A method for measuring the concentration of a solution
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ABSTRACT
When a linearly polarized light beam enters a surface-plasmon-resonance (SPR) apparatus and is incident on the boundary surface with tested solution at the resonant angle, the phase difference between s- and p- polarizations is changed with the concentration of the solution. And the phase difference can be measured accurately by the heterodyne interferometry. Based on these effects, a method for measuring the concentration of a solution is presented.

Keywords: Surface-plasmon resonance, Heterodyne interferometry, Concentration.

SUMMARY
A schematic diagram of our method is designed and shown in Fig. 1. The heterodyne light source consists of a linearly polarized laser, a half-wave plate H, and an electro-optical modulator EO driven by a function generator FG. The angular frequency difference between s- and p- polarizations is $\omega$. A light beam coming from this heterodyne light source enters the SPR apparatus being an isosceles right-angle prism with a thin metal film deposited on the hypotenuse surface being contacted with the test solution. If the light beam is incident at the resonant angle $\theta_0$ on this boundary surface, then the reflection coefficients $r_p$ and $r_s$ of p- and s- polarizations of the reflected light can be derived from the Airy’s formulas, and can be written as

$$r_q = \frac{r_{10}^d + r_{20}^d e^{2i\phi_d}}{1 + r_{00}^d r_{12}^d e^{2i\phi_s}} = |r_q| e^{i\phi_q} \quad (q = p, s),$$

where $r_{01}$ and $r_{12}$ are the reflection coefficients of prism-metal boundary and metal-solution boundary, respectively, $d$ is the thickness of metal film, $k$ is the wave vector in metal, $\phi$ and $\phi_s$ are the phases of s- and p- polarizations of the reflected light, and $\phi$ is the phase difference between s- and p- polarizations, respectively.

From these equations, it is clear that $\phi$ is dependent on the refractive index of the tested solution and the refractive index is related to its concentration. Consequently it can be seen that $\phi$ is dependent on the concentration. Next the reflected light passes an analyzer AN with the transmission axis at $\alpha$ to the horizontal axis and is detected by a photodetector D. Then the intensity measured by D is the test signal and can be derived as

$$I_t = |E_t|^2 = \frac{1}{4} \left[ r_p^2 \cos^2 \alpha + r_s^2 \sin^2 \alpha + 2 r_p r_s \cos \alpha \sin \alpha \cos (\phi_d + \phi) \right].$$

On the other hand, the electrical signal generated by the function generator FG is filtered and becomes the reference signal. It has the form as

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\[ I_s = \frac{1}{2}[1 + \cos(\omega t)]. \]  

Both of these two sinusoidal signals are sent to a phase meter PM, \( \phi \) can be measured accurately. The concentration can be estimated from its associated data of \( \phi \) with the specified relation curve of phase difference versus concentration.

In order to demonstrate the feasibility of this method, an SPR apparatus consisting of a BK7 prism and a thin gold film of thickness 35nm to measure the concentration of glucose solution. Their refractive indices are measured with an ellipsometer and they are 1.5151 and 0.1973+i3.5631, respectively. A high-resolution rotation stage with angular resolution 0.001° is used to mount the SPR apparatus. A phase meter with angular resolution 0.01° is used and a personal computer is used to record and analyze the data. The frequency difference of the heterodyne light source is 1 kHz and \( \theta_{\text{sp}} \) equals to 71.03°. The measured results and its fitting curve are shown in Fig. 2. And the concentration of other glucose solution can be estimated from its associated data of \( \phi \) with this fitting curve.

Because \( r_p \) is very small in this method, so the fast axis of the half-wave plate should be located moderately to enhance the contrast of the test signal. Considering the angular resolution of the phase meter, second harmonic error, and polarization-mixing errors, the angular resolution is decreased to is 0.03°, the sensitivity of our method is \( 3.3 \times 10^2 \) (mg/ml). This method has the advantages of both the common-path interferometry and the heterodyne interferometry, such as high stability, high resolution, and easy operation.

REFERENCES