Forecasting nonlinear time series of energy consumption using a hybrid dynamic model

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Abstract

Energy consumption is an important index of the economic development of a country. Rapid changes in industry and the economy strongly affect energy consumption. Although traditional statistical approaches yield accurate forecasts of energy consumption, they may suffer from several limitations such as the need for large data sets and the assumption of a linear formula. This work describes a novel hybrid dynamic approach that combines a dynamic grey model with genetic programming to forecast energy consumption. This proposed approach is utilized to forecast energy consumption because of its excellent accuracy, applicability to cases with limited data sets and ease of computability using mathematical software. Two case studies of energy consumption demonstrate the reliability of the proposed model. Computational results indicate that the proposed approach outperforms other models in forecasting energy consumption.

Keywords:
Energy consumption
Grey forecasting model
Genetic programming
Hybrid dynamic approach

1. Introduction

Energy-related issues are a priority owing to the major role that energy sources, such as coal, oil, gas, and wind, play in daily life and the global economy. Energy consumption has greatly increased owing to a burgeoning population growth and elevated living standards [1,2]. For instance, the Energy Information Administration (EIA) of the United States has forecast that global energy consumption will increase by 49% from 2007 to 2035 [3]. Also, the energy consumption of public buildings is increasing in proportion to overall national use [4]. In economics, energy consumption has significantly and positively affected Asian economic growth [5]. Accordingly, a highly precise model for forecasting energy consumption must be developed. Based on such a model, energy policy makers can either implement an energy conservation policy or allocate a certain amount of energy to public buildings.

The autoregressive integrated moving average (ARIMA) model is extensively used to forecast time-series data [6]. However, the forecasting accuracy of the ARIMA model is poor when data are few or nonlinear [7]. Forecasting models (such as the ARIMA model) that are based on conventional statistical methods are limited because real-world data are commonly few or fail to satisfy statistical assumptions.

Forecasting models can also be developed using data-mining approaches such as artificial neural networks (ANNs), evolutionary algorithms (EAs), and mixed-integer programming [8,9]. However, the hidden layers in ANNs are difficult to explain, and the relationship between input and output variables in ANNs is difficult to express as a clear forecasting equation. To solve this problem and compare the forecasting accuracy with ANNs, some studies [7] have applied genetic programming (GP) to construct a clear forecasting equation and compared the forecasting accuracy with other models. GP is more accurate than ANNs in forecasting or classification problems [7,10,11]. In forecasting energy consumption, Togun and Baysec [12] found that GP performs as well as ANNs. In contrast to the ANNs model, GP uses symbolic regression to derive a clear forecasting equation [7,10,12–14].

The grey model (GM) of grey system theory has been adopted in many forecasting studies [15–18] with only four or more observations. Real-word data sets are often difficult to collect and data sets include a few observations. Although linear regression (such as the ARIMA model) is often utilized to forecast time-series data, it is inaccurate when observations are few or do not satisfy statistical assumptions. GM(1,1), the first-order one-variable GM, has been widely applied in various fields [15–18]. Although capable of forecasting using small time-series data accurately, GM(1,1) may fail to do so for nonlinear time-series data.

Many researchers have been developed GM models to increase their forecasting accuracy. For instance, Hsu and Wang [17] estimated the parameters of a grey differential function using the Bayesian method to increase the accuracy of GM(1,1). Wang and Hsu [18] estimated the parameters of grey differential function using genetic algorithms (GAs) to increase the forecasting accuracy of GM(1,1). To improve further the performance of GM(1,1) models,
some studies have developed innovative approaches to forecast the residual series of GM(1,1). For instance, Hsu and Chen [15] combined residual modification with residual ANN sign estimation to forecast the residual series of GM(1,1). Hsu [16] combined residual modification with residual Markov-chain sign estimation to forecast residual series of GM(1,1). To increase the predictive accuracy of the method of Hsu and Chen [15], Lee and Tong [10] combined residual modification with residual GP sign estimation to increase the effectiveness of ANN in estimating the residual signs of GM(1,1). When the time-series data are nonlinear, the forecasting accuracy of GM(1,1) or an improved GM(1,1) may be poor. Hence, Zhou and Hu [19] developed a hybrid GM(1,1) model that combines GM(1,1) modeling in original time-series data with ARIMA modeling in residual series to increase forecasting accuracy of GM(1,1). However, their approach adopts a linear model (ARIMA) to forecast the residual series. Small or nonlinear residual-series data may obtain inaccurate outcomes using ARIMA model.

GM(1,1) is normally constructed using an entire data set. Akay and Atak [20] developed a grey predictive model with a rolling mechanism (GPRM), in which only a minimal amount of recent data are used, to increase forecasting accuracy. Based on the structure of GM(1,1), GPRM can be used efficiently to increase the forecasting accuracy of GM(1,1) in each rolling process when applied to exponential or chaotic data sets. Although capable of increasing the forecasting accuracy of GM(1,1), GPRM does not model the residual series in each rolling process to increase forecasting accuracy. Furthermore, improved forecasting models [10,15,16,19] fail to enhance significantly the accuracy of GM(1,1) modeling [10,15,16] or ARIMA modeling [19] in forecasting the residual series. To enhance the accuracy of the residual series, heuristic methods, such as symbolic regression, must be utilized since they perform well in forecasting [13]. Lee and Tong [7] claimed that the conventional linear time-series model (ARIMA model) cannot easily be used to fit nonlinear time-series data and therefore developed a heuristic approach to improve the accuracy of residual series.

To increase the accuracy of GM(1,1) applied to original time-series data and to prevent inaccurate forecasting using conventional linear time-series models when residual series are complex patterns (such as nonlinear patterns), this work develops a novel hybrid dynamic forecasting model in which dynamic grey prediction is applied to the time-series data and GP prediction is applied to the residual-series data of the dynamic grey prediction, to ensure high forecasting accuracy.

The rest of this paper is organized as follows. Section 2 reviews available models for forecasting energy consumption. Section 3 then describes the proposed novel hybrid dynamic GM for forecasting energy consumption. Next, based on real-world examples, Section 4 evaluates the forecasting accuracy of the proposed model, and compares it to other energy consumption models. Section 5 draws conclusions.

2. Energy consumption models

This section describes three models that are used in forecasting energy consumption. The first one, the GM(1,1) model, is commonly adopted when only a few time-series data are available. The second one, the dynamic GM(1,1) model, is known for its robustness in forecasting each rolling time-series data. The third one, the GP model, is often used either to forecast nonlinear time-series data [12,14] or to elucidate a complex data-structure.

2.1. GM(1,1) forecasting model

GM(1,1) has been applied in many fields [10,16,17,21,22] such as energy distribution [10] and the integrated circuit industry [16,17,21,22]. This model can be constructed as follows [10,15–18,20–23].

Step 1: Obtain positive time-series data as follows.

\[
y^{(0)}(0) = y^{(0)}(1), y^{(0)}(2), y^{(0)}(3), \ldots, y^{(0)}(n), \quad n \geq 4
\]

Step 2: Apply the accumulated generating operator (AGO) to the original time-series data (i.e. \( y^{(0)}(t) \)) to obtain the accumulated time-series \( y^{(1)}(t) \) as follows.

\[
y^{(1)}(t) = [y^{(1)}(0), y^{(1)}(2), y^{(1)}(3), \ldots, y^{(1)}(n)]
\]

where \( y^{(1)}(0) = y^{(0)}(1) \) and \( y^{(1)}(n) = \sum_{m=0}^{n} y^{(0)}(m) \).

Step 3: Construct GM(1,1) using a grey differential equation, \( y^{(0)}(t) + a y^{(1)}(t) = u \), where \( a \) and \( u \) denote the grey parameters of the GM(1,1) model, and \( z^{(1)}(t) \) represents the average of \( y^{(1)}(t - 1) \) and \( y^{(1)}(t) \). Also, the grey parameters of the grey differential equation can be estimated using the ordinary least squares (OLS) method.

Step 4: Replace the estimated parameters (\( a \) and \( u \)) in the grey differential equation and then obtain the GM(1,1) forecasting equation using the inverse AGO (IAGO) technique, in the following exponential form.

\[
y^{(0)}(t) = y^{(0)}(1) - \left(1 - e^{-\frac{u}{a}}\right) e^{-\frac{t}{a}}, \quad t = 1, 2, \ldots
\]

2.2. Dynamic GM(1,1) model

Most works have constructed GM(1,1) using an entire data set [15–18,21]. However, GM(1,1) should be applied using only recent data to increase its forecasting accuracy [20]. Akay and Atak [20] developed GPRM, a dynamic forecasting method, in which the forecasting accuracy of a grey prediction scheme was increased using recent data when the time-series data were exponential or chaotic in a rolling mechanism. Some studies [20,23] have developed dynamic GM(1,1) models (DGM(1,1)) to increase the forecasting accuracy of GM(1,1). In the DGM(1,1) model, \( y^{(0)}(k + 1) \) is predicted using GM(1,1) and \( y^{(0)}(k) = [y^{(0)}(1), y^{(0)}(2), y^{(0)}(3), \ldots, y^{(0)}(k)] \), where \( k < n \). Following the determination of \( y^{(0)}(k + 1) \), \( y^{(0)}(k + 1) \) is added to the original time-series, and \( y^{(0)}(1) \) is removed from the original time-series to yield a new series \( y^{(0)} = [y^{(0)}(2), y^{(0)}(3), y^{(0)}(4), \ldots, y^{(0)}(k + 1)] \). The predicted value of \( y^{(0)}(k + 2) \) can be obtained using the new series \( y^{(0)} \). The evaluation procedure is continued to obtain \( y^{(0)}(k + l) \) for \( l = 3, 4, 5, \ldots, n - k - 1 \). The index of forecasting accuracy in the \( (k + 1) \) period of GM(1,1) is defined as follows [20]:

\[
e(k + 1) = \frac{|y^{(0)}(k + 1) - y^{(0)}(k + 1)|}{y^{(0)}(k + 1)} \times 100\%.
\]

Furthermore, the average rolling error of GM(1,1) can be determined using Eq. (4) as follows [20].

\[
e = \frac{1}{n - 4} \sum_{k=4}^{n-1} e(k + 1) \times 100\%.
\]

Based on Eq. (5), the accuracy of DGM(1,1) can be evaluated as \((100 - e)\%\).

2.3. GP model

Koza [24] developed GP as a novel automatic programming algorithm that exploits the concept of evolution to identify the structure of forecasting model. GP constructs a forecasting model by symbolic regression [12,13]. The basic concepts of GP resemble those of GAs, including mutation, crossover and reproduction [10–12,14]. Parkins and Nandi [25] described how GAs and GP
3. Hybrid dynamic grey forecasting

This section describes a novel nonlinear hybrid dynamic forecasting model that combines the dynamic grey model with GP. The proposed model is derived as follows.

Step 1: Assume that original time-series of energy consumption data is \( y_1 \) (\( n \) data points), and that \( y_1 \) is predicted using a novel DGM(1,1) model (NDGM(1,1)). Because GM(1,1) requires at least four data points to construct the forecasting model, the NDGM(1,1) model utilizes the most recent four data points to predict the next data point: \( k = 4 \) is used in each rolling cycle for the dynamic forecasting process.

Therefore, in the first rolling, \( y^{(0)}(k + 1) \) can be determined from the series \( (y^{(0)}(k), y^{(0)}(k - 1), y^{(0)}(k - 2), y^{(0)}(k - 3)) \); in the second rolling, \( y^{(0)}(k + 2) \) can be determined from \( (y^{(0)}(k), y^{(0)}(k - 1), y^{(0)}(k - 2)) \). Moreover, in each rolling cycle, the newly predicted values of original data \( (y^{(0)}(k + 1), y^{(0)}(k + 2), \ldots) \) are determined using the GM(1,1) model. The residual series of the NDGM(1,1) model can be expressed as \( \varepsilon_i = y_i - \hat{y}_i \).

Step 2: In each rolling cycle of NDGM(1,1), construct the model for forecasting the error \( (r_{ij}) \) using the nonlinear function, determined by GP as follows:

\[
\hat{r}_{ij} = f(r_{i1,j-1}, r_{i2,j-2}, r_{i3,j-3}, r_{i4,j-4}) + \varepsilon_{ij}, \quad (i,j) = (1,5), (2,6), \ldots, (n - 4, n),
\]

where \( r_{ij} \) denotes the \( j \)-th point estimate of NDGM(1,1) that is conditioned in the \( i \)-th rolling cycle; \( (r_{i1,j-1}, r_{i2,j-2}, r_{i3,j-3}, r_{i4,j-4}) \) represent the errors of the \( i \)-th rolling cycle and can be obtained using the GM(1,1) model in the four periods; \( \varepsilon_{ij} \) represents a random error. Therefore, during the first rolling cycle of the NDGM(1,1) model, the forecasted error \( r_{ij} \) can be written as \( \hat{r}_{15} = f(r_{11,4}, r_{13}, r_{12}, r_{11}) + e_{15} \). In the GP model, the input variables are the lagging residual series \( (r_{ij-1}, r_{ij-2}, r_{ij-3}, r_{ij-4}) \) and the output variable is \( r_{ij} \).

To reduce the forecasting error, the fitness function in GP is defined as follows [10):

\[
\text{Minimize } \sum_{j=5}^{n} |r_{ij} - r_{ij}|, \quad i = 1, 2, \ldots, n - 4.
\]

Some studies [26] had adopted the minimization of root mean square error (RMSE) as the fitness function of GP. Adoption of the heuristic method (described by Eq. (7)) can prevent the problems of conventional optimization, such as the setting of initial values and the use of a differential function. The settings of initial values in conventional optimization approaches affect forecasting performance. In practice, the heuristic method can be conveniently applied in various engineering problems (given a clear fitness function). The function in GP utilizes operators \( +, -, \times, \div, \log, \sin, \cos, \exp, \text{constant} \). The parameters of GP, population size, maximum number of generations, crossover rate and mutation rate are set to 150, 1000, 0.9, and 0.01, respectively. The values of the GP parameters are determined by trial-and-error.

Step 3: Express the hybrid dynamic forecasting model that combines the NDGM(1,1) model and the GP model as follows.

\[
y = \hat{y} + \hat{e},
\]

where \( y \) denotes the forecasted value of \( y \); \( \hat{y} \) represents the series \( (\hat{y}^{(0)}(5), \hat{y}^{(0)}(6), \ldots, \hat{y}^{(0)}(n)) \), and \( \hat{e} \) represents the series \( (\hat{r}_{15}, \hat{r}_{26}, \ldots, \hat{r}_{n-4}) \). Notably, the proposed model differs from the model in the literature [10] as follows:

1. The proposed model combines the dynamic GM(1,1) model in time-series data with the GP model in the residual series of dynamic GM(1,1). The energy consumption model [10] (GP-GM(1,1) model) is constructed using GM(1,1) (both in observations and residual values); GP is used to predict the residual sign (0 or 1) rather than residual values in the GP-GM(1,1) model [10]. Strictly, the GP-GM(1,1) model [10] is not a hybrid forecasting model because only GM(1,1) is applied to observations and residual values.

2. When the absolute residual series has complex patterns, the forecasting accuracy of GM(1,1) is unsatisfactory. Hence, the proposed model increases the forecasting accuracy by using GP to construct the forecasting equation of the absolute residual series.

This work adopts MATLAB software and DTREG software [27] to construct the proposed forecasting model. The matrix operation of MATLAB is utilized to determine the results of Step 1 \( (\hat{y}^{(0)}(k + 1), \hat{y}^{(0)}(k + 2), \ldots, k = 4, 5, \ldots) \). The gene expression programming (GEP) package of DTREG software is utilized to determine the results of Step 2 \( (\hat{r}_{ij} = 1, 2, \ldots, n - 4; j = 5, 6, \ldots, n) \). Hence, the forecasts of hybrid dynamic model can be obtained from the outcomes of Steps 1 and 2.

4. Computational results

To demonstrate the effectiveness of the proposed hybrid dynamic GM, two energy consumption data sets from the United States [28] and China [29] are used to evaluate the accuracy of the proposed model. Energy consumption data from the United States from 1970 to 2008 provide a total of 39 observations. Annual energy consumption data from the US from 1974 to 1998 form a testing set (25 observations), and data from 1999 to 2008 form a testing set (10 observations). The China energy consumption data range from 1957 to 2007, providing a total of 51 observations. The annual energy consumption data for China from 1961 to 1998 form the testing set (38 observations), and data from 1999 to 2007 form a testing set (9 observations).

This work compares the proposed model with a non-dynamic hybrid grey forecasting model by incorporating the hybrid GM(1,1) model as a benchmarked model. The hybrid GM(1,1) model uses the entire data set to forecast future data points. By assuming that the original time sequence with \( n \) data points is \( y = (y(1), y(2), \ldots, y(n)) \), this work constructs a GM(1,1) model using \( n \) data points. The original data that are forecasted using GM(1,1) are denoted as \( \hat{y}(1), \hat{y}(2), \ldots, \hat{y}(n) \). Let \( c(i) = (\hat{y}(i) - \hat{y}(i)), \quad i = 1, 2, \ldots, n \), be the absolute residual data points. \( c(i) \) can be estimated using the GP model, in which the independent variables are \( e_{1}, e_{2}, e_{3}, e_{4} \) and the dependent variable is \( e_{i} \). The estimated value of \( e_{i} \) is represented as \( c(i) \). Finally, the non-dynamic forecasting model can be expressed as \( \hat{y}(i) + c(i) \times \hat{e}(i) \), where \( c(i) \) denotes the residual sign (\( c(i) = 1 \) if the ith residual is positive and \( c(i) = -1 \) if the ith residual is negative). Additionally, the method in the literature [10] is also used as a benchmark model.
The accuracy of the proposed forecasting model is compared with other models using four indices. The first index is the RMSE, which compares the forecasted time-series data with the real time-series data. The RMSE is defined as,

$$\text{RMSE} = \sqrt{\frac{1}{N} \sum_{t=1}^{N} (f_t - o_t)^2},$$  \hspace{1cm} (9)$$

where $f_t$ denotes the forecasted value for the $t$th year, and $o_t$ denotes the real value for the $t$th year. The second index is the mean absolute percentage error (MAPE). This index statistically specifies the accuracy of the fitted time-series data. The MAPE is defined as,

$$\text{MAPE} = \frac{1}{N} \sum_{t=1}^{N} \frac{|f_t - o_t|}{o_t} \times 100\%,$$  \hspace{1cm} (10)$$

Lewis [30] developed MAPE criteria for evaluating the effectiveness of a forecasting model. The third index is the mean absolute error (MAE), which is defined as,

$$\text{MAE} = \frac{1}{N} \sum_{t=1}^{N} |f_t - o_t|.$$  \hspace{1cm} (11)$$

The fourth index is the percentage error (PE), which compares the forecasted and real of the time-series data. The PE is defined as,

$$\text{PE} = \frac{|f(t) - o(t)|}{o(t)} \times 100\%.$$  \hspace{1cm} (12)$$

Fig. 1 summarizes the US energy consumption data from 1974 to 2008, and the data fitted using GM(1,1), NDGM(1,1), the ARIMA model, the GP model, the method of Lee and Tong [10], the hybrid GM(1,1) model, and the hybrid dynamic GM (proposed). Fig. 2 displays the PE obtained when these models are used to forecast the US energy consumption data set. Table 1 summarizes the forecasts and errors for all forecasting models in US energy consumption data. Based on the MAPE index, the proposed hybrid dynamic GM has the lowest forecasting error (0.062%) among all of the forecasting models when applied to the US energy consumption data. Fig. 3 summarizes the China energy consumption data from 1961 to 2007, and the corresponding results obtained using different forecasting models. Fig. 4 displays the PE obtained when these models are used to forecast the China energy consumption data set. Table 2 summarizes the forecasts and errors for all models in the China data. According to the MAPE index, the proposed model has the lowest forecasting error (0.4%) among all of the models.

Using the proposed model to forecast the residual series greatly increases the predictive accuracy over other models. Accordingly, the hybrid dynamic GM forecasts energy consumption more accurately than the other models. Moreover, the proposed hybrid dynamic GM outperforms the hybrid GM(1,1) model and has lower prediction errors. The proposed model adopts dynamic...
Table 1
Forecasted values and errors of various forecasting models in US energy consumption (unit: Quadrillion Btu).

<table>
<thead>
<tr>
<th>Year</th>
<th>Actual value</th>
<th>GM(1,1)</th>
<th>NDCM(1,1)</th>
<th>ARIMA</th>
<th>Lee and Tong [10]</th>
<th>Hybrid GM(1,1)</th>
<th>Hybrid dynamic GM</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Model value</td>
<td>Errora</td>
<td>Model value</td>
<td>Errora</td>
<td>Model value</td>
<td>Errora</td>
</tr>
<tr>
<td>1999</td>
<td>96.82</td>
<td>93.97</td>
<td>2.94</td>
<td>95.71</td>
<td>1.14</td>
<td>96.06</td>
<td>0.78</td>
</tr>
<tr>
<td>2000</td>
<td>98.98</td>
<td>95.01</td>
<td>3.40</td>
<td>96.17</td>
<td>2.84</td>
<td>96.95</td>
<td>2.06</td>
</tr>
<tr>
<td>2001</td>
<td>96.33</td>
<td>96.06</td>
<td>0.28</td>
<td>96.68</td>
<td>0.36</td>
<td>97.83</td>
<td>1.56</td>
</tr>
<tr>
<td>2002</td>
<td>97.86</td>
<td>97.12</td>
<td>0.76</td>
<td>97.16</td>
<td>0.72</td>
<td>98.71</td>
<td>0.87</td>
</tr>
<tr>
<td>2003</td>
<td>98.21</td>
<td>98.19</td>
<td>0.02</td>
<td>97.66</td>
<td>0.56</td>
<td>99.59</td>
<td>1.41</td>
</tr>
<tr>
<td>2004</td>
<td>100.35</td>
<td>99.28</td>
<td>1.07</td>
<td>98.15</td>
<td>2.19</td>
<td>100.48</td>
<td>0.13</td>
</tr>
<tr>
<td>2005</td>
<td>100.48</td>
<td>100.38</td>
<td>0.1</td>
<td>98.66</td>
<td>1.81</td>
<td>101.36</td>
<td>0.88</td>
</tr>
<tr>
<td>2006</td>
<td>99.88</td>
<td>101.49</td>
<td>1.61</td>
<td>99.16</td>
<td>0.72</td>
<td>102.24</td>
<td>2.37</td>
</tr>
<tr>
<td>2007</td>
<td>101.55</td>
<td>102.61</td>
<td>1.04</td>
<td>99.66</td>
<td>1.86</td>
<td>103.13</td>
<td>1.55</td>
</tr>
<tr>
<td>2008</td>
<td>99.3</td>
<td>101.74</td>
<td>4.47</td>
<td>100.17</td>
<td>0.67</td>
<td>104.01</td>
<td>4.74</td>
</tr>
</tbody>
</table>

RMSE  2.22  1.52  2.02  2.73  1.64  1.53  0.092
MAPE (%)  1.63  1.31  1.63  1.93  1.23  1.21  0.062
MAE  1.61  1.30  1.62  1.90  1.21  1.21  0.06

*ER = \( \frac{E_{FM} - E_{FM'}}{E_{FM}} \times 100\% * 

Fig. 3. The distributions of forecast values and real values from 1961 to 2007 in China.

Fig. 4. The percentage of predicting error for forecasting models from 1999 to 2007 in China.
programming in predicting both original observations and residual series. It differs from the hybrid GM(1,1), which adopts non-dynamic programming to predict both original observations and residual values. Similarly, the proposed model is more precise than the methodology of Lee and Tong [10]. Computational results further demonstrate that combining two forecasting methods (hybrid dynamic GM(1,1) and hybrid GM(1,1)) to forecast observations and residual values, yields a higher forecasting accuracy than adopting just one of them during the construction of the energy consumption model (GM(1,1), GP, the methodology of Lee and Tong [10], or NGDGM(1,1)). The proposed hybrid dynamic GM model is practically applicable when the structure of the energy consumption data is complex.

5. Conclusions

Developing a high-precision energy consumption model is rather complex owing to various uncertain factors that affect it. Methodologies in the literature often use whole data sets to construct an energy consumption model. However, many uncontrolled factors affect annual energy consumption data. The use of available data to construct a forecasting model may be unreliable when historical observations of energy consumption vary significantly. This work develops a novel hybrid dynamic GM which combines the dynamic grey model with an improved residual equation using GP to forecast energy consumption. Computational results indicate that the hybrid dynamic GM is more accurate and reliable than other forecasting models. Moreover, the proposed model can accurately predict annual energy consumption because it uses dynamic programming. In energy field applications, the proposed method can handle situations in which energy consumption varies greatly in certain years. Additionally, the proposed algorithm can be used to forecast energy consumption using mathematical software. The proposed approach can be used by energy utilities for accurate forecasting.

References