3.** The reflectance versus wavelength with different starting radius for (a) \( E \)-polarization and (b) \( H \)-polarization at azimuthal mode \( m = 0 \), and for (c) \( E \)-polarization and (d) \( H \)-polarization at azimuthal mode \( m = 1 \), respectively. (a) Type 1, (b) Type 2 and (c) Photonic quantum well structure.
molecular orientation of LC is disordered. At this state, the refractive index is \( n_{LC} = \sqrt{2n_o^2 + n_e^2} / C_0 / C_1 = 3.0 \), where the subscripts stand for ordinary and extraordinary, respectively. When the LC is homeotropic state, the molecular orientation of LC will parallel with the direction of applied electric field. Therefore, the refractive index of LC will change from \( n_{LC} \) to \( n_{LC}' \) and the refractive index can be considered as \( n_{LC}' = n_o = 1.49 \), where the subscripts stand for ordinary [18,19]. In Fig. 7(a), the PBG was located at wavelength \( \lambda = 420 \text{ nm} \) to \( \lambda = 910 \text{ nm} \) and there are two pass-bands around \( \lambda = 575 \text{ nm} \). When the electric field was applied, the pass-band at longer wavelength will disappear but something special happened to another pass-band. As indicated in Fig. 7(b), the reflectance of the pass-band at shorter wavelength decreases from 0.75 to 0.05 in the TM mode, but it’s almost the same as in the TE mode. After that, we changed the azimuth mode from \( m = 0 \) to \( m = 1 \), the PBG was nearly invariant as \( m = 0 \) shown in Fig. 8(a). When the external electric field is turned on, the reflectance of the defect mode remains unchanged in the TM mode but the reflectance was decrease substantially from 0.75 to 0.05 in the TE mode. The result of azimuth mode \( m = 1 \) is contrary to azimuth mode \( m = 0 \). If the starting radius is getting larger, the feature will disappear. Therefore, this phenomenon is caused by the effect of small starting radius.

4. Conclusion

In summary, we take advantage of the conception of PQW structure to broaden the PBG of cylindrical PCs. The PBG of our structure was demonstrated to be widened for azimuthal mode \( m = 0 \) and \( m = 1 \). Moreover, we created a defect mode by partially matching the PBG of type 1 and pass-band of type 2, then replaced air with LCs for tunability. According to the property of LCs, the phase of LCs under an applied electric field will change from pseudo-isotropic to homeotropic state and the refractive index will also change from \( n_{LC} = 1.56 \) to \( n_{LC}' = 1.49 \). From reflectance spectrum of CPQW structure with LCs, there are two narrow pass-bands around wavelength...
575 nm when the LCs are pseudo-isotropic and the azimuthal mode \( m = 0 \). If the electric field is turned on, the reflectance of the shorter wavelength pass-band decreases from 0.75 to 0.05 in the TM mode but the reflectance in the TE mode is almost unchanged. For the mode order \( m = 1 \) and the LCs are pseudo-isotropic state, the PBG and the defect mode remain the same as \( m = 0 \). However, the result was reversed when the state of LCs are changed from pseudo-isotropic to homeotropic state. The reflectance of the defect mode in the TM mode is the same as the reflectance of pseudo-isotropic state, but the reflectance in the TE mode decreases substantially from 0.75 to 0.05. This novel feature will become insignificant when the starting radius \( r_0 \) is increased. The notable feature of the switchable tunneling mode may potentially be applied to communication components or optical devices such as sensor, filter and polarizer.

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