A subwaveform threshold retracker for ERS-1 altimetry: A case study in the Antarctic Ocean

Yuande Yang, Cheinway Hwang, Hung-Jui Hsu, Dongchen E, Haihong Wang

A subwaveform threshold retracker for ERS-1 altimetry is developed and coded in FORTRAN for satellite altimetry to determine the leading edge and retracking gate, and to improve the precision of sea surface heights (SSHs) and gravity anomalies (GAs). Using ERS-1/ERM waveforms, the subwaveform threshold retracker outperforms full-waveform threshold retrackers at the tide gage Port Station. A direct comparison between retracked SSHs and in situ SSHs is made at tide gage Port Station. Here the subwaveform retracking improves SSH precision from 0.241 to 0.193 m, yielding an improvement percentage (IMP) of 20%. Using ERS-1/GM waveforms, the subwaveform threshold retracker outperforms the Beta-5 and full-waveform threshold retrackers over the Bellingshausen and Amundsen Seas (BAS) in the Antarctic Ocean. The standard deviations of raw and retracked SSHs are 0.157 and 0.070 and 1.836 and 0.220 m over the ice-free and ice-covered oceans, corresponding to IMPs of 54.4% and 88%, respectively. Use of retracking improves the precision of GAs by up to 46.6% when comparing altimeter-derived and shipborne GAs.

1. Introduction

Satellite altimetry has been widely used in many disciplines of Earth science. A summary of altimetric theories and applications is given by Fu and Cazenave (2001). For a pulse-limited radar, the return waveform is the basic measurement. The waveform is used to derive the range between the satellite antenna and the Earth's surface, which in turn yields surface topography at sea and land. Over oceans, the radar ranging accuracy can normally meet the mission-required accuracy due to the reflecting surface of the ocean that result in an ideal waveform, i.e., the Brown waveform (Brown, 1977; Sandwell and Smith, 2005). The ranging accuracy is quickly degenerated as the observation is near coasts or over nonocean surfaces, largely due to waveform contamination (Deng, 2003; Deng et al., 2003; Deng and Featherstone, 2006; Hwang et al., 2006).

Over oceans, the waveform contamination can happen not only near coastal areas, but also over areas covered with sea ice. A postprocessing technique, known as waveform retracking, can be used to retrack the corrupted waveform and in turn improve the ranging accuracy of altimeter-derived sea surface height (SSH). For geodetic and geophysical applications, SSHs from altimetry are often used to derive gravity anomalies (GAs). For example, Brooks et al. (1997), Sandwell and Smith (2005), Deng and Featherstone (2006), Hwang et al. (2006), and Sandwell and Smith (2009) show that waveform retracking can improve the accuracies of SSHs and GAs over both open and shallow waters.

Several algorithms have been developed to retrack waveforms over different reflecting surfaces, such as land/sea ice, land, and coastal waters (Gommenginger et al., 2011). For example, the Beta retracker (Martin et al., 1983), the threshold retracker (Wingham et al., 1986), and the surface/volume retracker (Davis, 1993) have been used over ice. A review of waveform retracking methods for different reflecting surfaces can be found in Deng and Featherstone (2006). These algorithms are based on either a statistical model or a deterministic model.

This paper presents a subwaveform retracker to compute range corrections for satellite altimetry. A FORTRAN computer program is developed to implement this retracker. This retracker first identifies the leading edge based on subwaveform correlation analysis, and then computes the retracking gate using a threshold method. This retracker will be compared with the Beta-5 and threshold retrackers to assess its performance in the Antarctic Ocean. Improvements in SSHs and GAs due to retracking by this method will be presented.
2. Algorithm for leading edge determination and retracking

2.1. Diffuse and specular waveforms

Since our case study will be carried out in the Antarctic Ocean, a waveform classification is presented here. A waveform can be specular over an ice-covered ocean and diffuse over an ice-free ocean. A specular waveform is characterized by an initial sharp rise, followed by a rapid fall off in power. For a diffuse waveform, the rise of the leading edge and the trailing edge depends largely on a significant wave height (SWH). Fig. 1 shows a typical specular waveform and a typical diffuse waveform of ERS-1. The peak power of a specular waveform can be up to 3 orders of magnitude greater than that of a diffuse waveform (Laxon, 1994; Peacock and Laxon, 2004). A waveform classification is to distinguish specular waveforms from diffuse ones. In the classification, the pulse peakiness (PP) is computed as (Peacock and Laxon, 2004). A waveform classification is to distinguish specular waveforms from diffuse ones. In the classification, the pulse peakiness (PP) is computed as (Peacock and Laxon, 2004; Lee, 2008)

\[ PP = \frac{31.5 \times P_{\text{max}}}{\sum_{i=5}^{14} P(i)} , \]  

where \( P_{\text{max}} \) is the waveform peak power, and \( P(i) \) is the power of the \( i \)th gate. A waveform with PP < 1.8 is regarded as a diffuse waveform (Peacock and Laxon, 2004); otherwise it is a specular one. Over oceans, this classification can be used to distinguish the ice-free area from the ice-covered area; see the case study in Section 4.

2.2. Brown waveform model and waveform correlation

The return power of a Brown waveform, \( P(t) \), can be expressed as (Brown, 1977; Sandwell and Smith, 2005)

\[ P(t) = A \left[ \text{erf} \left( \frac{t-\tau}{\sqrt{2} \sigma} \right) + 1 \right] \left\{ \begin{array}{ll} 1 & t < \tau \\ \exp(-t-\tau/\sigma) & t \geq \tau \end{array} \right. \]  

where \( A \) is the amplitude of the power, \( \sigma \) is associated with the slope of the leading edge governed by SWH, \( \tau \) is the time of gate, \( \alpha \) is the center of the leading edge, \( \lambda \) is an exponential decay parameter in the trailing edge, and \( \text{erf} \) is the error function. For the ERS-1 waveform, \( \alpha \) can be regarded as a constant (137 ns) (Sandwell and Smith, 2005). Therefore, the parameters \( A, \tau, \) and \( \sigma \) govern the shape of the waveform. The rise width \( \sigma \) is a convolution of the effective width of the point target response and the vertical distribution of ocean surface waves, usually parameterized in terms of SWH. For a theoretical ERS-1 waveform, \( \tau \) is 32.5 in dimensionless unit of sample gate width and can be converted to time by 3.03 ns (Fu and Cazenave, 2001). Therefore, a waveform shape with \( A=1 \) is determined by the parameter \( \sigma \). Correlation is a statistical method used to describe the dependence between two observed arrays. This method is adapted to analyze the relationship between two waveforms. A correlation coefficient is computed as

\[ r = \frac{S_{xy}}{\sqrt{S_{xx} S_{yy}}} \]  

with

\[ S_{xx} = \frac{1}{k-1} \sum_{i=1}^{k} (P_r(i) - \bar{P}_r)^2, \]  

\[ S_{yy} = \frac{1}{k-1} \sum_{i=1}^{k} (P_x(i) - \bar{P}_x)^2, \]  

\[ S_{xy} = \frac{1}{k-1} \sum_{i=1}^{k} (P_r(i) - \bar{P}_r)(P_x(i) - \bar{P}_x), \]  

where \( P_r(i) \) and \( P_x(i) \), \( i=1, ..., k \) are the return powers of the reference waveform and an arbitrary waveform, respectively. \( \bar{P}_r \) and \( \bar{P}_x \) are the average powers, \( S_{xx} \) and \( S_{yy} \) are the standard deviations of the powers, and \( S_{xy} \) is the covariance of the two time series of powers from the reference and arbitrary waveforms. For an ERS-1 waveform, \( k \) is 64. A waveform is composed of thermal noise, leading edge, and trailing edge. In these three parts, the samples of the leading edge are more accurate than the other
parts. The sharp increase of return power in Fig. 1 corresponds to
the leading edge of the waveform. This leading edge is over a
subwaveform of this full waveform. The objective of our analysis
is to derive the leading edge to reduce the error in the estimated
arrival time of the pulse, hence to improve the precision of SSHs
and GAs. This is achieved in four steps. The first is to obtain an
accurate reference leading edge from the Brown model. Then, the
subwaveform correlation is used to derive the optimal subwave-
form. Third, the leading edge is determined after analyzing the
optimal subwaveform. Finally, the retracking correction is derived
from the leading edge with the threshold retracking.

2.3. Determining the leading edge by matching with a reference
subwaveform
The determination the leading edge of a waveform is critical to
the correlation method. Two experiments are done with theore-
etical ERS-1 waveform data from the Brown model: experiment ‘A’
does the subwaveform correlation; experiment ‘B’ is to analyze the
affect of on subwaveform correlation. Fig. 2 shows the
theoretical waveforms of ERS-1 based on Eq. (2), with SWHs
ranging from 1 to 19 m at a 2-m interval. The slope of the leading
does the subwaveform correlation; experiment ‘B’ is to analyze
the affect of on subwaveform correlation. Fig. 2 shows the
theoretical waveforms of ERS-1 based on Eq. (2), with SWHs
ranging from 28.5 to 36.5 at a 1-s interval and SWH=5 m. The
CCs between the reference subwaveform and a moving subwave-
form are computed. The reference subwaveform is the same as
that used in experiment A, with =32.5. The maximal CC (exclud-
ing the case =32.5) is almost 1 and occurs at gates between
16 and 24. When increases by 1, the gate of maximal CC also increases by 1. Hence the CC can reflect the change of .

However, for an observed waveform of ERS-1, the CCs can be
different from the ideal values given in Fig. 3. The reason is that
the shape of the waveform (for both diffuse and specular wave-
forms) can deviate from the ideal shape following Eq. (2). For
example, Fig. 4 shows the CCs between the reference subwave-
form (same as the one used in Fig. 2) and the 43 subwaveforms of
an observed diffuse and an observed specular waveform over the
Antarctic Ocean. For the diffuse waveform (Fig. 4a), the maximal
CC is about 1 and occurs at gate=19, around which the CCs
fluctuate rapidly. The zero crossing gate is 32. For the specular
waveform (Fig. 4b), the maximal CC is about 0.55 and occurs at
gate=17, with the zero crossing gate at 22. The variation of CC in
the case of the specular waveform is also quite irregular.

In fact, the optimal subwaveform is not always the same as the
leading edge. The first and the last gates of the leading edge are
determined using an algorithm given in Fig. 5. In Fig. 5, is the
gate corresponding to the maximal CC, is the gate of zero
crossing, and is the gate difference. In general, the
gate difference increases with SWH. Since the reference subwave-
form is based on SWH=5 m, the ideal gate difference is 12. Thus,
if the gate difference is 12, the leading edge of the observed
subwaveform contains 22 samples as in the reference subwave-
form. If the gate difference is not 12, an empirical method,
detailed in Fig. 5, was used to select the first and the last gates of
the leading edge. The leading edge contains the return powers
from gate to gate. We must stress that, although the reference
subwaveform is based on Eq. (2), our algorithm of the leading
ege identification is effective for both specular and diffuse
waveforms. Also, if an observed waveform contains multiple
ramps, the subwaveform identified by the method presented in
this section is again the optimal one; see also Section 4 for the
assessments.

Fig. 2. Theoretical waveforms with SWH from 1 to 19 m. The waveform in blue is
based on SWH=5 m, and its subwaveform over gates from 20 to 42 is the
reference subwaveform.

Fig. 3. Correlation coefficients between the reference subwaveform and the
subwaveforms of varying SWH (including SWH=5 m).
2.4. Retracking the leading edge using threshold retracker

Once the leading edge is identified, the retracking gate, which must fall within this subwaveform, is determined by the threshold retracking (Davis, 1997). This method computes retracking gate using the formulas

\[ A = \sqrt[\text{sample}]{\frac{\sum_{i=1}^{\text{sample}} P_i^2(t)}{\sum_{i=1}^{\text{sample}} P_i(t)}}, \]  
\[ P_N = \frac{1}{5} \sum_{i=1}^{5} P_i, \]  
\[ T_i = (A - P_N) \cdot \text{Th} + P_N, \]  
\[ G_r = G_{k-1} + (G_k - G_{k-1}) \frac{T_k - P_k}{P_k - P_{k-1}} + \delta_{\text{first}}, \]

where \( \text{sample} \) is the number gates of the leading edge, \( A \) is the amplitude of the leading edge, \( P_i(t) \) is the normalized power of waveform at the \( i \)th gate, \( P_N \) is the averaged value of the first five normalized powers, \( \text{Th} \) is a threshold value, \( G_k \) is the \( k \)th gate whose normalized power is greater than \( T_i \), \( G_r \) is the retracking gate.

Note that if \( P_k \) equals \( P_{k-1} \), then \( k \) is replaced by \( k+1 \). The range correction is then computed by

\[ C = (G_r - G_T) \Delta R, \]

where \( G_r \) is the theoretical retracking gate and \( \Delta R \) is the range corresponding to one gate. The method for computing \( A \) is the same as the method for the OCOG retracking (Appendix A). For the ERS-1 altimeter, \( G_T = 32.5 \), and \( \Delta R = 0.4545 \text{ m} \). The optimal threshold value is obtained using certain criterion and this is discussed in Section 4.

3. A FORTRAN program for leading edge determination and retracking

A FORTRAN program, called subwave.f, was developed to implement the theory of subwaveform threshold retracking. This program first computes CCs between the reference waveform (cf. Fig. 2) and the subwaveforms of a full waveform (containing all return powers) to determine the leading edge for retracking. The retracking gate of this subwaveform is then determined by the threshold retracking (Section 2.3, but using only return powers within the leading edge). The computer program also includes the Beta-5 and OCOG retractors (the methods are given in Appendix A) and the full-waveform threshold retracker. The full-waveform threshold retracker in subwave.f uses the full waveform and its theory is given in Section 2.3. This program accepts command-line arguments and operates in the UNIX and the Microsoft DOS environment. The usage of this program, which also appears as comments in the program, is presented below

NAME
subwave: program to compute the centers of leading edge or retracking corrections of waveform using subwaveform threshold retracking, Beta-5, OCOG, and full-waveform threshold retracking

SYNOPSIS
subwave -F waveform_file -Gofile2 [-T retracker -O data_type -H threshold_value -Cofile1]
- -F: this input file contains return powers of waveform
- -G: this output file contains retracking corrections (or the centers of leading edge)

![Fig. 4. Correlation coefficients between the reference subwaveform and the 43 subwaveforms for an observed diffuse waveform and an observed specular waveform.](image-url)
4. A case study in the Antarctic Ocean

4.1. The waveform data of ERS-1 and the study area

We assessed the subwaveform retracker using ERS-1 altimeter data. ERS-1 is the first European Remote Sensing Mission, launched on 17 July 1991 and ended in 1996, with a 98.5° inclination angle at an altitude between 782 and 785 km. ERS-1/GRM data were collected during two 168-day geodetic phases, and the cross-track spacing is 8 km at the equator. The ERS-1/GRM data were collected during the repeat phase after the geodetic phases. The ERS-1 altimeter transmits radar pulses at a frequency of 1020 Hz, and the onboard processor averages 50 return echoes to generate 20 sets of waveforms in 1 s. This results in the 20 Hz waveforms and subsequently the 20-Hz SSHs. Each set of waveforms contains 64 return powers. The geophysical corrections for the ERS-1 instantaneous SSHs include solid-Earth, pole and ocean tides, tropospheric and ionospheric corrections, inverse barometer effect, sea state bias, and ocean tides based on NAO99b (Matsumoto et al., 2000).

Our experiments were carried out over two areas around the Antarctic Ocean, one bounded by 55°S–latitude < 82°S, 225°E–longitude < 270°E, and the other by 51°S–latitude < 52°S, 301°E–longitude < 303°E. The first area is one of the world’s southernmost seas, including a large portion of Bellingshausen and Amundsen Seas (BAS) lying offshore in West Antarctica. An accurate, high-resolution gravity field over BAS from satellite altimetry can reveal important details about the tectonic history of this region, such as the plate tectonic behavior since 18 Ma of West Antarctica and the Campbell Plateau—New Zealand microcontinent (McAdoo and Laxon, 1997). Moreover, some parts of BAS are covered with perpetual and seasonal sea ice. Due to varying surface reflecting properties, BAS is an ideal region to test different waveform retrackers. The tide gage Port Station, shown in Fig. 6, is located at 51.45°S, 302.0667°E in the second area. Here selected tidal records will be used to assess the performances of different retrackers.

4.2. Direct assessment using tide gage data

The range correction from retracking can be used to compute improved SSH, which is called retracked SSH below. There are many methods for assessing the quality of retracked SSH, e.g., analysis of crossover differences of SSH and comparison between retracked SSH with a well-defined field. In the Antarctic Ocean, crossover points may be sparse and interrupted by sea ice, and the crossover differences of SSH can be easily amplified by spurious SSH (even with retracking).

As a direct method of quality assessment, we compared retracked SSHs with hourly tidal records at Port Station gage data from University of Hawaii Sea Level Center (UHSLC, http://ilikai.soest.hawaii.edu/uhslc/datai.html) over 1992 to 2011. The closest subsatellite point of ERS-1/GRM is about 3 km from Port Station (Fig. 6). Only data from passes 19444, 19945, 20446, 20947, 21448, and 21949 of ERS-1/GRM are available for assessment near Port Station. Because the tide gage and altimetry records are not on the same vertical datum, the demeaned SSHs, named sea level anomalies (SLAs), from tide gage and altimetry near Port Station were compared; see also Fenoglio-Marc (2002). Affected by land mass near Port Station, Beta-5 fails to retrack most of the
waveforms and the corresponding result is not shown in this assessment. Fig. 7 compares the raw and retracked SSHs near Port Station. Table 1 shows the statistics of the differences between tide gage- and ERS-1-derived SSHs. Table 1 suggests that retracking cannot always reduce the differences between tide gage- and ERS-1-derived SSHs around a coastal station such as Port Station, where waveforms can be seriously corrupted by land mass. However, the subwaveform threshold retractors (with different threshold values) always outperform the full waveform threshold retracker. The comparison in Table 1 suggests that a threshold value of 0.5 is the optimal value for the subwaveform threshold retracker. The use of retracked SSHs reduces the standard

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### Table 1

<table>
<thead>
<tr>
<th>Altimeter SSHs</th>
<th>Mean</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Raw</td>
<td>0.605</td>
<td>0.241</td>
</tr>
<tr>
<td>Subwaveform threshold 0.1</td>
<td>0.850</td>
<td>0.364</td>
</tr>
<tr>
<td>Subwaveform threshold 0.2</td>
<td>0.571</td>
<td>0.284</td>
</tr>
<tr>
<td>Subwaveform threshold 0.3</td>
<td>0.392</td>
<td>0.239</td>
</tr>
<tr>
<td>Subwaveform threshold 0.5</td>
<td>0.075</td>
<td>0.193</td>
</tr>
<tr>
<td>Threshold(^a)</td>
<td>-0.244</td>
<td>0.998</td>
</tr>
</tbody>
</table>

\(^a\) Full waveform is used.

---

### Table 2

<table>
<thead>
<tr>
<th>Ocean</th>
<th>Beta-5</th>
<th>Threshold(^a)</th>
<th>Subwaveform threshold</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ice-free</td>
<td>0.100</td>
<td>0.100</td>
<td>0.059 0.062 0.067 0.088</td>
</tr>
<tr>
<td>Ice-covered</td>
<td>NA</td>
<td>0.404</td>
<td>0.192 0.229 0.261 0.368</td>
</tr>
</tbody>
</table>

\(^a\) Full waveform is used.
deviation of the SSH differences from 0.241 to 0.193 m, yielding an improvement percentage (IMP) of 20%. IMP is the ratio between the difference of the standard deviations of the raw and retracted SSHs and the standard deviation of the raw SSHs (Hwang et al., 2006). The mean difference is also reduced from 0.605 to 0.075 m by retracking.

4.3. Indirect assessment using along-track differenced residual SSH

In this section, we use an indirect quality assessment of retracted SSHs as follows. We first computed the difference between the retracted SSH and the geoidal height from the EGM2008 geopotential model (Pavlis et al., 2008), called residual SSH, as

$$N_{\text{res}} = N - N_{\text{long}}.$$  \hspace{1cm} (12)

where $N_{\text{long}}$ is the geoidal height from EGM2008 using the all-harmonic coefficients provided by the EGM2008 (complete to spherical harmonic degree and order 2159, and contains additional coefficients extending to degree 2190 and order 2159). Furthermore, to reduce the effect of the ocean dynamic height and the long wavelength error in the altimeter ranging, we computed the differenced residual SSH between two successive points along satellite ground tracks as

$$\Delta N_{\text{res}} = N_{\text{res2}} - N_{\text{res1}}.$$  \hspace{1cm} (13)

where $N_{\text{res1}}$ and $N_{\text{res2}}$ are two successive residual SSHs. The standard deviation of $\Delta N_{\text{res}}$, denoted as $S_{\Delta N}$, over the interested area was then computed to serve as the descriptor of the improvement of SSH due to retracking. It is clear that $S_{\Delta N}$ will decrease with improved SSHs.

As an example, Table 2 compares $S_{\Delta N}$ values for different retrackers along ERS-1/GM pass 14501 (Fig. 8). Table 2 shows that the subwaveform threshold retracker outperforms the Beta-5 and the full-waveform threshold retrackers over both the ice-free and ice-covered areas.

**Table 3**
Standard deviations of differenced SSHs (in m) over BAS.

<table>
<thead>
<tr>
<th>Ocean</th>
<th>Beta-5</th>
<th>Threshold*</th>
<th>Subwaveform threshold</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.118</td>
<td>0.124</td>
<td>0.070 0.074 0.083 0.110</td>
</tr>
<tr>
<td>Ice-free</td>
<td>NA</td>
<td>0.349</td>
<td>0.220 0.232 0.253 0.322</td>
</tr>
<tr>
<td>Ice-covered</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* Full waveform is used.

**Table 4**
Statistics of range corrections (in m) from the subwaveform retracker.

<table>
<thead>
<tr>
<th>Ocean</th>
<th>Max</th>
<th>Min</th>
<th>Mean</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ice-free</td>
<td>5.844</td>
<td>-7.236</td>
<td>-0.003</td>
<td>0.233</td>
</tr>
<tr>
<td>Ice-covered</td>
<td>10.688</td>
<td>-12.131</td>
<td>0.094</td>
<td>1.646</td>
</tr>
</tbody>
</table>

Fig. 9. Distribution of range corrections (in m) over (left) the ice-free ocean and the ice-covered ocean.
ocean and the ice-covered ocean. The optimal threshold value for
the subwaveform threshold retracker was found to be 0.1, while
for the full-waveform threshold retracker, the value was 0.5. The
$S_{AN}$ values from the subwaveform retracker (0.059 and 0.193 m
over ice-free and ice-covered oceans, respectively) are about 50%
of the $S_{AN}$ from the full-waveform threshold retracker. The
comparison in Table 2 suggests that the subwaveform retracker
with a threshold value of 0.1 outperforms the other two retrack-
ers. Also, the retracked SSHs over the ice-free ocean yield an $S_{AN}$
that is about 25% of the $S_{AN}$ over the ice-covered ocean. This
indicates that retracked SSHs over the ice-free oceans are more
accurate than those over the ice-covered ocean.

A further assessment was carried out over BAS using 2278
passes of ERS-1/GM. A comparison of the $S_{AN}$ values from
different retrackers is shown in Table 3. The results over BAS
are similar to the result along pass 14501. Compared with pass
14501, the $S_{AN}$ values of the subwaveform threshold retracker
(again with 0.1 threshold value) over the ice-free and ice-covered
oceans are 0.070 and 0.220 m, which are larger than that of pass
14501. The above two experiments (Tables 2 and 3) show that the
subwaveform threshold retracker with a 0.1 threshold is the
optimal retracker.

Table 4 shows the statistics of the range corrections from the
subwaveform retracking over both the ice-free and ice-covered
oceans. Table 4 suggests that the average range correction over
the ice-covered ocean is much larger than that over the ice-free
ocean (1.646 vs. 0.233 m in terms of standard deviation). Since

<table>
<thead>
<tr>
<th>Ocean</th>
<th>Raw (m)</th>
<th>Retracked (m)</th>
<th>IMP (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ice-free</td>
<td>0.157</td>
<td>0.070</td>
<td>55.4</td>
</tr>
<tr>
<td>Ice-covered</td>
<td>1.836</td>
<td>0.220</td>
<td>88.0</td>
</tr>
</tbody>
</table>

The difference in SSH between the altimeter-derived and shipborne
GA is larger than three times the standard deviation of the
residual SSHs from the raw SSH and from retracked SSH. The
standard deviation of differenced SSHs from raw and retracked SSHs,
and improvement percentage (IMP).

Possible causes of the large differenced residual SSHs in
Fig. 11 are poor geophysical correction models and the fact that
the sea ice surface is not identical to the ocean surface.

4.4 Improved gravity anomaly from retracked SSH

An indirect assessment of the retracked SSHs was also carried
out: comparison between altimeter-derived GAs with shipborne
GAs. We chose to use the inverse Vening Meinesz (IVM) formula
(Hwang, 1998) to compute GAs from SSHs. For gravity derivation,
the 20-Hz SSHs are resampled at a 2-Hz rate by a polynomial
fitting and smoothing. Details of the resampling technique is
given by Hwang et al. (2006). In the gravity derivation, along-
track SSHs were first converted to geoid gradients, which were
then used to create two $2\times2$ grids of geoid gradients in the
north–south and the west–east directions. The standard remove-
restore procedure was applied in the gravity derivation, in which
the EGM2008 using the all-harmonic coefficients provided by the
model was adopted as the reference gravity field.

Two $2\times2$ grids of GAs over BAS were derived, one from the
retracked SSHs and another from the raw SSHs of ERS-1/GM. The
shipborne GAs over BAS from the National Geophysical Data
Center (NGDC) (http://www.ngdc.noaa.gov) were then compared
with the altimeter-derived GAs. Before comparison, the bias and
drift along any of the cruises of the NGDC shipborne GAs were
removed by the method described in Hwang and Parsons (1995),
and outliers were deleted using the three-sigma criterion.
A shipborne GA was deleted if the difference between the
shipborne GA and the altimeter-derived GA exceeds three times
that of the sigma. Table 6 shows the statistics of the differences
between the altimeter-derived and shipborne GAs. The use of
retracked SSHs has reduced the standard deviation of the

![Fig. 10. Histograms of range corrections (in m) over the ice-free ocean (top) and the ice-covered ocean.](image-url)
The mean difference is also reduced due to retracking. In terms of standard deviation, the IMP due to the subwaveform retracking is 46.6\%. Fig. 12 shows the distribution of the differences. The reduction of the differences due to retracking is evident in Fig. 12. In particular, the large differences over the ice-covered ocean have been significantly reduced due to the use of retracked SSHs.

5. Discussion and conclusions

The novel idea of the correlation method of retracking is matching a reference waveform (the ideal waveform, see Section 2.2) with selected sets of subwaveforms to determine the optimal subwaveform (also the leading edge) for retracking. Retracking over such a leading edge is found to produce the best improved SSH compared to the cases of using other subwaveforms. Our retracking is simply based on the threshold method that exists in the literature. Such a retracker is called subwaveform threshold retracker, which outperforms the full-waveform threshold and the Beta-5 retrackers. In addition to ERS-1, we expect that this retracker is applicable to waveforms from missions such as Geosat, TOPEX/Poseidon, ERS-2, Envisat, and Geosat-follow-on for best possible improved SSH. However, the optimal threshold values may vary with waveforms from different missions and with different applications, and selecting such values will be important for the success of this retracker. For example, the optimal threshold value for the subwave retracker is 0.5, while the optimal threshold value is 0.1 in the indirect assessments of residual SSHs and GAs.

The direct assessment of retracked SSHs at tide gage Port Station suggests that retracking can reduce the uncertainty of altimetry SSH by about 20\%. Also, we use an “indirect” assessment of retracked SSH based on a comparison between altimeter-derived gravity and in situ shipborne gravity. Because SSH is essential to altimeter-derived gravity, the improvement in the former will naturally lead to the improvement in the latter. The assessment results show that retracking by the subwaveform threshold retracker improves the altimeter-derived gravity up to 46\%.

It is important to understand that retracking alone cannot improve the quality of SSH. Retracking will only improve the ranging accuracy at short wavelengths (if the waveform is sufficiently good). Ranging accuracy at all wavelengths may be degraded by poor geophysical and environmental corrections. Finally, compared to the case over an ice-free sea surface, a more sophisticated data processing technique over an ice-covered sea surface is needed to obtain good results.

Table 6
Statistics of differences (in mgal) between altimeter-derived and shipborne gravity anomalies using raw and retracked SSHs.

<table>
<thead>
<tr>
<th>SSH</th>
<th>Maximum</th>
<th>Minimum</th>
<th>Mean</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Raw</td>
<td>117.526</td>
<td>−81.525</td>
<td>1.052</td>
<td>15.038</td>
</tr>
<tr>
<td>Retracked</td>
<td>50.045</td>
<td>−47.567</td>
<td>0.147</td>
<td>8.028</td>
</tr>
</tbody>
</table>

Fig. 11. Distributions of differenced residual SSH (in m) from specular waveforms before (left) and after retracking.
Fig. 12. Differences (in mgal) between altimeter-derived and ship gravity anomalies from raw SSHs (left) and retracked SSHs along ship tracks.

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**Appendix A. Methods of Beta-5 and OCOG retracking**

The following summarizes the methods of Beta-5, OCOG retracking. The FORTRAN computer codes for these methods are also included in our retracking program “subwave.f.”

**A.1. Beta-5 retracker**

The Beta-5 retracker method was developed by Martin et al. (1983), which is the first retracking algorithm based on a functional fit to Brown’s surface scattering model for waveforms over continental ice sheets. The mathemetic model between the powers of a waveform and the times is (Martin et al., 1983; Zwally and Brenner, 2001)

\[ y(t) = \beta_1 + \beta_2 \left( 1 + \beta_3 t \right) \frac{t - \beta_4}{\beta_5}, \quad (A-1) \]

with

\[ Q = \begin{cases} 0 & \text{for } t < \beta_3 + 0.5\beta_4 \\ t - (\beta_3 + 0.5\beta_4) & \text{for } t \geq \beta_3 + 0.5\beta_4 \end{cases} \quad (A-2) \]

\[ P(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} \exp \left( -\frac{q^2}{2} \right) dq, \quad (A-3) \]

where \( y \) is the power of waveform sample at time \( t \), \( q \) the variable of the normal distribution function \( \exp(-q^2/2) \), \( \beta_1 \) the thermal noise level of the return waveform, \( \beta_2 \) the amplitude of return signal, \( \beta_3 \) the gate corresponding to the center of the leading edge (retracking gate), \( \beta_4 \) the half ascending time of the leading edge, \( \beta_5 \) the slope of the trailing edge, and \( \varepsilon \) error in the observable.

The five parameters in Eq. (A-1) can be estimated using the least-squares method making the target function (the sum of weighted squares of \( \varepsilon \)) a minimum. The observation equation is based on Eq. (A-1). In the case of a sharp leading edge, the least-squares method of Beta-5 retracking may result in singularity of the normal matrix, and the retracking will fail. The range correction is computed by

\[ C = (\beta_3 - G_T) \Delta R, \quad (A-4) \]

where \( G_T \) is the theoretical tracking gate and \( \Delta R \) is the range corresponding to one gate. For the ERS-1 altimeter, \( G_T = 32.5 \), and \( \Delta R = 0.4545 \) m.

**A.2. OCOG retracker**

The OCOG retracker first estimates the amplitude (\( A \)), width (\( W \)), and center of gravity (COG) of a waveform as (Wingham et al., 1986)

\[ A = \sqrt{\sum_{i=1+n_0}^{64-n_0} P^2_1(t)/ \sum_{i=1+n_0}^{64-n_0} P^2_i(t)} \quad (A-5) \]

\[ W = \left( \sum_{i=1+n_0}^{64-n_0} P^2_1(t) \right)^2 / \sum_{i=1+n_0}^{64-n_0} P^2_i(t) \quad (A-6) \]

\[ \text{COG} = \sum_{i=1+n_0}^{64-n_0} i P^2_1(t) / \sum_{i=1+n_0}^{64-n_0} P^2_i(t) \quad (A-7) \]

where \( n_0 \) is the gate number before which the return powers are neglected. The retracking gate is computed by

\[ \text{LEG} = \text{COG} - \frac{W}{2} \quad (A-8) \]

where \( P_i(t) \) is the power of waveform at the \( i \)th gate. The range correction is computed by Eq. (A-4) by replacing \( \beta_3 \) by LEG. In subwave.f, the amplitude in Eq. (A-5) is used for the threshold retrackers (Section 2.3).

**Appendix B. Supplementary material**

Supplementary data associated with this article can be found in the online version at doi:10.1016/j.cageo.2011.08.017.
References


