A new method for tool steel materials selection under fuzzy environment

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Abstract

This paper presents a new method to solve the tool steel materials selection problem under fuzzy environment, where the importance weights of different criteria and the ratings of various alternatives under different criteria are assessed in linguistic terms represented by fuzzy numbers. Because the proposed method uses simple arithmetic operations rather than the complicated arithmetic operations for aggregation and ranking of fuzzy numbers described in Wang and Chang (1995), its execution is much faster than the one presented in Wang and Chang (1995). © 1997 Elsevier Science B.V.

Keywords: Fuzzy number; Tool steel; Multiple criteria; Material selection

1. Introduction

In [17], Wang and Chang pointed out that tool steels can be used for cutting tools, locators, clamps, and gauges. There are various kinds of tool steels, such as air-hardening die steels, high-carbon high-chromium die steels, water-hardening tool steels, oil-hardening tool steels, hot-work die steels, shock-resisting tool steels, etc. [1]. In recent years, some studies on the problem of effective tool steel material selection have begun [3, 16, 17]. In [16], Tang presented an analytical model of material selection based on material cost and material quality. In [16], Chen et al. presented a model for raw material selection, where two decision factors were considered in the model, i.e., raw material cost and additional material cost for the inappropriateness of raw material quality. In [17], Wang et al. pointed out that the methods presented in [3, 16] tend to use analytical approach to determine the most suitable material, and they also pointed out that it is difficult to make a suitable tool material selection by the conventional quantitative analysis due to the fact that the characteristics of different materials stated in engineering design handbooks [1, 2] are multi-dimensional and qualitative, and the assessments of these attributes by experts are given in linguistic terms. Thus, in [17], Wang et al. introduced a fuzzy set multiple criteria decision-making approach to help the designer to select the most suitable steel materials. The method presented in [17] allows the importance weights of different criteria assigned by the designers and the material evaluation data under different criteria...
from the design handbooks to be assessed in linguistic terms represented by fuzzy numbers. However, the method presented in [17] is not efficient enough due to the fact that it needs to perform the complicated aggregation and ranking operations of fuzzy numbers to determine the most suitable tool steel material. Thus, it is necessary to develop a more efficient method to help the designer to select the most suitable tool steel materials systematically.

In this paper, we propose a new method to solve the tool steel materials selection problem under fuzzy environment, where the importance weights of different criteria and the ratings of various alternatives under different criteria are assessed in linguistic terms represented by fuzzy numbers. Because the proposed method uses simple arithmetic operations rather than the complicated arithmetic operations for aggregation and ranking [11, 14, 15] of fuzzy numbers presented in [17], its execution is much faster than that presented in [17].

The rest of this paper is organized as follows. In Section 2, we briefly review the theory of fuzzy sets from [4–10, 12, 13, 18]. In Section 3, we briefly review the Wang and Chang [17] method for tool steel materials selection. In Section 4, we present a new method to solve the tool steel materials selection problem under fuzzy environment. In Section 5, we use an example to illustrate the tool steel materials selection process. The conclusions are discussed in Section 6.

2. Fuzzy set theory

In [18], Zadeh proposed the theory of fuzzy sets. Roughly speaking, a fuzzy set is a class with fuzzy boundaries. Let $U$ be the universe of discourse, $U = \{u_1, u_2, \ldots, u_n\}$. A fuzzy set $A$ of $U$ is a set of ordered pairs $\{(u_1, f_A(u_1)), (u_2, f_A(u_2)), \ldots, (u_n, f_A(u_n))\}$, where $f_A$ is the membership function of $A$, $f_A : U \rightarrow [0, 1]$, and $f_A(u_i)$ indicates the grade of membership of $u_i$ in $A$. A fuzzy set $A$ of the universe of discourse $U$ is convex if and only if for all $u_1, u_2$ in $U$,

$$f_A(\lambda u_1 + (1 - \lambda)u_2) \geq \min(f_A(u_1), f_A(u_2)), \quad (1)$$

where $\lambda \in [0, 1]$. A fuzzy set $A$ of the universe of discourse $U$ is called a normal fuzzy set implying that

$$\exists u_i \in U, \quad f_A(u_i) = 1. \quad (2)$$

A fuzzy number is a fuzzy subset in the universe of discourse $U$ that is both convex and normal. According to [13], a fuzzy number $M$ of the universe of discourse $U$ may be characterized by a trapezoidal distribution parametrized by $(a, b, c, d)$ shown in Fig. 1.

In the following, we introduce a defuzzification method of trapezoidal fuzzy numbers [6, 7, 13]. Let us consider a trapezoidal fuzzy number $M$ shown in Fig. 2, where $e$ is a defuzzification value of the
trapezoidal fuzzy number \( M \). From Fig. 2, we can see that
\[
(e - b)(1) + \frac{1}{2}(b - a)(1) = (c - e)(1) + \frac{1}{2}(d - c)(1)
\]
\[
\Rightarrow (e - b) + \frac{1}{2}(b - a) = (c - e) + \frac{1}{2}(d - c)
\]
\[
\Rightarrow (e - b) - (c - e) = \frac{1}{2}(d - c) - \frac{1}{2}(b - a)
\]
\[
\Rightarrow 2e = \frac{d - c - b + a}{2} + \frac{2b + 2c}{2}
\]
\[
\Rightarrow 2e = \frac{a + b + c + d}{2}
\]
\[
\Rightarrow e = \frac{a + b + c + d}{4}.
\] (3)


In [17], Wang and Chang presented a fuzzy set approach to solve the tool steel materials selection problem under fuzzy environment, where the importance weights of the criteria and the performance ratings of criteria assessed by the decision makers are described in fuzzy terms represented by fuzzy numbers. The criteria considered in [17] include:

1. nondeforming properties for materials
2. safety in hardening for materials
3. toughness for materials
4. resistance to softening effect of heat for materials
5. wear resistance for materials
6. machinability for materials
7. cost for materials

where the importance weights of the criteria are assessed in linguistic terms represented by fuzzy numbers, such as very low (VL), low (L), medium (M), high (H), and very high (VH), and the membership functions of the five linguistic terms are shown in Fig. 3.

In [17], Wang and Chang also assumed that the decision-makers can assign the ratings of different tool steels under different selection criteria using linguistic terms represented by fuzzy numbers, such as worst (W), poor (P), fair (F), good (G), and best (B), where the membership functions of the five linguistic terms are shown in Fig. 4.

In [17], Wang and Chang used the aggregation and ranking operations of fuzzy numbers to solve the tool steel materials selection problem. The method is briefly reviewed as follows. Assume that a committee of \( n \) decision-makers is formed to conduct the task of tool steel materials selection, then the \( n \) decision-makers' opinions can be aggregated by

\[
W_t = (1/n) \otimes (W_{t1} \oplus W_{t2} \oplus \ldots \oplus W_{tn}),
\] (4)

where \( W_t \) is the aggregated weighting for criterion \( t \), \( W_{tn} \) is the importance weighting given by decision-maker \( n \) to criterion \( t \), \( k \) is the number of criteria, \( \oplus \) and \( \otimes \) are the addition operator and the multiplication operators of fuzzy numbers, respectively. Furthermore, assume that the performance ratings of different tool steels under different selection criteria are evaluated. If the ratings are assigned by different decision-makers, then their opinions can be aggregated by

\[
R_{it} = (1/n) \otimes (R_{it1} \oplus R_{it2} \oplus \ldots \oplus R_{itn}),
\] (5)
where $i = 1, 2, \ldots, m$, $t = 1, 2, \ldots, k$, $R_{it}$ is the aggregated rating of alternative $i$ under criterion $t$, $R_{ti}$ is the assigned rating of alternative $i$ under criterion $t$ by decision-maker $n$. After the weights and ratings have been assigned and aggregated, $R_{it}$ can further be weighted by the aggregated weight $W_t$ to obtain the final rating $F_i$,

$$F_i = (1/k) \otimes [(R_{i1} \otimes W_1) \oplus (R_{i2} \otimes W_2) \oplus \cdots \oplus (R_{ik} \otimes W_k)],$$

(6)

where $F_i \approx (Y_i, Q_i, R_i, Z_i)$ is the approximated fuzzy number of the fuzzy suitability index of alternative $i$. Finally, Wang et al. used Chen's method [11] of maximizing set and minimizing set to rank the final ratings of different materials to obtain the most suitable tool steel material, where the maximizing set is defined as $M = \{(x, f_M(x)) | x \in \mathbb{R} \}$ with the membership function

$$f_M(x) = \begin{cases} 
(x - x_1)/(x_2 - x_1), & x_1 \leq x \leq x_2, \\
0, & \text{otherwise},
\end{cases}$$

(7)

and the minimizing set is defined as $G = \{(x, f_G(x)) | x \in \mathbb{R} \}$ with the membership function

$$f_G(x) = \begin{cases} 
(x - x_2)/(x_1 - x_2), & x_1 \leq x \leq x_2, \\
0, & \text{otherwise},
\end{cases}$$

(8)

where $x_1 = \inf D$, $x_2 = \sup D$, $D = \bigcup_{i=1}^m D_i$, $D_i = \{x | f_{D_i}(x) > 0\}$, and $i = 1, 2, \ldots, m$. The right utility $U_M(F_i)$ and the left utility $U_G(F_i)$ of each fuzzy rating $F_i$ are defined by

$$U_M(F_i) = \sup_x (f_R(x) \land f_M(x))$$

(9)

and

$$U_G(F_i) = \sup_x (f_R(x) \land f_G(x)),$$

(10)

where $F_i$ is the fuzzy suitability index of alternative $i$, for $i = 1, 2, \ldots, m$. The ranking value $U_T(F_i)$ of $F_i$ is obtained by combining the right and the left utilities,

$$U_T(F_i) = [U_M(F_i) + 1 - U_G(F_i)]/2,$$

(11)

for $i = 1, 2, \ldots, m$. The ranking order is determined by the ranking value. The greater the ranking value $U_T(F_i)$, the higher priority the alternative $i$ will be, where $i = 1, 2, \ldots, m$.

However, the Wang and Chang method presented in [17] is not efficient enough due to the fact that it needs a lot of computation time to perform the aggregation and ranking operations of fuzzy numbers. Thus, it is necessary to develop a more efficient method to solve the tool steel materials selection problem.

### 4. A new method for tool steel materials selection

In this section, we present a new method to solve the tool steel materials selection problem under fuzzy environment. Because the proposed method uses simple arithmetic operations rather than the complicated aggregation and ranking operations of fuzzy numbers mentioned in [17], its execution is much faster than the one presented in [17].

Let us consider the membership functions of the linguistic terms "very low", "low", "medium", "high", and "very high" shown in Fig. 3, where the linguistic terms and their corresponding quadruple representations of fuzzy numbers are shown in Table 1.

By applying formula (3), the defuzzified values of the fuzzy numbers shown in Table 1 can be evaluated as follows.

(1) The defuzzified value of the fuzzy number "very low" is equal to

$$\frac{0 + 0 + 0 + 0.3}{4} = 0.075.$$

(2) The defuzzified value of the fuzzy number "low" is equal to

$$\frac{0 + 0.3 + 0.3 + 0.5}{4} = 0.275.$$

<table>
<thead>
<tr>
<th>Linguistic terms</th>
<th>Fuzzy numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very low (VL)</td>
<td>(0, 0, 0.3)</td>
</tr>
<tr>
<td>Low (L)</td>
<td>(0.3, 0.3, 0.3)</td>
</tr>
<tr>
<td>Medium (M)</td>
<td>(0.2, 0.5, 0.5, 0.8)</td>
</tr>
<tr>
<td>High (H)</td>
<td>(0.5, 0.7, 0.7, 1)</td>
</tr>
<tr>
<td>Very high (VH)</td>
<td>(0.7, 1, 1, 1)</td>
</tr>
</tbody>
</table>
(3) The defuzzified value of the fuzzy number "medium" is equal to
\[
\frac{0.2 + 0.5 + 0.5 + 0.8}{4} = 0.5.
\]

(4) The defuzzified value of the fuzzy number "high" is equal to
\[
\frac{0.5 + 0.7 + 0.7 + 1}{4} = 0.725.
\]

(5) The defuzzified value of the fuzzy number "very high" is equal to
\[
\frac{0.7 + 1 + 1 + 1}{4} = 0.925.
\]

Furthermore, let us consider the membership functions of the linguistic terms, worst (W), poor (P), fair (F), good (G), and best (B) shown in Fig. 4, where the linguistic terms and their corresponding quadruple representations of fuzzy numbers are shown in Table 2.

By applying formula (3), the defuzzified values of these fuzzy numbers can be evaluated as follows.

(1) The defuzzified value of the fuzzy number "worst" is equal to
\[
\frac{0 + 0 + 0 + 0.3}{4} = 0.075.
\]

(2) The defuzzified value of the fuzzy number "poor" is equal to
\[
\frac{0 + 0.3 + 0.3 + 0.5}{4} = 0.275.
\]

(3) The defuzzified value of the fuzzy number "fair" is equal to
\[
\frac{0.2 + 0.5 + 0.5 + 0.6}{4} = 0.5.
\]

(4) The defuzzified value of the fuzzy number "good" is equal to
\[
\frac{0.5 + 0.7 + 0.7 + 1}{4} = 0.725.
\]

(5) The defuzzified value of the fuzzy number "best" is equal to
\[
\frac{0.7 + 1 + 1 + 1}{4} = 0.925.
\]

Assume that there is a committee of n decision-makers (i.e., D1, D2, ..., Dn) to conduct the tool steel selection, and further assume that there are m candidate tool steel materials (i.e., A1, A2, ..., Am) to be considered. Furthermore, assume that there are k basic tool steel characteristics evaluation criteria (i.e., C1, C2, ..., Ck). Let Wtj be the importance weights given by decision-maker Dj to criterion Ct, where Wtj is a linguistic term represented by a fuzzy number parametrized by a quadruple \((a_{tj}, b_{tj}, c_{tj}, d_{tj})\), \(1 \leq j \leq n\) and \(1 \leq t \leq k\), and let \(R_{it}\) be the assigned rating of alternative \(A_i\) under criterion \(C_t\) by the committee of \(n\) decision-makers, where \(R_{it}\) is a linguistic term represented by a fuzzy number parametrized by a quadruple \((p_{it}, x_{it}, y_{it}, z_{it})\), \(1 \leq i \leq m\), and \(1 \leq t \leq k\). By applying formula (3), the defuzzified values of the fuzzy numbers \(W_{tj}\) and \(R_{it}\) are \(w_{tj}\) and \(r_{it}\), respectively,

\[
\begin{align*}
\text{(12)} & \quad w_{tj} = \frac{a_{tj} + b_{tj} + c_{tj} + d_{tj}}{4}, \\
\text{(13)} & \quad r_{it} = \frac{p_{it} + x_{it} + y_{it} + z_{it}}{4},
\end{align*}
\]

\(1 \leq j \leq n, 1 \leq t \leq k, \text{ and } 1 \leq i \leq m.\)

Assume that the importance weight of each criterion assigned by each decision-maker is shown in Table 3, where \(W_{tj}\) is a linguistic term represented by a fuzzy number parametrized by a quadruple \((a_{tj}, b_{tj}, c_{tj}, d_{tj})\), \(1 \leq j \leq n\), and \(1 \leq t \leq k\).

By applying formula (12), the fuzzy number \(W_{tj}\) shown in Table 3 can be defuzzified into a crisp value \(w_{tj}\) shown in Table 4, where \(1 \leq j \leq n, \text{ and } 1 \leq t \leq k.\)

Furthermore, the aggregated importance weight of each criterion \(C_t\) assessed by the committee of
Table 3
The importance weights of the criteria

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Decision-makers</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$D_1$</td>
</tr>
<tr>
<td>$C_1$</td>
<td>$W_{11}$</td>
</tr>
<tr>
<td>$C_2$</td>
<td>$W_{21}$</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>$C_k$</td>
<td>$W_{k1}$</td>
</tr>
</tbody>
</table>

Table 4
Defuzzified values of the importance weights of the criteria

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Decision-makers</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$D_1$</td>
</tr>
<tr>
<td>$C_1$</td>
<td>$w_{11}$</td>
</tr>
<tr>
<td>$C_2$</td>
<td>$w_{21}$</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>$C_k$</td>
<td>$w_{k1}$</td>
</tr>
</tbody>
</table>

$n$ decision-makers can be evaluated by the function $T$:

$$T(C_i) = \frac{w_{i1} + w_{i2} + \cdots + w_{in}}{n}, \quad (14)$$

where $1 \leq i \leq k$.

According to [17], the evaluation criteria can be distinguished into two categories:

1. Subjective criteria (e.g., $C_1, C_2, \ldots, C_{k-1}$).
2. Objective criteria (e.g., $C_k$).

The ratings of different tool steels under different subjective criteria can be assessed by the decision-makers based on the available assessment information given in the design handbook. For example, assume that the ratings of each candidate tool steel material under the subjective criteria (i.e., $C_1, C_2, \ldots, C_{k-1}$) assigned by the committee of $n$ decision-makers are shown in Table 5, where $R_{it}$ is a linguistic term represented by a fuzzy number parametrized by a quadruple $(p_{it}, x_{it}, y_{it}, z_{it})$, $1 \leq i \leq m$, and $1 \leq t \leq k - 1$.

Table 5
Ratings of the tool steel materials under the subjective criteria

<table>
<thead>
<tr>
<th>Subjective criteria</th>
<th>Alternatives</th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>...</th>
<th>$C_{k-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>$R_{11}$</td>
<td>$R_{12}$</td>
<td>...</td>
<td>$R_{1(k-1)}$</td>
<td></td>
</tr>
<tr>
<td>$A_2$</td>
<td>$R_{21}$</td>
<td>$R_{22}$</td>
<td>...</td>
<td>$R_{2(k-1)}$</td>
<td></td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td></td>
</tr>
<tr>
<td>$A_m$</td>
<td>$R_{m1}$</td>
<td>$R_{m2}$</td>
<td>...</td>
<td>$R_{m(k-1)}$</td>
<td></td>
</tr>
</tbody>
</table>

By applying formula (13), the fuzzy number $R_{it}$ shown in Table 5 can be defuzzified into a crisp real value $r_{it}$ shown in Table 6, where $1 \leq i \leq m$, and $1 \leq t \leq k - 1$.

The rating of the objective criteria $C_k$ (e.g., material cost) can be assessed by the following method. Assume that the cost of each tool steel material is represented by fuzzy numbers parametrized by quadruplets shown in Table 7.

Table 6
Ratings of the tool steel materials under the subjective criteria

$A_1$ | $R_{11}$ | $R_{12}$ | ... | $R_{1(k-1)}$ |
$A_2$ | $R_{21}$ | $R_{22}$ | ... | $R_{2(k-1)}$ |
... | ... | ... | ... | ...
$A_m$ | $R_{m1}$ | $R_{m2}$ | ... | $R_{m(k-1)}$ |

By applying formula (3), the defuzzified values of the fuzzy numbers shown in Table 7 can be obtained and are also shown in Table 7, where

$$v_i = \frac{a_{i1} + a_{i2} + a_{i3} + a_{i4}}{4}, \quad (15)$$

and $1 \leq i \leq m$. Then, based on the converting function $F$ [17], the converted rating $F(A_i)$ of the alternative $A_i$ under the objective criteria $C_k$ (i.e., "material cost") can be evaluated as follows:

$$F(A_i) = \frac{1}{v_1 * \left(1 + \frac{1}{v_2} + \cdots + \frac{1}{v_m}\right)}, \quad (16)$$

where $F(A_i) \in [0, 1]$, $1 \leq i \leq m$, and $\sum_{i=1}^{m} F(A_i) = 1$.

Based on Tables 4, 6, 7, and formulas (14)–(16), the ranking values of the tool steel material alternatives $A_1, A_2, \ldots, A_m$ can be evaluated by the function $R$,

$$R(A_i) = r_{i1} * T(C_1) + r_{i2} * T(C_2) + \cdots + r_{i(k-1)} * T(C_{k-1}) + F(A_i) * T(C_k), \quad (17)$$
Table 6
Defuzzified values of the ratings of the tool steel materials under the subjective criteria

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>...</th>
<th>$C_{k-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>$r_{11}$</td>
<td>$r_{12}$</td>
<td>...</td>
<td>$r_{1(k-1)}$</td>
</tr>
<tr>
<td>$A_2$</td>
<td>$r_{21}$</td>
<td>$r_{22}$</td>
<td>...</td>
<td>$r_{2(k-1)}$</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>$A_m$</td>
<td>$r_{m1}$</td>
<td>$r_{m2}$</td>
<td>...</td>
<td>$r_{m(k-1)}$</td>
</tr>
</tbody>
</table>

Table 7
Cost of the tool steel materials

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>Material cost (per piece)</th>
<th>Defuzzified values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>$(a_{11}, a_{12}, a_{13}, a_{14})$</td>
<td>$v_1$</td>
</tr>
<tr>
<td>$A_2$</td>
<td>$(a_{21}, a_{22}, a_{23}, a_{24})$</td>
<td>$v_2$</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>$A_m$</td>
<td>$(a_{m1}, a_{m2}, a_{m3}, a_{mk})$</td>
<td>$v_m$</td>
</tr>
</tbody>
</table>

where $C_1, C_2, ..., C_{k-1}$ are subjective criteria; $C_k$ is an objective criteria; $T(C_j)$ is the aggregated importance weight of criterion $C_j$, where $1 \leq j \leq k$; $r_{11}, r_{12}, ..., r_{1(k-1)}$ are the defuzzified values of the ratings of the alternative $A_i$ under the subjective criteria $C_1, C_2, ..., C_{k-1}$, respectively; $F(A_i)$ is the converted rating of the alternative $A_i$ under the objective criterion $C_k$, and $1 \leq i \leq m$. The larger the value of $R(A_i)$, the higher priority the alternative $A_i$ will have. If $R(A_i)$ is the largest value among the values $R(A_1), R(A_2), ..., R(A_m)$, then the best selection is $A_i$, where $1 \leq i \leq m$.

5. Example

In this section, we use an example to illustrate the proposed method. In order to compare the proposed method with the one shown in [17], we use the example shown in [17] to illustrate the tool steel materials selection process.

Assume that a committee of three decision-makers conduct the task of tool steel materials selection, where the AISI steel numbers are $W_1, A_2, D_2, J_1$, and $F_1$, respectively. The tool steel selection criteria are shown in Table 8. The importance weights of the criteria are shown in Table 9. The ratings of the tool steel materials under the six criteria are shown in Table 10. The cost data of different candidate tool steel materials are shown in Table 11.

Based on Table 1, the linguistic terms shown in Table 9 can be represented by fuzzy numbers parametrized by quadruples shown in Table 12. By
Table 11
Cost of the tool steel materials

<table>
<thead>
<tr>
<th>AISI steel number</th>
<th>Material cost (per piece)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W_1$</td>
<td>(1.5, 1.6, 1.6, 1.7)</td>
</tr>
<tr>
<td>$S_2$</td>
<td>(1.8, 2.0, 2.0, 2.2)</td>
</tr>
<tr>
<td>$D_2$</td>
<td>(1.0, 1.0, 1.0, 1.0)</td>
</tr>
<tr>
<td>$F_1$</td>
<td>(1.2, 1.4, 1.4, 1.6)</td>
</tr>
<tr>
<td>$F_2$</td>
<td>(2.5, 3.0, 3.0, 3.5)</td>
</tr>
</tbody>
</table>

Table 12
The importance weights of the criteria represented by fuzzy numbers

<table>
<thead>
<tr>
<th>Decision-makers</th>
<th>Criteria</th>
<th>$D_1$</th>
<th>$D_2$</th>
<th>$D_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_1$</td>
<td>(0.5,0.7,0.7,1.0)</td>
<td>(0.5,0.7,0.7,1.0)</td>
<td>(0.7,1.0,1.0,1.0)</td>
<td></td>
</tr>
<tr>
<td>$C_2$</td>
<td>(0.2,0.5,0.5,0.8)</td>
<td>(0.5,0.7,0.7,1.0)</td>
<td>(0.2,0.5,0.5,0.8)</td>
<td></td>
</tr>
<tr>
<td>$C_3$</td>
<td>(0.7,1.0,1.0,1.0)</td>
<td>(0.7,1.0,1.0,1.0)</td>
<td>(0.5,0.7,0.7,1.0)</td>
<td></td>
</tr>
<tr>
<td>$C_4$</td>
<td>(0.5,0.7,0.7,1.0)</td>
<td>(0.5,0.7,0.7,1.0)</td>
<td>(0.2,0.5,0.5,0.8)</td>
<td></td>
</tr>
<tr>
<td>$C_5$</td>
<td>(0.2,0.5,0.5,0.8)</td>
<td>(0.2,0.5,0.5,0.8)</td>
<td>(0.2,0.5,0.5,0.8)</td>
<td></td>
</tr>
<tr>
<td>$C_6$</td>
<td>(0.5,0.7,0.7,1.0)</td>
<td>(0.5,0.7,0.7,1.0)</td>
<td>(0.7,1.0,1.0,1.0)</td>
<td></td>
</tr>
<tr>
<td>$C_7$</td>
<td>(0.7,1.0,1.0,1.0)</td>
<td>(0.7,1.0,1.0,1.0)</td>
<td>(0.7,1.0,1.0,1.0)</td>
<td></td>
</tr>
</tbody>
</table>

Table 13
Defuzzified values of the importance weights of the criteria and the aggregated importance weight of each criterion

<table>
<thead>
<tr>
<th>Criteria</th>
<th>$D_1$</th>
<th>$D_2$</th>
<th>$D_3$</th>
<th>Aggregated importance weights</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_1$</td>
<td>0.725</td>
<td>0.725</td>
<td>0.925</td>
<td>0.7917</td>
</tr>
<tr>
<td>$C_2$</td>
<td>0.725</td>
<td>0.725</td>
<td>0.925</td>
<td>0.8583</td>
</tr>
<tr>
<td>$C_3$</td>
<td>0.725</td>
<td>0.725</td>
<td>0.725</td>
<td>0.65</td>
</tr>
<tr>
<td>$C_4$</td>
<td>0.725</td>
<td>0.725</td>
<td>0.725</td>
<td>0.8583</td>
</tr>
<tr>
<td>$C_5$</td>
<td>0.725</td>
<td>0.725</td>
<td>0.925</td>
<td>0.7917</td>
</tr>
<tr>
<td>$C_6$</td>
<td>0.725</td>
<td>0.725</td>
<td>0.925</td>
<td>0.7917</td>
</tr>
<tr>
<td>$C_7$</td>
<td>0.725</td>
<td>0.725</td>
<td>0.925</td>
<td>0.7917</td>
</tr>
</tbody>
</table>

By applying formula (12), the defuzzified values of the fuzzy numbers shown in Table 12 can be evaluated as shown in Table 13. By applying formula (14), the aggregated weight for criterion $C_i$ ($i = 1, 2, \ldots, 7$) is also shown in Table 13.

Based on Table 2, the linguistic terms shown in Table 10 can be represented by fuzzy numbers parametrized by quadruples, and by applying formula (13), the defuzzified values of the fuzzy numbers can be evaluated as shown in Table 14.

By applying formula (3), the defuzzified values of the fuzzy numbers shown in Table 11 can be evaluated as shown in Table 15. Furthermore, by applying formula (16), the converted rating values of the candidate tool steel materials can be evaluated as shown in Table 15.

By applying formula (17), the ranking values of the five candidate tool steel materials can be evaluated as follows:

$$R(W_1) = 0.275 \times 0.7917 + 0.5 \times 0.575 + 0.725 \times 0.8583 + 0.275 \times 0.65 + 0.5 \times 0.5$$

$$= 2.8066,$$
Table 14
Defuzzified values of the ratings of the tool steel materials under the six criteria

<table>
<thead>
<tr>
<th>AISI steel number</th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>$C_3$</th>
<th>$C_4$</th>
<th>$C_5$</th>
<th>$C_6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W_i$</td>
<td>0.275</td>
<td>0.5</td>
<td>0.725</td>
<td>0.275</td>
<td>0.5</td>
<td>0.925</td>
</tr>
<tr>
<td>$Y_2$</td>
<td>0.925</td>
<td>0.925</td>
<td>0.5</td>
<td>0.5</td>
<td>0.725</td>
<td>0.5</td>
</tr>
<tr>
<td>$Y_3$</td>
<td>0.925</td>
<td>0.925</td>
<td>0.725</td>
<td>0.5</td>
<td>0.5</td>
<td>0.275</td>
</tr>
<tr>
<td>$Y_4$</td>
<td>0.5</td>
<td>0.725</td>
<td>0.725</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>$Y_5$</td>
<td>0.725</td>
<td>0.725</td>
<td>0.5</td>
<td>0.925</td>
<td>0.725</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Table 15
Defuzzified values of the cost of tool steel materials and the converted rating of the tool steel materials

<table>
<thead>
<tr>
<th>AISI steel number</th>
<th>Defuzzified values of material cost</th>
<th>Converted rating</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W_i$</td>
<td>1.6</td>
<td>0.1970</td>
</tr>
<tr>
<td>$Y_2$</td>
<td>2.0</td>
<td>0.1576</td>
</tr>
<tr>
<td>$Y_3$</td>
<td>1.0</td>
<td>0.3152</td>
</tr>
<tr>
<td>$Y_4$</td>
<td>1.4</td>
<td>0.2251</td>
</tr>
<tr>
<td>$Y_5$</td>
<td>3.0</td>
<td>0.1051</td>
</tr>
</tbody>
</table>

\[
R(Y_1) = 0.725 \times 0.7917 + 0.725 \times 0.575 + 0.5 \\
\times 0.8583 + 0.925 \times 0.65 + 0.725 \times 0.5 \\
+ 0.5 \times 0.7917 + 0.1051 \times 0.925 \\
= 0.5740 + 0.4169 + 0.4292 + 0.6013 \\
+ 0.3625 + 0.3959 + 0.0972 \\
= 2.877.
\]

From the above calculations, we can see that the ranking order is $Y_2$, $Y_3$, $Y_1$, $Y_4$, and $W_i$. Thus, the best selection is steel number $W_i$. It is obvious that this result coincides with the one presented in [17]. We can see that the proposed method is more simple in calculations than the one presented in [17], and it can be executed much faster than the one presented in [17].

6. Conclusions

In this paper, we have presented a new method to solve the tool steel materials selection problem under fuzzy environment. We also use an example to illustrate the tool steel materials selection process. From the example, we can see that the proposed method is more simple in calculations than the one presented in [17], and its execution is much faster than the one presented in [17]. In this paper, we have assumed that the importance weights of different criteria and the ratings of various candidate tool steel materials under different criteria are assessed in linguistic terms represented by trapezoidal fuzzy numbers. However, the proposed method also can be used in the situation that the importance weights and the rating of the candidate alternatives under different criteria are directly described by trapezoidal fuzzy numbers. The proposed method can provide a useful way to solve the tool steel materials selection problem.

Acknowledgements

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References


