Laboratory cranking wave functions and ground-state moments of inertia of heavy deformed nuclei

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Nuclear ground-state moments of inertia for heavy deformed nuclei are calculated using a rotating intrinsic wave function in the laboratory system. Numerical calculations are reported for rare-earth nuclei. The results show that the agreements between the theoretical values and the experimental observations have been improved.

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I. INTRODUCTION

Consider the motion of nuclear rotation as a many-body problem, nearly all of the fully microscopic theories are based on or related to some version of the cranking model of Inglis [1]. A very large amount of progress has been made along the way, see, for instance, [2–9]. Denote the Hamiltonian in the lab system by $H_0$, the semiclassical equation of motion for the nuclear intrinsic state $|n\rangle$ in the rotating frame is given as

$$ (H_0 - \omega j_z) \Phi_n = E_n \Phi_n \ .$$

(1)

The same form of Eq. (1) can also be obtained by a constraint variational method where $H_0 - \omega j_z$ is the auxiliary constrained Hamiltonian. $\Phi_n$ is then a solution of Eq. (1) with the condition $\langle \Phi_n | j_z | \Phi_n \rangle =$ const.

Usually, when a Hartree-Bogoliubov method without any special technique is applied to solve the equation

$$ H_0 \Psi_n = \varepsilon_n \Psi_n \ ,$$

(2)

we obtain a set of independent particle wave functions localized in the lab system. In order to study the rotational effects of the system on the individual particles, the cranking idea can be used, and the problem is reduced to solve Eq. (1) in the rotating frame. However, the physical experiments can only be performed in the lab system. Furthermore, when we are observing the nuclear intrinsic system which is rotating in the lab system, there is no Coriolis force acting on the particles in the nucleus. Therefore, it should be more appropriate if we could describe the nuclear intrinsic system in our lab system. In a recent paper, the author [10] has suggested a method of finding a simplified Hartree-Bogoliubov type of wave function for Eq. (2) to represent a rotating nuclear intrinsic state in the lab system where experiments are performed. The purpose of this work is to apply this wave function to calculate the nuclear ground-state moments of inertia for rare-earth nuclei.

With self-consistent Hartree-Fock wave functions, the cranking model gives results in a rigid-body value for the nuclear moments of inertia which is two to three times larger than that of the experimental observation. It was pointed out by Bohr and Mottelson [11] that residual two-body forces, not included in the one-body self-consistent field, would lower the moment, and that correlations due to pairing would be the most important. Subsequently, Belyaev [12] showed that residual interactions of the pairing type indeed lower the moment of inertia from the rigid-body value through an increased energy denominator and a reduction of the $j_z$ matrix element in the numerator by a factor $(uv' - vu')$, where $u$ and $v$ are coefficients of the Bogoliubov transformation [13]. Numerical calculations were performed by Griffin and Rich [14] and by Nilsson and Prior [15]. With the "best" choice of parameter values, they obtained remarkable agreements with experiments for both the rare-earth and the actinide nuclei. However, the theoretical values for the moments of inertia were systematically still about 20–30% too small on the average. Corrections due to residual interactions were derived by Migdal [16] and Belyaev [17]. Numerical calculations by Mayer et al. [18] showed that the effects of the residual particle-hole and particle-particle interactions on the moments of inertia of all rare-earth nuclei nearly cancelled each other and left the simple cranking value approximately unchanged. Calculations by Birbrair and Nikolaev [19] and by Kam Murray and Kusuno [20] on the same subject did not take into account the effects of rotation.

Mottelson and Valatin [21] observed that, in a rotating reference system, the Coriolis forces act in opposite directions on particles forming a time-reversed pair and tend to decouple the pairing correlations. The authors showed that, in the second-order approximation, the reduction of the pairing interaction energy is proportional to the square of the rotational frequency $\omega$. This is reflected in a decrease in the pairing-gap parameter $\Delta$ and, consequently, an increase in the moment of inertia of the nucleus. As we shall see below, this Mottelson-Valatin type of Coriolis antipairing effect is indeed included in the rotating solution of Eq. (2), which represents the intrinsic state of the nucleus observed in the lab system. For the completeness of this present work, we shall give a brief derivation of the relevant formalism in the following.

II. MOTTELSON-VALATIN EFFECT

To study the Mottelson-Valatin Coriolis antipairing effect, we shall follow an approach derived by Lin and Faessler [22] for studying strongly deformed nuclei in
which state-dependent antipairing forces were treated. In the constrained system, we take a Hamiltonian consisting of an axially symmetric Nilsson potential plus a pairing force, and a constraint Coriolis term, neglecting terms connecting \( m = \pm \frac{1}{2} \) states in the matrix elements of \( j_x \). Numerical calculations show that the contribution of these terms to the moments of inertia should be no more than 5\%. This would not change the final results significantly. This approximation reduces the sizes of matrices to be diagonalized by a factor of 2. Furthermore, it introduces a twofold degeneracy which allows a separation of the single-particle space into a space \( | \alpha \rangle \) with positive projection \( m_\alpha > 0 \), and a space \( | \bar{\alpha} \rangle \) with \( m_\alpha < 0 \). In this approximation, the constrained Hamiltonian is given as

\[
H = \sum_{\alpha > 0} (\epsilon_\alpha - \lambda)(C_\alpha^\dagger C_\alpha + C_\alpha C_\alpha^\dagger) - \frac{G}{4} \sum_{\alpha, \beta} C_\alpha^\dagger C_\beta C_\alpha C_\beta \]

\[-\hbar \omega \sum_{\alpha, \beta > 0} \langle \alpha | j_x | \beta \rangle (C_\alpha^\dagger C_\beta - C_\beta C_\alpha^\dagger) . \tag{3}
\]

Here \( \alpha \) and \( \bar{\alpha} \) are the Nilsson state and its time-reversal conjugate, \( \epsilon_\alpha \) the single-particle energy, \( \lambda \) the Fermi energy, \( G \) the pairing strength, and \( \omega \) the angular frequency. We use Greek subscripts for states in the Nilsson representation.

In order to take into account the Coriolis force directly, we first diagonalize the Nilsson part and the Coriolis force of the Hamiltonian. Let the transformation be

\[
C_\alpha = \sum_i B_{\alpha i} b_i, \quad C_\alpha = \sum_i B_{\bar{\alpha} i} b_i . \tag{4}
\]

We designate states in the \( b \) representation by Roman subscripts. It can be shown by simple matrix algebra that the eigenvalues \( \epsilon_i \) and \( \epsilon_i \) are equal and

\[
B_{\alpha k} = B_{\bar{\alpha} k} \text{ for } \alpha = k ,
\]

\[
B_{\alpha k} = -B_{\bar{\alpha} k} \text{ for } \alpha \neq k ,
\]

\[
B_{\bar{\alpha} k} = B_{\alpha k}^2 . \tag{5}
\]

This greatly simplifies the formalism and the numerical calculation. It enables us to write the Hamiltonian in the \( b \)-representation as

\[
H = \sum_{k > 0} (\epsilon_k - \lambda)(b_k^\dagger b_k + b_k b_k^\dagger)
\]

\[-G \sum_{ijk \neq 0} R_{ij} R_{ki} b_i^\dagger b_j^\dagger b_k b_i , \tag{6}
\]

where

\[
R_{ij} = \sum_{\alpha > 0} B_{\alpha i} B_{\alpha j}^* ,
\]

\[
R_{ij} = \sum_{\alpha > 0} B_{\alpha i} B_{\alpha j} . \tag{7}
\]

The \( R_{ij} \)'s have been computed as a function of \( \omega \). The result indicates that \( R_{ij} \) is strongly state dependent, and that \( R_{ii}^2 \gg R_{ij} \) by an order of magnitude for rotational frequency up to the backbending region corresponding to \( \hbar \omega \approx 0.3 \text{ MeV} \). There is no guarantee that \( R_{kk} \)'s are all positive definite, but this can be simply accomplished by redefining the phase of the single-particle states in the \( b \) representation.

When the rotation frequency \( \omega \) is zero, the transformation (4) is a unity transformation with \( B_{\alpha k} = \delta_{\alpha k} \) for all \( k \). For small \( \omega \), the \( R_{kk} \)'s are likewise small for \( \alpha \neq k \). From the orthonormalization conditions for the transformation (4) and the relation (5), we have

\[
R_k = R_{kk} = 1 - 2 \sum_{\alpha \neq k} B_{\alpha k}^2 . \tag{8}
\]

Therefore, as long as \( \omega \) remains sufficiently small, all the \( R_{kk} \)'s are positive.

From the above consideration, we can neglect all small terms in the Hamiltonian (6) so that it can be reduced to a standard form in the constrained system as

\[
H = \sum_{k > 0} (\epsilon_k - \lambda)(b_k^\dagger b_k + b_k b_k^\dagger)
\]

\[-G \sum_{i, k \neq 0} R_{ik} R_{ki} b_i^\dagger b_j^\dagger b_k b_i . \tag{9}
\]

The neglected terms can be included in the residual interaction which can be taken into account when we are improving the Hamiltonian. For small \( \omega \), the transformation coefficients \( B_{\alpha i} \) of Eq. (4) can be expanded in terms of \( \omega \) by perturbation method as

\[
B_{ik} = (i j \omega | k \rangle \omega \epsilon_i - \epsilon_k , \tag{10}
\]

\[
B_{i k} = (i j \omega | k \rangle \omega \epsilon_i - \epsilon_k .
\]

Using Eq. (8), we have

\[
R_k = 1 - 2 \sum_{i \neq k} \left[ (i j \omega | k \rangle \omega \epsilon_i - \epsilon_k \right]^2 . \tag{11}
\]

With Eq. (11), take the average over states for \( R_i R_k \), and sum up terms to the second order in \( \omega \), the Hamiltonian (9) is then given as

\[
H = \sum_{k > 0} (\epsilon_k - \lambda)(b_k^\dagger b_k + b_k b_k^\dagger) - \tilde{G} \sum_{i, k \neq 0} b_i^\dagger b_j^\dagger b_k b_i , \tag{12}
\]

where

\[
\tilde{G} = G \left\{ 1 - 4(\hbar \omega)^2 \sum_k \left( \frac{i j \omega | k \rangle \omega}{\epsilon_i - \epsilon_k} \right)^2 \right\} . \tag{13}
\]

This is the well-known Mottelson-Valatin Coriolis antipairing effect for a rotating system.

It is standard procedure to diagonalize the Hamiltonian (12) by the quasiparticle approximation. The results are the following:
\[ H = U_0 + \sum_{k>0} E_k (a_k^\dagger a_k + a_k^\dagger a_k^\dagger) , \]  
(14)

\[ U_0 = \sum_{k>0} [\eta_k + \frac{1}{2} \mu] v_k^2 - \frac{1}{2} u_k v_k \Delta ] , \quad E_k = \sqrt{\eta_k + \Delta^2} , \]

Note that the states \(|i\rangle\) and \(|k\rangle\) are single-particle states in the constrained frame and, to first order in \(\omega\), we have
\[ \langle i | j \chi | k \rangle = \langle i | j \chi | k \rangle \quad \text{for} \quad i = k , \]  
(22)
\[ \langle i | j \chi | k \rangle = -\langle i | j \chi | k \rangle \quad \text{for} \quad i \neq k . \]  
(23)

In the constrained frame, \(j \chi\) has nonvanishing diagonal matrix elements.

To diagonalize the Hamiltonian (17), we shall follow a method used by [23] and write the equations of motion for \(a_i, a_i^\dagger\) (\(i = 1, 2, \ldots, N\), where \(N\) is the dimensionality of the single-particle space \(|i\rangle\)),
\[ [a_i, H_0] = E_i a_i + \omega \sum_{k>0} (R_{ik} a_k + T_{ik} a_k^\dagger) , \]  
(24a)
\[ [a_i^\dagger, H_0] = -E_i a_i^\dagger + \omega \sum_{k>0} (T_{ki} a_k - S_{ki} a_k^\dagger) . \]  
(24b)

When we use a real representation for \(j \chi\), Eq. (24) is a symmetrical matrix equation for the vector \((a_1, a_2, \ldots, a_N, a_1^\dagger, a_2^\dagger, \ldots, a_N^\dagger)\). It can thus be diagonalized by an orthonormal transformation. Let the normal modes be written as
\[ \beta_i = \sum_{k>0} (f_{ik} a_k + g_{ik} a_k^\dagger) , \]  
(25a)
\[ \beta_i^\dagger = \sum_{k>0} (f_{ik} a_k^\dagger + g_{ik} a_k) . \]  
(25b)

The coefficients, \(f_i\)'s and \(g_i\)'s, can be determined by a simple diagonalization program for a real symmetric matrix. The Hamiltonian \(H_0\) can be written in the \(\beta\) representation as
\[ H_0 = W_0 + \sum_{k>0} W_k (\beta_k^\dagger \beta_k + \beta_k \beta_k^\dagger) , \]  
(26)

where \(W_k\) is the eigenvalue of the \(k\)th normal mode of the matrix in Eq. (24), and \(W_0\) is a constant of the new ground-state energy in the laboratory system which is
\[ W_0 = U_0 + \omega \cdot J_\chi + \omega \cdot \sum_{i,k>0} \left[ E_i (g_{ik}^2 + g_{ik}^2) + g_{ik}^2 \right] + T_{ik} \sum_{j>0} (g_{ji} f_{jk} - f_{ji} g_{jk}) . \]  
(27)

The ground state \(|\Psi_0\rangle\) of \(H_0\) (for an even-mass system) is defined as
\[ \beta_k |\Psi_0\rangle = 0 \quad \text{for any} \quad k , \]  
(28)

and accordingly, we have
\[ |\Psi_0\rangle = A \prod_k (\beta_k \beta_k^\dagger) |0\rangle , \]  
(29)
where $A$ is the normalization constant. When $\omega = 0$, $|\Psi_0\rangle$ is reduced to the Nilsson-BCS ground state (without rotation), and $W_0 = \mathcal{U}_0(\omega = 0)$, the Nilsson-BCS ground-state energy. $|\Psi_0\rangle$ can be expressed in terms of the basis states of the constrained system as

$$|\Psi_0\rangle = |\Phi_0\rangle + \omega \cdot (2 \text{ qp states}) + \omega^2 \cdot (4 \text{ qp states}) + \cdots.$$  

(30)

IV. NUCLEAR GROUND-STATE CRANKING MOMENT OF INERTIA

Suppose the nuclear Hamiltonian $\mathcal{H}$ can be transformed into an intrinsic part $H_0$, a rotational part $H_r$, and a coupling term $H_c$

$$\mathcal{H} = \frac{R^2}{2\mathcal{F}} + H_0 + H_c,$$  

(31)

where $\mathcal{F}$ is an operator involving the intrinsic coordinates of the various particles and $H_0$ depends only on the intrinsic particles. We are now considering the nuclear structure near the ground state, the coupling term $H_c$ can be neglected. To find the connection between the intrinsic wave functions $\Psi_n$ and the moment of inertia, we follow a derivation of de Shalit and Feshback [24], introduce an angle variable $\phi$ depending on the coordinates of all the particles and let $\phi$ satisfy the following assumptions.

(a) The angle $\phi$ describes a collective orientation of the nucleus in the sense that it commutes with all intrinsic coordinates

$$[\mathcal{F}, \phi] = [H_0, \phi] = 0.$$  

(32)

(b) $\phi$ is the conjugate variable to the collective angular momentum $\mathcal{R}$ in the sense that

$$[\mathcal{R}, \mathcal{F}](\phi) = -i\mathcal{F}'(\phi).$$  

(33)

(c) $[\mathcal{R}, \mathcal{F}] = 0$, 

$$[H_0, \mathcal{F}] = 0.$$  

(34)

With these assumptions, de Shalit and Feshback [24] have shown that the moment of inertia of the ground state $|\Psi_0\rangle$ is given as

$$\langle \Psi_0|\mathcal{F}|\Psi_0\rangle = 2\hbar^2 \sum_{W_n \neq W_0} \left| \frac{\langle \Psi_0|R|\Psi_n\rangle}{W_n - W_0} \right|^2,$$  

(35)

where $R$ is $j_x$ because it is a rotation around the direction perpendicular to the nuclear symmetry axis. Substitute Eq. (29) and $|\Psi_n\rangle = \beta^n_1\beta^n_k|\Psi_0\rangle$ into (35) we have

$$\langle \Psi_0|\mathcal{F}|\Psi_0\rangle = 2\hbar^2 \sum_{i, k > 0} \frac{|\langle \Psi_0|j_x\beta^n_1\beta^n_k|\Psi_0\rangle|^2}{W_i + W_k}.$$  

(36)

For small $\omega$, all the quantities can be expressed in terms

| TABLE I. Theoretical and experimental values of the ground-state moments of inertia for rare-earth nuclei. |
|---|---|---|---|---|---|---|---|
| Nuclide | $\frac{2}{3} \theta_{\exp}$ (MeV) | $G_n \times A$ (MeV) | $G_p \times A$ (MeV) | $\Delta_n$ (MeV) | $\Delta_p$ (MeV) | $\frac{2}{3} \theta_{\text{th}}$ (MeV) | $\frac{2}{3} \theta_{\text{th}}$ (MeV) |
| Sm | 152 | 49.2 | 216.24 | 16.63 | 11.08 | 1.083 | 40.3 | 0.026 | 0.982 | 1.017 | 48.9 |
| | 154 | 73.2 | 216.22 | 19.83 | 1.009 | 1.015 | 53.0 | 0.034 | 0.929 | 0.921 | 56.0 |
| Gd | 154 | 48.8 | 16.46 | 19.59 | 1.131 | 1.082 | 37.0 | 0.026 | 0.983 | 0.986 | 45.5 |
| | 156 | 67.4 | 16.45 | 19.79 | 1.051 | 0.021 | 49.4 | 0.031 | 0.941 | 0.924 | 52.6 |
| | 158 | 75.5 | 16.43 | 19.98 | 0.978 | 1.004 | 55.3 | 0.034 | 0.983 | 0.902 | 62.3 |
| | 160 | 79.7 | 16.42 | 20.18 | 0.903 | 1.009 | 58.7 | 0.039 | 0.787 | 0.882 | 68.9 |
| Dy | 160 | 69.0 | 16.65 | 19.94 | 1.011 | 1.029 | 47.7 | 0.031 | 0.772 | 0.914 | 62.9 |
| | 162 | 74.4 | 16.64 | 20.14 | 0.922 | 1.024 | 54.2 | 0.035 | 0.764 | 0.893 | 66.9 |
| | 164 | 81.8 | 16.63 | 20.33 | 0.839 | 1.027 | 59.2 | 0.042 | 0.652 | 0.856 | 75.3 |
| Er | 164 | 66.7 | 16.86 | 20.10 | 0.954 | 1.046 | 50.9 | 0.034 | 0.726 | 0.918 | 67.0 |
| | 166 | 74.5 | 16.84 | 20.29 | 0.870 | 1.050 | 56.8 | 0.042 | 0.632 | 0.879 | 76.7 |
| | 168 | 75.2 | 16.83 | 20.48 | 0.847 | 1.062 | 56.9 | 0.044 | 0.605 | 0.889 | 76.6 |
| | 170 | 75.6 | 16.82 | 20.67 | 0.844 | 1.079 | 56.6 | 0.044 | 0.625 | 0.872 | 76.5 |
| Yb | 170 | 71.2 | 17.05 | 20.44 | 0.892 | 1.111 | 52.6 | 0.037 | 0.715 | 0.965 | 66.8 |
| | 172 | 76.2 | 17.03 | 20.63 | 0.870 | 1.121 | 54.4 | 0.041 | 0.666 | 0.948 | 72.0 |
| | 174 | 78.5 | 17.02 | 20.82 | 0.860 | 1.130 | 54.5 | 0.043 | 0.641 | 0.945 | 73.6 |
| | 176 | 73.1 | 17.01 | 21.00 | 0.898 | 1.139 | 50.8 | 0.039 | 0.725 | 0.973 | 64.6 |
| Hf | 176 | 67.9 | 17.23 | 20.78 | 0.910 | 1.156 | 48.2 | 0.038 | 0.710 | 0.981 | 64.9 |
| | 178 | 64.4 | 17.22 | 20.97 | 0.927 | 1.155 | 44.5 | 0.038 | 0.682 | 0.935 | 62.2 |
| | 180 | 64.3 | 17.21 | 21.15 | 0.967 | 1.158 | 40.5 | 0.044 | 0.536 | 0.873 | 70.3 |
| W | 182 | 60.0 | 17.42 | 21.12 | 1.010 | 1.130 | 36.5 | 0.041 | 0.602 | 0.834 | 62.4 |
| | 184 | 54.1 | 17.41 | 21.30 | 1.041 | 1.126 | 32.4 | 0.036 | 0.776 | 0.855 | 49.7 |
| | 186 | 49.0 | 17.40 | 21.48 | 1.059 | 1.129 | 28.8 | 0.035 | 0.806 | 0.844 | 44.6 |
of functions of $\omega$ [23]. It can be shown in a straightforward manner that by taking only the lowest-order terms of (36) we have

$$\langle \Psi_0 | \mathcal{F} | \Psi_0 \rangle = 2\hbar^2 \sum_{i, k > 0} \frac{|u_i u_k \tilde{j}_k + u_k u_i \tilde{j}_k|^2}{E_i + E_k}.$$ (37)

V. NUMERICAL CALCULATIONS, RESULTS, AND DISCUSSION

In performing numerical calculations, we take an axially symmetric Nilsson model from [25] and neglect the $P_4$ force. The major shells included are $N = 3, 4, 5,$ and 6 for protons and $N = 4, 5, 6,$ and 7 for neutrons. This larger dimensionality is important for the calculation since otherwise the single-particle basis states may not be complete enough. Especially for rare-earth nuclei with higher mass number, the occupation of single-particle orbitals may easily go up to $j_{15/2}$. For the deformation parameter $\delta$ we take from [15]. For the pairing strength parameter $G$ we use the prescription of [25] as

$$G \times A = g_0 \pm g_1 \frac{N - Z}{A},$$ (38)

where the plus sign holds for protons and the minus sign for neutrons. In [25] the authors also emphasize the surface dependence of the pairing strength $G$ as the following. In order to reproduce the indicated large energy gap at the fission saddle point [26] one must assume [27] an increase of $G$ with surface area. For a small deviation in $G$ from a “normal” value $G_0$, they put

$$G = G_0 + \delta G \approx G_0 + G_0 \frac{\delta S}{S},$$ (39)

where $S$ is the surface area of the nucleus. If the nature of Eq. (39) exists for the $G$ in a nucleus, there is no reason that it would not exist for nuclei with different mass number $A$, even the effect is small. We shall suggest that this small variation of $G$ should be included in Eq. (38) with $S \sim A^{2/3}$.

The general form of this dependence of $G$ on $S$ may be complicated which may depend on the shell structure. However, we are not only interested in making an estimated correction of this dependence within a region of nuclei which the shell structure of those nuclei are similar. The simplest way is to directly take Eq. (39) and obtain

$$G \times A = \left[ g_0 \pm g_1 \frac{N - Z}{A} \right] \left( 1 + g_2 \frac{A^{2/3} - A_0^{2/3}}{A_0^{2/3}} \right).$$ (40)

It was found that, with our single-particle basis space, we could reproduce reasonably well the empirical odd-even mass differences by using Eq. (40) from $A = 152$ to 189. The values of $G \times A$, $\Delta N$ and $\Delta \rho$ for rare-earth nuclei when $\omega = 0$ are given in Table I, using $g_0 = 18.7$ MeV, $g_1 = 9.6$ MeV, and $g_2 = 0.6$.

The experimental values of nuclear moments of inertia $\Theta$ are extracted according to

$$2\Theta/h^2 = \frac{\Delta[J(J + 1)]}{\Delta E(J)}.$$ (41)

Since only integral values of $J$ can exist and $\Delta J = 2$, Eq. (41) for ground-state moments of inertia becomes

$$2\Theta/h^2 = \frac{2(2 + 1)}{E(J = 2) - E(J = 0)}.$$ (42)

For theoretical calculation, we cannot solve the complete Hamiltonian $\mathcal{H}$ of Eq. (31) to calculate the moment of inertia for Eq. (42). We have to use Eq. (37) through the use of the intrinsic wave function $\Psi_0$. For $\omega = 0$, we have $(\tilde{j}_k) = 0$, there is no rotation and it is meaningless to talk about moment of inertia. Therefore we calculate (37) by using the ground state with the lowest $J$, which is $J = 2$, to find the corresponding $\Theta$ for Eq. (42). That is, we have to find the value of $\omega$ such that we have the

![Graph](image_url)

**FIG. 1.** Comparison of the calculated ground-state cranking moments of inertia and the experimental observations for rare-earth nuclei.
ground state $\Psi(\omega, J = 2)$ of the intrinsic system. To do this, we construct a $\Psi(\omega)$ and calculate $\langle j_\omega | \Psi \rangle$. Then we vary $\omega$ until $\langle j_\omega | \Psi \rangle = 2$ if the magnitude of the angular moment is $\sqrt{J(J + 1)}$, its maximum projection in one direction is $J$.

The calculated values of the $\Delta$'s and $\Theta$'s for rare-earth nuclei at $\omega = 0$ and $\omega = \omega(J = 2)$, together with the experimental values for $\Theta$ are shown in Table I. Those values for $\Theta(\omega)$, $\Theta(0)$, and $\Theta(\text{exp})$ are shown in Fig. 1. In Table I, we can see, as it is expected, that the values of $\Delta$'s are decreased as a result of the Mottelson-Valatin effect due to rotation. We can also see that the overall behavior of the calculated values of ground-state moments of inertia for rare-earth nuclei have actually been improved. If one wishes to improve the calculation further, solutions of $\mathcal{H}$ with good angular momenta and energies should be obtained by using angular momentum projection and $\Theta$ should be calculated according to Eq. (41). In this way, the results should be, in principle, better and quantitatively more accurate. However, all of the quantities would be computed numerically and the simple analytical form of the cranking formula would no longer exist. Nevertheless, a treatment like the present work can be considered as an approximation method which has more analytical aspects of a problem such that we can use it easier to study the physical qualitative features of a system.

ACKNOWLEDGMENT

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[10] L. Lin, Nucl. Phys. A 570, 211C (1994). (In this reference, the equation number $N$ should be corrected as $N \rightarrow 1$ for $3 \leq N \leq 8$.)