Type-2 fuzzy controller design using a sliding-mode approach for application to DC–DC converters


Abstract: Fuzzy controllers and fuzzy sliding-mode controllers have found extensive use in a variety of applications. Generally, type-1 fuzzy sets are used for the membership functions of these controllers. However, in real-time applications, uncertainty associated with the available information always occurs. A type-2 fuzzy controller and a type-2 fuzzy sliding-mode controller are proposed that are able to solve the problem that the words used in the inference rules can mean different things to different people. Since the membership functions use type-2 fuzzy sets then, the proposed control schemes can handle the rule uncertainties when the operation is extremely uncertain and/or the membership grades cannot be exactly determined. The proposed control systems are applied to control a buck DC–DC converter. A comparison between a PI controller, a type-2 fuzzy controller and a type-2 fuzzy sliding-mode controller is made. The experimental results show that the type-2 fuzzy sliding-mode controller achieves the best control performance.

1 Introduction

Type-1 fuzzy controllers (T1FC) include those based on a fuzzifier, rules, an inference engine or a defuzzifier. They have been successfully used in numerous applications many of which are too complex to be analysed using conventional mathematical techniques [1, 2]. Many of the operations in a T1FC system use the error and change-of-error as the fuzzy input variables. However, the large number of fuzzy rules required by a T1FC system makes the analysis complex. In order to reduce the number of required fuzzy rules, approaches based on sliding-mode control, referred to as type-1 fuzzy sliding-mode control (T1FSMC) have been proposed [3–5]. Since in this case only one variable is required to be defined as the fuzzy input variable in the fuzzy rules, the main advantage of T1FSMC is that its number of fuzzy rules is smaller than that for T1FC. Moreover, the use of sliding-mode control, results in the system being more robust against parameter variation and external disturbances [4]. The design of T1FC and T1FSMC requires the experience and knowledge of human experts to decide both the membership functions and the fuzzy rules. Since the membership grade of the T1FC and T1FSMC is a crisp number in [0,1], they are unable to directly handle rule uncertainties. In addition, in real-time applications, the words that are used in the fuzzy rules can often mean different things to different people. This will result in rule uncertainty with the available information.

To tackle this problem, Zadeh [6] proposed the concept of a type-2 fuzzy system which is an extension of a type-1 fuzzy system. A type-2 fuzzy system is again characterised by IF-THEN rules, but its membership functions are now type-2 fuzzy sets. The structures of type-1 and type-2 fuzzy systems are shown in Figs. 1a and 1b, respectively. The structure of a type-2 fuzzy system is very similar to the structure of a type-1 fuzzy system with differences only occurring in the output processing. The output processor includes a type reducer and a defuzzifier to generate a type-1 fuzzy system output from the type reducer or a crisp number from the defuzzifier. Thus, the type reduction captures more information about rule uncertainties than does the defuzzified value (a crisp number). A type-2 fuzzy system is characterised by a fuzzy membership function, i.e. the membership grade for each element is a fuzzy set in [0, 1], unlike the type-1 fuzzy system in which the membership grade is a crisp number in [0, 1]. Thus, a type-2 fuzzy system is very useful in circumstances in which the membership grades are difficult to exactly determine [6–10].

DC–DC converters are power electronic systems that convert one level of electrical voltage into another level using a switching action [11, 12]. They are used extensively in personal computers, computer peripherals, and adapters for consumer electronic devices. A control technique for DC–DC converters must not only cope with their wide input voltage and load variations to ensure stability in any operating condition but also still provide a fast transient response. The control of DC–DC converters has been attempted using: (i) output feedback linearisation theory [12, 13]; (ii) a sliding-mode control approach [14]; and (iii) a fuzzy control technique [14–16]. In the feedback linearisation control design, although the controller is simple to implement and easy to design, its performance generally depends on the working point. However, the control parameters which ensure proper behaviour in any operating conditions are difficult to obtain. In sliding-mode control design, a system model is required for the controller design. The main disadvantage of this approach is control chattering. In fuzzy control design the fuzzy controller is able to regulate the output voltage to a desired value...
Fig. 1 The structures of the fuzzy systems
a A type-1 fuzzy system
b A type-2 fuzzy system

without steady-state oscillations despite changes in the load resistance or input voltage. However, too many fuzzy rules need to be constructed to create a successful design.

In [14–16], since the membership functions of the fuzzy controller for the DC–DC converters use type-1 fuzzy sets, the grades of the membership function need to be determined using a time-consuming trial-and-error tuning procedure to achieve a satisfactory performance. To tackle this problem, we intend to propose a type-2 fuzzy controller (T2FC) for a DC–DC converter. Moreover, to reduce the number of fuzzy rules and to strengthen the robust characteristics, a type-2 fuzzy sliding-mode controller (T2FSMC) will also be developed. The T2FC and T2FSMC will be able to handle any uncertainties due to linguistic interpretation by using type-2 fuzzy sets to determine the membership functions. Thus, they will be suitable to control a buck DC–DC converter. We will perform experiments to demonstrate that the T2FC and T2FSMC can achieve robust characteristics and regulation performance for the input voltage and load resistance variations. The use of the T2FSMC will not only reduce the implementation complexity but will also achieve a better regulation performance by defining a sliding surface as the fuzzy input variable. Thus, the proposed T2FSMC will be highly suitable to control a buck DC–DC converter.

2 Converter modelling and control objective

A buck DC–DC converter is used to drop DC voltages. The circuit of a buck DC–DC converter is shown in Fig. 2, where \( C \) is output capacitor, \( L \) is the inductor, \( R \) is the load resistor, \( r_L \) is the inductor series resistance, \( r_C \) is the capacitor series resistance, \( V_i \) is the input voltage, and \( V_o \) is output voltage. The state equation for a buck DC–DC converter can be written as [11]:

\[
\dot{x} = Ax + Bu \quad V_o = Cx \tag{1}
\]

where \( x = [i_L \ v_C]^T \), and \( A, B, \) and \( C \) are system matrices of the constituent linear circuits. The system matrices represent different operating modes (a subscript ‘1’ stands for a transistor being on, and a subscript ‘0’ stands for a transistor being (off) of the converter circuit. The system matrices can be obtained for different operating modes as:

\[
\begin{align*}
A_1 = A_2 &= \begin{bmatrix}
-1/(L(R + r_C)) + r_L & -1/R \\
1/C(R + r_C) & -1/C(R + r_C)
\end{bmatrix} \\
B_1 &= \begin{bmatrix} 1 \\ 0 \end{bmatrix} \\
B_2 &= \begin{bmatrix} 0 \\ 0 \end{bmatrix} \\
C_1 = C_2 &= \begin{bmatrix} 0 & R \\
R & R + r_C \end{bmatrix}
\end{align*}
\tag{3}
\]

The state-space averaging method is a very useful technique to analyse the low-frequency small-signal performance of switch circuits [11, 12]. It is applicable when the converter switching period is short as compared to the response time of the output voltage. Using the state-space averaging method, the state equation can be obtained as:

\[
\dot{x} = Ax + Bu \quad V_o = Cx \tag{7}
\]

where \( A = dA_1 + (1 - d)A_2, B = dB_1 + (1 - d)B_2, C = dC_1 + (1 - d)C_2 \) and \( d \) is the switching duty cycle. The control problem is to control the duty cycle so that the output voltage can supply a fixed voltage under the occurrence of uncertainties such as a wide input voltage and load resistance variations. The error voltage is defined as:

\[
e = V_{ref} - V_o \tag{9}
\]

where \( V_{ref} \) is the reference output voltage. The control law of the switching duty cycle is determined by the error voltage signal to provide a fast transient response. The output of the designed controller \( \delta d(k) \) is the change in the duty cycle. The duty cycle \( d(k) \), at the kth sampling time, is determined by adding the previous duty cycle \( d(k - 1) \) to the calculated change in duty cycle:

\[
d(k) = d(k - 1) + \delta d(k) \tag{10}
\]

The calculated duty cycle signal is then sent to a pulse width modulation (PWM) output stage that generates the appropriate switching pattern for the switch in the DC–DC converter. In addition, a ramp waveform voltage \( V_{ramp} \) in the PWM output stage should be limited to be a constant voltage at the operation point.

3 Design of type-2 fuzzy controller

The block diagram of a T2FC for a DC–DC converter is shown in Fig. 3. Assume that there are \( M \) rules in the type-2 fuzzy system, each of which has the following form

Rule \( i: \) IF \( e \) is \( \tilde{G}_e^i \) and \( \dot{e} \) is \( \tilde{G}_e^i \) THEN \( \delta d(k) \) is \([\tilde{u}_1^i, \tilde{u}_2^i] \)

\[
\text{Rule } i: \text{IF } e \text{ is } \tilde{G}_e^i \text{ and } \dot{e} \text{ is } \tilde{G}_e^i \text{ THEN } \delta d(k) \text{ is } [\tilde{u}_1^i, \tilde{u}_2^i] \tag{11}
\]
The THEN-part a The IF-part

\[ F^i = \left[ \begin{array}{c} f^i \\ \bar{f}^i \end{array} \right] \] (12)

where

\[ f^i = \mu_{G^i}^1(e) \times \mu_{G^i}^2(e) \] (13)

\[ \bar{f}^i = \pi_{G^i}^1(e) \times \pi_{G^i}^2(e) \] (14)

in which \( \mu(\cdot) \) and \( \pi(\cdot) \) denote the grade of the lower membership function and the upper membership function, respectively. A singleton fuzzification with a minimum \( t \)-norm is used in this work and is shown in Fig. 5. The output can be expressed as:

\[ \delta d_{\text{cos}} = [\delta d_1, \delta d_L] \] (15)

where \( \delta d_{\text{cos}} \) is an interval type-1 set determined by the left and right end points (\( \delta d_1 \) and \( \delta d_L \)), which can be derived from the consequent centroid set \( [w_t^r, w_t^l] \) and firing strength \( f^i \in F^i = [f^i, \bar{f}^i] \). The interval set \( [w_t^r, w_t^l] \) (i = 1, ..., M) should be computed or set first before the computation of \( \delta d_{\text{cos}} \). The left-most point \( \delta d_1 \) and the right-most point \( \delta d_L \) can be expressed as [8, 9]:

\[ \delta d_1 = \frac{\sum_{i=1}^{M} f^i w_t^{ri}}{\sum_{i=1}^{M} f^i} \] (16)

and

\[ \delta d_L = \frac{\sum_{i=1}^{M} f^i w_t^{li}}{\sum_{i=1}^{M} f^i} \] (17)

We briefly state the procedure to compute \( \delta d_1 \) and \( \delta d_L \). First of all, we compute the right-most point \( \delta d_L \). Without loss of generality, assume that the \( w_t^r \) are arranged in ascending order, i.e. \( w_t^1 \leq w_t^2 \leq \cdots \leq w_t^M \).

Step 1: Compute \( \delta d_1 \) in (17) by initially using \( f^i = (f^i + \bar{f}^i)/2 \) for \( i = 1, 2, \ldots, M \), where \( f^i \) and \( \bar{f}^i \) are pre-computed by (13) and (14); and let \( \delta d_L = \delta d_1 \).

Step 2: Find \( R (1 \leq R \leq M - 1) \) such that \( w_t^R \leq \delta d_L \). Without loss of generality, assume that \( \delta d_L = \delta d_L^r \). We have to do the fuzzification and defuzzification for the interval case.

Step 3: Compute \( \delta d_1 \) in (17) with \( f^i = f^i \) for \( i \leq R \), and \( f^i = \bar{f}^i \) for \( i > R \). Let \( \delta d_L^r = \delta d_L \).

Step 4: If \( \delta d_L^r \neq \delta d_1^r \), then go to step 5. If \( \delta d_L^r = \delta d_1^r \), then set \( \delta d_1 = \delta d_L^r \) and go to step 6.

Step 5: Set \( \delta d_1^r = \delta d_L^r \) and return to step 2.

Step 6: End.

Hence, \( \delta d_1 \) in (17) can be re-expressed as:

\[ \delta d_1 = \delta d_1(f^1, \ldots, f^R, \bar{f}^1, \ldots, \bar{f}^M, w_t^1, \ldots, w_t^M) \]

\[ = \frac{\sum_{i=1}^{R} f^i w_t^i + \sum_{i=R+1}^{M} \bar{f}^i w_t^i}{\sum_{i=1}^{R} f^i + \sum_{i=R+1}^{M} \bar{f}^i} \] (18)

The procedure to compute \( \delta d_1 \) is similar to that for \( \delta d_1 \), with slight modifications as stated below. In step 2, we need to find \( L (1 \leq L \leq M - 1) \), such that \( w_t^L \leq \delta d_1^L \leq w_t^{L+1} \). In step 3, let \( f^i = \bar{f}^i \) for \( i \leq L \), and \( f^i = f^i \) for \( i > L \). Therefore, \( \delta d_1 \) in (16) can be expressed as:

\[ \delta d_1 = \delta d_1(\bar{f}^1, \ldots, \bar{f}^L, f^{L+1}, \ldots, \bar{f}^M, w_t^1, \ldots, w_t^M) \]

\[ = \frac{\sum_{i=1}^{L} \bar{f}^i w_t^i + \sum_{i=L+1}^{M} f^i w_t^i}{\sum_{i=1}^{L} \bar{f}^i + \sum_{i=L+1}^{M} f^i} \] (19)

Then, the defuzzified crisp output from an interval type-2 fuzzy system is the average of \( \delta d_1 \) and \( \delta d_L \), i.e.
The fuzzy rules for a T2FC are summarised in Fig. 6, which is constructed for the scenario in which \( e \) and \( \dot{e} \) approach zero with a fast rise time and without a large overshoot. Generally, determination of these rules comes from human knowledge and via some trial-and-error processes.

### 4 Design of a T2FSMC

The sliding surface plays a very important role in the design of a T2FSMC. It can dominate the dynamic behaviour of the control system as well as reduce the size of the fuzzy rule base. The sliding surface is chosen as the input variable of the control system as well as reduce the size of the fuzzy rule base. The sliding surface is chosen as the input variable of the fuzzy inference rules so that the number of fuzzy rules can be less than those where the state error variables \( e \) and \( \dot{e} \) are used as the input variables. A sliding surface is defined by the following scale function:

\[
s = \dot{e} + \lambda e
\]

where \( \lambda > 0 \) is a given positive constant. A block diagram of a T2FSMC for a DC–DC converter is shown in Fig. 7. Assume that there are \( N \) rules for the T2FSMC, each of which has the following form:

\[
\text{Rule } j: \text{ IF } s \text{ is } G^j_s, \text{ THEN } \delta d_{\text{fsmc}} = [r^j_1, r^j_2]
\]

where \( j = 1, 2, \ldots, N \), \( G^j_s \) is the interval type-2 fuzzy sets of the antecedent part, and \( r^j_1 \) and \( r^j_2 \) are the singleton upper and lower control actions. The fuzzy rules for the T2FSMC are summarised in Fig. 8, which is constructed using the basic idea that if the state is far away from the sliding surface then a large control effort needs to be applied, and if the state is near the sliding surface then only a small control effort needs to be applied. Therefore, the state can quickly reach the sliding surface without a large overshoot. Based on the above discussion, the controller output is accomplished.

### 5 Experimental results

The experimental system for the computer control of a buck DC–DC converter is shown in Fig. 9. A servo control card was installed in the control computer which has D/A, A/D, PIO and encoder interface circuits. The control problem consists in the control of the duty cycle so that the output voltage can supply a fixed voltage \( (V_{\text{ref}} = 10 \text{ V}) \) despite the occurrence of uncertainties such as a wide input voltage and load variations. The proposed control algorithms were realised for the Pentium chip using the Turbo C language. Two experimental cases were addressed and they are as follows: (i) the nominal case (the input voltage is set as \( V_s = 20 \text{ V} \)); (ii) the input variation case (the input voltage is changed to \( V_s = 25 \text{ V} \)). In both cases, some load resistance variations with step changes were tested: (i) from 20 to 5 \( \Omega \) at 300 ms; (ii) from 5 to 20 \( \Omega \) at 500 ms; and (iii) from 20 to 5 \( \Omega \) at 700 ms. The circuit parameter values of the buck DC–DC converter were chosen to be \( R = 20 \Omega \), \( L = 500 \mu\text{H} \) and \( C = 2200 \mu\text{F} \). The converter ran at a switching frequency of 20 kHz and the controller ran at a sampling frequency of 2 kHz. The duty cycle was generated using a PWM IC SG1825. The generated duty cycle is directly proportional to the analog output of the controller.

To compare the regulation efficiency, first a proportional-integral (PI) controller proposed in [13] is applied to the buck DC–DC converter. The controller output is computed as:

\[
\delta d_{\text{PI}} = 0.2 \dot{e} + 0.05 \ddot{e}
\]

The experimental results for the PI controller for the two studied two cases are shown in Fig. 10. The converter responses are shown in Figs. 10a and 10c; and the

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**Fig. 7** Block diagram of T2FSMC for a DC–DC converter system

**Fig. 8** Fuzzy rules of T2FSMC for a buck DC–DC converter

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associated control efforts are shown in Figs. 10b and 10d, respectively. The experimental results, show that the PI controller can achieve a fast tracking performance, however, there exists a 10% overshoot and the state feedback gain should be constructed using trial-and-error techniques to ensure proper behaviour in the operating conditions. The T2FC was then applied to the buck DC–DC converter. The experimental results for the T2FC system for the two considered cases are shown in Fig. 11. The converter responses are shown in Figs. 11a and 11c; and the associated control efforts are shown in Figs. 11b and 11d, respectively. It can be seen that the regulation performance of the T2FC is better than that of the PI controller, and no overshoot do not appears in the T2FC system. However, the large number of fuzzy rules required by the T2FC system results in a complex analysis and difficult implementation. Finally, the T2FSMC with \( \lambda = 5 \) was applied to the buck DC–DC converter. The experimental results of the T2FSMC system for the two considered cases are shown in Fig. 12. The converter responses are shown in Figs. 12a and 12c; and the associated control efforts are shown in
It can be seen that the T2FSMC can achieve a better regulation performance than the PI controller and the T2FC, and that again to overshoot occurred. A settling time comparison between the PI controller, the T2FC and the T2FSMC is made in Fig. 13. For the nominal case, as shown in Fig. 13a, the settling times of the PI controller, the T2FC and the T2FSMC are 33, 21 and 20 ms, respectively. For the input variation case, as shown in Fig. 13b, the settling times of the PI controller, the T2FC and the T2FSMC are 27, 19 and 18 ms, respectively. In conclusion, the T2FC and T2FSMC can achieve a better performance than the PI controller. In addition, the number of fuzzy rules can be minimised by use of the T2FSMC. This in turn reduces the complexity of the analysis and case of implementation when using the sliding surface. Thus, the T2FSMC design method is highly suitable for application to DC-DC converters.

6 Conclusions

We have clearly demonstrated that T2FC and T2FSMC can effectively control a buck DC-DC converter. A type-2 fuzzy system was used to handle the rule uncertainties when the operation is extremely uncertain and/or the membership grades cannot be exactly determined. A comparison between a PI controller, a T2FC and a T2FSMC was performed. Experimental results show that the proposed T2FC and T2FSMC are more robust against input voltage and load resistance variations than the PI controller. Moreover, the T2FSMC can reduce the complexity of analysis and implementation by using the sliding surface. Thus, the proposed T2FSMC is highly suitable for applications to the control of a buck DC-DC converter.

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8 References