THE ELIMINATION OF NOISE BY THE
OPTIC DUPLEX MODULATION

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Abstract

The optic duplex modulation performed by using two kerr cells as modulator is analyzed. It is possible to modulate a laser beam simultaneously by applying two independent signals to kerr cells at the optic transmitter. The signals are recoverable at the receiver by using photodetector to respond the appropriate polarization components of light. The application of duplex modulation is to eliminate the noises due to atmospheric turbulence. The result of experiments shows that the atmospheric turbulent noises are really eliminated by the method. Finally the advantages and disadvantages to use kerr-effect rather than pockel effect for optic modulation system are compared.
I. INTRODUCTION

Certain crystals and liquids become birefringent in the presence of electric field. Therefore, the index of refraction of these media can be modified by applying electric field on them. By using this electro-optic (e-o) effect, many modulation techniques were developed for laser communication. The history of e-o effect might begin with kerr effect discovered in 1875. However, it was not until 1943 that the large e-o effects of potassium dihydrogen phosphate (KDP) and ammonium dihydrogen phosphate (ADP) were discovered. Sufficient data of the e-o effects were published in 1949 by Billings. Since then, the linear e-o (pockel) effect had been widely employed in the optic modulation system.

Buhrer et al. used zinc sulfide, a pockel effect crystal having three fold rotation axis, as a modulator for duplex modulation and single side band modulation in 1963. In their analysis, a light beam can be modulated simultaneously by two independent signals. And it was recoverable at the receiver by photodetectors responding to appropriate polarization components of light beam. The work done in this paper is to perform the same duplex modulation but using kerr (quadratic e-o) effect instead of pockel (linear e-o) effect in the modulation system.

The analysis of duplex modulation and its application is carried out detailly in section II. Section III shows the experimental setup which contain the structure of the kerr cell. The result of the elimination of noise is discussed in section IV. Finally the advantages of kerr effect is compared with pockel effect in section V.

II. THEORETICAL ANALYSIS

The method of analysis of this paper is based upon a series of papers developed by R. Clark Jones in 1941. The function of kerr cell is regarded as a retardator in an optic modulation system. The phase of laser beam is modulated by kerr cell. By using some other optic components, such as polarizer, quarter wave plate etc, we can change the phase modulation into intensity modulation.

2.1 Optic Duplex Modulation

The arrangement of duplex modulation is shown in Fig. 1. A left circularly polarized light propagates through two kerr cells. The optic axis of the first cell is adjusted parallel to x axis whereas that of the second cell bisected to the x and y axis. Let the left circularly polarized wave be

\[ E_x = E_{i\theta} \cos \omega t \]

\[ E_y = E_{i\theta} \sin \omega t \]
where $E_{10}$ is the amplitude of laser beam. Expressed in complex vector form, the electric field of the light beam is

$$E = \begin{pmatrix} E_x \\ E_y \end{pmatrix} = E_{10} \begin{pmatrix} 1 \\ j \end{pmatrix} e^{j\omega t}$$

Let the electric field $\varepsilon_1$ and $\varepsilon_2$ applied to cell one and cell two respectively be

$$\varepsilon_1 = \varepsilon_{dc} + S_1$$
$$\varepsilon_2 = \varepsilon_{dc} + S_2$$

Here $\varepsilon_{dc}$ is the d.c electric field and $S_i$ is the signal, $i=1,2$. Assume $S_i \ll \varepsilon_{dc}$ such that the phase retardation $\Gamma_i$ is

$$\Gamma_i = 2\pi K_1 \varepsilon_{dc}^2 + 4\pi K_1 S_i$$

(1)

In Fig. 1, the fast axis of cell one is parallel to $y$ axis. The matrix of retardator one can be expressed as

$$M_1 = \begin{pmatrix} e^{j(\Gamma_1/2)} & 0 \\ 0 & e^{-j(\Gamma_1/2)} \end{pmatrix}$$

(2)
The fast axis of cell two bisected to \(x\) and \(y\) axis, the retardation matrix should be transformed \(\frac{\lambda}{4}\), i.e.

\[
M_2 = \begin{pmatrix}
\cos \frac{\pi}{4} & -\sin \frac{\pi}{4} \\
\sin \frac{\pi}{4} & \cos \frac{\pi}{4}
\end{pmatrix} \left[
\begin{array}{c}
e^{i(\Gamma_x/2)} \\
e^{-i(\Gamma_y/2)}
\end{array}
\right] \begin{pmatrix}
\cos \frac{\pi}{4} & \sin \frac{\pi}{4} \\
-\sin \frac{\pi}{4} & \cos \frac{\pi}{4}
\end{pmatrix}
\]

(3)

Then the transmitted light to the receiver is

\[
\rightarrow E_0(t) = M_2M_1 \begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} E_{0x}(t) \\ E_{0y}(t) \end{pmatrix}
\]

\[
= \frac{E_{ix}^2}{2} \left[
\begin{array}{c}
e^{i(\Gamma_3^1+\Gamma_1^1)/2} + e^{i(\Gamma_1^1-\Gamma_3^1)/2} + j(e^{i(\Gamma_2^1-\Gamma_1^1)}/2 - e^{-i(\Gamma_1^1+\Gamma_3^1)/2}) \\
e^{i(\Gamma_1^1+\Gamma_2^1)/2} - e^{-i(\Gamma_1^1-\Gamma_2^1)/2} + j(e^{i(\Gamma_1^1+\Gamma_2^1)/2} + e^{-i(\Gamma_1^1+\Gamma_2^1)/2})
\end{array}
\right] e^{i\omega_0 t}
\]

(4)

The time average of the carrier intensity of \(x\) polarization component of light over a period \(T\) is

\[
\overline{I_{ox}} = \frac{1}{T} \int_0^T E_{ox} E_{ox}^* dt
\]

\[
= \frac{E_{ix}^2}{4} \left[4 - 2 \sin (\Gamma_1 + \Gamma_2) + 2 \sin (\Gamma_1 - \Gamma_2) \right]
\]

for small signal approximation

\[\sin (\Gamma_1 + \Gamma_2) \approx \Gamma_1 + \Gamma_2\]
\[\sin (\Gamma_1 - \Gamma_2) \approx \Gamma_1 - \Gamma_2\]

then \(\overline{I_{ox}} \approx E_{ix}^2 (1 + \Gamma_1)\)

(5a)

similarly \(\overline{I_{oy}} \approx E_{iy}^2 (1 - \Gamma_1)\)

(5b)

If the axis of analyzer is rotated 45° such that it bisected to \(x\) and \(y\) axis.

\[
\overline{I_{ox/y}} = \frac{1}{T} \int_0^T E_{ox/y} E_{ox/y}^* dt = E_{ix}^2 (1 + \Gamma_2)
\]

(6a)

\[
\overline{I_{oy/x}} = E_{iy}^2 (1 - \Gamma_2)
\]

(6b)

Let \(I_{ox/\pm y} = E_{ix}^2 (1 \pm 2 \pi K_1 \varepsilon \delta \epsilon^2)\)

\[K' = 4 \pi K_1 \varepsilon \delta \epsilon E_{iy}^2\]

Substitute Eq. 1 into Eq. 5 and Eq. 6, we get

\[
\overline{I_{ox}} = I_{ox/\pm y} + K'S_1
\]
\[
\overline{I_{oy}} = I_{ox/\pm y} - K'S_1
\]
\[
\overline{I_{ox/y}} = I_{ox/\pm y} + K'S_2
\]
\[
\overline{I_{oy/x}} = I_{ox/\pm y} - K'S_2
\]

(7)
The photodetector produces an output current proportional to the instantaneous intensity of the carrier. It may be regarded as a linear intensity-to-current converter or a quadratic (square law) converter of optical electric field-to-detector current. The photodetector output current, \( i_p \), is proportional to the time average of the instantaneous carrier intensity \( I_c \), i.e.,

\[
i_p = DI_c
\]  

(8)

\( D \) is the detector conversion factor which is depend upon the type of photodetector employed. Thus we have two output current \( i_{pe} \propto S_1 \) and \( i_{pe/V} \propto S_2 \) at the output of the photomultipliers as shown in Fig. 1.

2.2 The Elimination of Noise

Let \( N \) be the noise current caused by atmospheric turbulence through which the laser beam propagates. In this case, we don't consider the background noise and detection noise due to the photodetector itself. At the output of the receiver, \( S_1 + N_1 \) and \( S_2 + N_2 \) will be detected. Since the responding current of photodetector is insensitive to the frequency, phase, or polarization of the carrier over its operating regime. The noise currents of \( S_1 + N_1 \) and \( S_2 + N_2 \) will be independent of the direction of the analyzer through which the laser beam passes. This means that \( N_1 \) is equal to \( N_2 \).

Now let only one signal modulate the laser beam, say \( S_2 = 0 \). The output currents of photodetector as shown in Fig. 2 will be proportional to \( S_1 + N \) and \( N \). Thus a differential amplifier may be used following the twindetector to form their difference and get just the signal without noise.
III. EXPERIMENTAL SETUP

In the experiment, the plane polarized light source is a Jodon model 1576 He-Ne laser. As shown in Fig. 1, a quarter wave plate is put in front of the kerr cells to change the linear polarized wave into left circularly polarized wave. The kerr cells were made of metal electrodes with which nitrobenzene filling between them. In Fig. 3, the half cylindrical electrode

Fig. 3 The Photography of kerr cell.

Fig. 4 The arrangement for measuring the kerr constant.
with plane face, 2.1×30 cm² were separated by a grain of glass with a distance 0.3 cm. Since the nitrobenzene we got doesn’t have standard quality, it is necessary to measure the kerr constant to make sure the quality of kerr cell. The same method as conventional amplitude modulation, as shown in Fig. 4, a power meter jodeon 450B is used to measure the half wave retardation voltage \( V_{1/2} \) i.e. \( V = V_{1/2} \) when \( \Gamma = \pi \). Now \( 1 = 30 \) cm, \( d = 0.3 \) cm, \( V_{1/2} = 2100 \) volt.

\[
\Gamma = 2\pi K1(V/d)^2
\]

\[
\therefore K = 3.5 \times 10^{-10} \text{ cm/V}^2
\]

Two audio test signals, one of them is a sine wave with frequency \( 2K \) \( H \). \( (\epsilon_1 = d.c (260 \text{ V}) + a.c (p-p 80 \text{ V}) \) \), the other is a square wave with period \( 5 \times 10^{-4} \) sec \( (\epsilon_2 = d.c (260 \text{ V}) + a.c (p-p 80 \text{ V}) \) \) were applied to the kerr cells respectively. At the receiver, the incoming light is divided by a beam splitter and brings each half through different analyzer into the photomultipliers (RCA 931A). By using a Tektronix 549 storage scope, in Fig. 5, an oscillogram shown, from top to bottom, the input sine wave, the detected output sine wave, the input square wave and the detected output square wave.
The experiment of the elimination of noise has been done with a longer optic path through the open air. Since the limitation of experimental equipment, the distance about 150 meter between transmitter and receiver is not long enough. After two times of reflection, the optic receiver is placed in the same room of which optic transmitter is set up. As shown in Fig. 6, the transmitted light should be collimated by two pieces of lens. At the receiver, the incoming light is collected by a telescope. Two pieces of optical bandpass filter are used to reject the background noise. The
distance of optic path in the experiment is too short to produce significant noise as shown in Fig. 7(a), various type of noise source such as a stationary heat source, transverse direction hot wind and longitudinal-direction hot wind are applied to the light beam. In Fig. 7, an oscillogram shows from top to bottom, modulating signal (trace one), the output of one of the photomultipliers ($S+N$, trace two), the output of the other photomultipliers ($N$, trace three) and the output of differential amplifier ($S$, trace four).

IV. EXPERIMENTAL RESULT AND DISCUSSION

![Fig. 7(a)](image)

![Fig. 7(b)](image)
Fig. 7 The oscillogram of elimination of noise experiment.

Fig. 7 The oscillograms of elimination of noise experiment done at outdoors with an optic path about 150 M. From top to bottom, modulating signal, signal plus noise, noise only and the output of diff. amp. (signal only). (a) without applying external noise, (b) a stationary heat source applied below optic path, (c) a cross direction hot wind blowing, (d) a longitudinal direction hot wind blowing.
The detected noises at the receiver originate both from the air through which the light propagates and from the photodetector itself. The major noises of the photodetector are thermal noise and photon fluctuation shot noise. The thermal noise voltage and current caused by thermal fluctuations of electrons in a resistor are Gaussian variables, whereas the photon fluctuation shot noise due to emission fluctuations of the optic radiation incident upon the photodetectors are Poisson random variables. The high frequency noises appearing in Fig. 5 and Fig. 7 are the photon shot noise which can be reduced by connecting a low pass filter after the photomultipliers.

Although the present work aims to eliminate the noise due to atmospheric turbulence, the method to form the difference of $S+N$ and $N$ by using a twin detector also do some favors to the optimigation of the threshold decision of the receiver. In general, a threshold value is decided in the sense that it minimizes the probability of detection error for PCM/IM system. By examining Fig. 7, trace four (output signal, $S$) of the oscillogram in each case shows a better (thinner) shape than that of trace two (output signal plus noise, $S+N$). It demonstrates that the photon shot noise is reduced. Further by comparing the waveform of trace two and trace four shows that the atmospheric turbulent noise almost has been eliminated completely. Thus we see that the elimination of noise by using kerr effect for duplexmodulation got a quite satisfactory result.

V. CONCLUSION

The object of the paper is to show that kerr effect is suitable for duplex modulation. To compare the kerr cell with conventional electro-optic crystal such as KDP, ADP etc, the former can be fabricated with longer optic path such that the voltage of modulating signal may be smaller. But it is not widely used because the difficulty in handling the kerr effect liquids which are generally caustic and poisonous. Also, application of fields to these liquids for prolonged period often causes decomposition. Nitrobengene has the largest kerr constant while carbon disulfide possess better properties for high frequency modulation and is regarded as the most suitable liquid material for microwave modulation.
Some new organic material, such as nematic liquid crystal
P-[(methoxybenzylidene) amino]-benzonitrile (MBAB) and
P-[(P-ethoxybenzylidene) amino]-benzonitrile (EBAB)
possess large kerr constant (about 30 times larger than nitrobenzene) and
fast time response (about $10\sim30$ nsec) are discovered. This is a strong
encouragement for kerr effect to be utilized in optic modulation system.

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1. INTRODUCTION

There has been considerable interest in the double-layered modulator
structures in Si devices because of their application in non-volatile
insulated-gate field-effect memory transistors. In particular, the MOS
(Metal/Silicon-oxide/Silicon-oxide/Semiconductor) and MAOS
(Metal/Aluminum-oxide/Silicon-oxide/Semiconductor) systems with
very thin oxide (CA 4) has been examined in detail.

MAOS system is essentially a Metal/Oxide/Semiconductor (or Metal/
Insulator/Semiconductor) structure. The characteristics of the MOS structure
has been described in its exponential voltage curve. The position of the C-V
curve depends upon the effective resistance of the oxide material. For
convenience, the flatband voltage ($V_{FB}$) evaluated at the oxide
surface, has been commonly used as a reference for analyzing the effects
about the flat energy band condition. Although the oxide surface, has been shown to

It is known that some trapping occurs at the interface between
these two oxide layers, and these traps can react chemically with the silicate
when a field is applied across the junction. Therefore, by applying an
appropriate positive voltage to the metal gate, electrons in the semiconductor
can be injected through the Si, this layer and get trapped at the trapping
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V. CONCLUSION

The object of the paper is to show that Kerr effect is suitable for dual modulation. To compare the Kerr cell with conventional electro-optic crystals such as KDP, AIP etc., the former can be fabricated with shorter optic paths such that the voltage or modulating signals may be smaller. But it is not widely used because the difficulty in handling the Kerr effect liquids which are generally cationic and poisonous. Also, application of fields in these liquids for prolonged periods cause serious decomposition. Nitrobenzene has the largest Kerr constant while certain diethyloxazane better properties for high frequency modulation and is regarded as the most suitable liquid material for microwave modulation.