THE SHAFT BEHAVIOR OF BTA DEEP HOLE DRILLING TOOL

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Abstract—This paper is a study on the shaft behavior of BTA deep hole drilling tool. The dynamics of tool shaft are often taken to be that of a second order lumped mass system for other cutting processes. This simplification does not apply for deep hole drilling because of its shaft length and the fact of fluid coupling. This paper constructed the general equations of motion for the pipe-like fluid conveying tool shaft of deep hole drill. The proposed general equations can be reduced to different specific forms of former works. Solutions for lateral and longitudinal motions were given. Series of experiments were designed and performed. Comparisons between theoretical and experimental results confirmed the validity of the constructed equations. The studies disclosed build the knowledge about the tool shaft and pave the way for future research concerning the correlation between the tool shaft and cutting process taking place on the cutting head.

Key words: deep hole drilling, shaft behavior, pipes, Bernoulli–Eulerian theory.

NOTATION

\( A_f, A_s \) area of the cross section of the fluid flow, drill tube, respectively, \( m^2 \)
\( c \) viscous damping coefficient in longitudinal and lateral vibration
\( C_f \) centroid of area \( A_f \) of the fluid flow
\( C_s \) centroid of area \( A_s \) of the drill tube
\( c_T \) viscous damping coefficient in torsional vibration
\( E \) Young’s modulus of the drill tube, Pa
\( F \) force resultant on the cross section of the drill tube, N
\( f_s \) fluid quality, l/min
\( G \) shearing modulus of the drill tube, Pa
\( \bar{g} \) gravity
\( I_{f, s} \) the moment of inertia of the area \( A_f, A_s \) respectively, \( m^4 \)
\( I_{x, y, z} \) the moment of inertia of the area \( A_f \) with respect to \( x, y, z \) axis, respectively, \( m^4 \)
\( I_{x, y} \) the moment of inertia of the area \( A_s \) with respect to \( x, y \) axis, respectively, \( m^4 \)
\( I_{x, y} \) the polar moment of inertia of the area \( A_f, A_s \) respectively, \( m^4 \)
\( J_1 \) mass moment of inertia of drill head, kg \( \cdot m^2 \)
\( k, k^1 \) components of curvature of the strained central line of the drill tube, l/m,
\( l \) length of drill tube, m
\( M_s \) coupled resultants about \( S_s \) on the cross section of the drill tube, N \( \cdot m \)
\( M_D \) coupled resultants about \( C_s \) on the cross section of the drill tube, N \( \cdot m \)
\( m_i \) mass of drill head, kg
\( N \) tension, N
\( N_t \) tension at the centroid of the cross section on the tool head, N
\( O_f \) mass center of the small element of the fluid flow
\( O_s \) mass center of the small element of the drill tube
\( P \) fluid pressure, N/m\(^2\)
\( P_s \) fluid pressure on the tool head, N/m\(^2\)
\( \bar{R} \) displacement of a point \( P \) on the central line axis of the drill tube, m
\( \tau \) location of a point \( P \) on the strained central line axis of the drill tube, m
\( \tau_s \) location of a point \( P \) on the central line axis of the drill tube, m
\( \tau_f \) location of fluid at the \( O_f \), m
\( s \) are length of the strained central line of the drill tube, m
\( S_s \) shear center of area \( A_s \) of the drill tube
\( t \) time s
\( T_1 \) torque at the centroid of the cross section on the tool head, N \( \cdot m \)
\( \bar{t}_1, \bar{t}_2, \bar{t}_3 \) unit vector in the direction \( x, y, z \) respectively
\( U \) fluid velocity, m/S
\( u, v, w \) component of \( \bar{R} \) in the direction \( X, Y, Z \) respectively, m
1. INTRODUCTION

A deep hole drilling machine can drill deep holes with depth-diameter-ratio above 100 due to its special design and construction. In a deep hole drilling operation, good hole tolerances with respect to bore diameter, roundness and straightness [1] can be obtained. The hole surface qualities equal those from reaming, honing and grinding, sometimes even surpassing them.

With such excellent machining performance, deep hole drillings are often applied in high precision manufacturing such as the military industry, machine tool and automobile industries. Applications examples are hydraulic cylinders, landing gears for aircraft, large holes in diesel truck applications, turbines, heat exchangers, and oil industry components, etc. [2].

There are three kinds deep hole drilling, namely, gun drilling, BTA(Boring and Trepanning Association) and ejector drilling. The gun drilling is used for a small size of hole and the BTA or ejector drilling is used for a large size of hole. In a deep hole drilling process the pressurized coolant is used.

1.1. Purpose of the study

Because of its length, the shaft dynamics of the deep hole drilling influences the quality of cutting process on the tool head. The deflection due to lateral bending and vibration worsens the axial hole deviation, tolerance, straightness, roundness, etc.

The purpose of this study is to establish a system model and investigate the dynamics of the shaft behavior of deep hole drilling tool theoretically and experimentally.

The behavior is correlative with the effect of fluid flow and structural dynamics. In this study we use the theories of pipes and tubes conveying fluid, with a velocity and Bernoulli–Eulerian theory containing symmetric bending and torsion to find the shaft behavior of the deep hole drilling.

1.2. Literature review

Flegel [1] published the quality of the deep hole drilling processes. Rudd and Hetherington [2] explain each of the three deep hole drilling processes. Rao and Shunmugam [3–6] made many experiments about hole size, axis deviation, roundness error, surface finish, axial and transverse profiles of the holes of drilled length, and wear of center, supporting pad of BTA drilling tool for many different machining conditions such as cutting speed, feed rate, cutting time, etc.. El-Khabeery, et al. [7] made experiments about surface integrity of deep drilling holes (such as surface roughness, hardness, micro hardness, plastic deformation of the surface and subsurface layers) for many machining conditions. For example, drilling speed, feed rate and work hole diameter in gun drilling processes. Corney and Griffiths [8] presented experiments analyzing the combined cutting and burnishing action in which the forces generated at a single cutting edge are balanced by the rubbing pad in the BTA drilling process. Sakuma et al. [9] made experiments and proposed simple formulas about the burnishing action of guide pads and influence on hole accuracies for machining conditions such as cutting forces, cutting torque, depth of deformation, and waves on burnished surface in the BTA process.
Sakuma et al. [10] found that the frequency of the bending vibration of the boring bar during machining of the holes corresponds to the number of the corners of the hole. Simple models of the support and boring bar was proposed by Sakuma et al. [10], but there were no complete considerations about motions, for example, the bending and torsional vibration of the shaft conveying fluid flow. Katsuti et al. [11] proposed comparison of single- and multi-edge tools for many conditions such as wall thickness, hardness of the bonded plate, etc. This paper presented simple models for cutting forces.

Yumshtyk and Kedrov [12] investigated the gun drill vibration with a very simple model which considered the lateral vibration by neglecting drill length, torsional vibration and the effect of the fluid flow. Sakuma et al. [13] presented the deflections of misalignment in tool pilot bush or bar support on the straightness error of a drilled hole, but there were no considerations about complete motion and fluid flow. Some guide values about gun drills, BTA drills and ejector drills, for example, cutting speed, feed, net power, cutting fluid quantity, cutting fluid pressure, etc. can be found in Ref. [14].

Chin et al. [15] proposed the theory of signal of chip formation and proved it experimentally. Detailed mathematics concerning chip signals were given by Chin and Wu [16]. Chin and Lin [17] discussed the stability of the drilling process by treating the tool shaft as a second order lumped mass system. Chandrashekhar et al. [18] proposed a three-dimensional model of the BTA machining system including the interaction between the workpiece and cutting tool. A physical model for stationary workpiece and rotating cutting tool was proposed. The modes method along with Lagrange's equation was used to obtain lateral and torsional vibration equations to represent the influence of axial force and torque. Chandrashekhar et al. [19] found the solutions and predicted the helical grooves which were observed on the drilled workpieces, and compared theories with experiments on roundness error, but there were obvious discrepancies between theoretical and experimental results. The radial and tangential forces were not considered in Ref. [19]; the velocity of fluid was held constant in the BTA drilling processes, and the vibration solution only considered the fundamental mode.

The papers below proposed the theories of pipes and tubes containing a fluid flow. Blevins [20] proposed the planar lateral motion of the pipes with constant velocity of fluid flow, and the critical flow velocity due to the buckling of the pipe and due to the flutter. Paidoussis [21] proposed the extension of gravity on hanging cantilever tubes. Paidoussis and Issid [22] proposed the extension of the fluctuation velocity of fluid flow. Yoshizawa et al. [23] proposed another method about the theory of the pipes conveying fluid with fluctuation velocity, and obtained critical flow velocity and maximum values of the static deflection of the buckling pipes. Thompson and Lunn [24] presented static elastic formulation and concluded that the net effect of the fluid flow in the static case was to add an end follower thrust to the mechanically applied forces. Lundgren et al. [25] investigated the three-dimensional lateral vibration of the tubes of uniform annular cross section containing fluid flow with constant velocity. The papers above investigated the lateral vibrational of the pipes with double symmetric cross section, and neglected rotary inertia. Edelstein et al. [26] applied the finite element method to the same equation described in [25] to obtain the oscillations. Hill and Swanson [27] proposed the lumped masses on the tubes conveying fluid. Sugiyama et al. [28] studied the spring effect on pipes conveying fluid and proposed the criterion of stability and critical flow velocity. The pipe theory above covered the stability and critical velocity due to flutter but did not consider the axial force, critical axial force and critical fluid pressure.

Literature investigation reveals that there are no rigorous equations of motion available that govern the tool shaft of the BTA deep hold drill. In this paper the three-dimensional general equations of motion for lateral, longitudinal and torsional motion of the shaft containing the fluid flow are constructed. Specific equations solutions are given for lateral and longitudinal motions. Finally, the proposed equations for lateral motions are verified by experiments.
2. EQUATIONS OF MOTION

The system under consideration consists of a shaft, conveying a fluid. The temperature and nozzle effect are neglected. The basic assumptions for the shaft are as follows:

1. Elastic homogeneous isotropic material is considered.
2. No shear deformation is considered.
3. Plane sections before deformation remain plane during deformation because of the Bernoulli–Euler theory.
4. The shaft is of uniform and thin-walled closed cross section and is initially straight.
5. Warping deformation is neglected and shaft deflection is small.

2.1. Bernoulli–Eulerian theory

The determination of orientation of shaft element conforms to Love [29]. Figure 1 shows the coordinates of a slightly displaced tool shaft. Let $P_1$ be a point on the central-line near $P_1$, and $P_1'$ be the displaced position of $P_1$. The length of the arc $P P_1'$ is designated by $\delta_r$.

The location of a point $P$ on the shaft axis is given by

$$\mathbf{r} = X(s, t)\mathbf{i} + Y(s, t)\mathbf{j} + Z(s, t)\mathbf{k}$$

where $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ are fixed orthogonal unit vectors. The location of a point $P_1$ is given by

$$\mathbf{r}_1 = s_1\mathbf{k}$$

and the displacement vector is as follows:

$$\mathbf{R} = \mathbf{r} - \mathbf{r}_1 = u(s, t)\mathbf{i} + v(s, t)\mathbf{j} + w(s, t)\mathbf{k}$$

The unit tangent vector is

$$\mathbf{t}_3 = \frac{\partial \mathbf{r}}{\partial s} = \frac{\partial X}{\partial s} \mathbf{i} + \frac{\partial Y}{\partial s} \mathbf{j} + \frac{\partial Z}{\partial s} \mathbf{k}.$$  (2)

Let the force and torque on the cross section of the shaft be $\mathbf{F}$ and $\mathbf{M}_g$ with

$$\mathbf{F} = V_x\mathbf{t}_1 + V_y\mathbf{t}_2 + N\mathbf{t}_3$$  (3)

where $V_x$, $V_y$ are shearing forces, and $N$ is the tension.

If the shearing forces $V_x\mathbf{t}_1 + V_y\mathbf{t}_2$ do not act through the shear center $S_s$, it can be replaced by a statically equivalent system shown in Fig. 2 [30]. We obtain

$$\mathbf{M}_s = \mathbf{M}_g + S_s\hat{\mathbf{C}}_s \times (V_x\mathbf{t}_1 + V_y\mathbf{t}_2)$$  (4)

where $C_s$ is the centroid.

Applying the Bernoulli–Eulerian theory yields

$$\mathbf{M}_s = EI_x k\mathbf{t}_1 + EI_y k\mathbf{t}_2 + C_r \tau$$  (5)
The shaft behavior of BTA deep hole drilling tool

\[ \mathbf{F} = V_x \mathbf{i}_x + V_y \mathbf{i}_y + N \mathbf{i}_z \]

Fig. 2. Statically equivalent system.

\[ \bar{F} = \frac{\partial \mathbf{P}}{\partial s_f} + \frac{\partial \mathbf{P}}{\partial s_f} \]

Fig. 3. The small element cut from the shaft.

where \( k \) and \( k' \) are the component of curvature of the strained central line, \( \tau \) is the twist of the shaft and \( C_r \) is the torsional with

\[ C_r = GJ. \]

2.2. Effect of fluid flow

Consider a small element of the shaft, with length \( \delta s_f \) (see Fig. 3). The corresponding mass of the enclosed fluid is \( \delta m_f \). \( \rho_f \) is the density of fluid. Then

\[ \delta m_f = \rho_f A_f \delta s_f. \]

The rate of change of momentum \( \bar{L}_f \) can be written as:

\[ \frac{d \bar{L}_f}{dt} = \rho_f A_f \delta s_f \left( \frac{\partial^2 \bar{r}_f}{\partial t^2} + 2 \frac{\partial^2 \bar{r}_f}{\partial t \partial s_f} + U^2 \frac{\partial^2 \bar{r}_f}{\partial s_f^2} + \frac{\partial U}{\partial t} \frac{\partial \bar{r}_f}{\partial s_f} \right) \]

\[ \frac{d \bar{L}_f}{dt} = \rho_f A_f \delta s_f \left( \frac{\partial}{\partial t} + U \frac{\partial}{\partial s_f} \right)^2 \bar{r}_f(s_f, t). \]

The results agree with the papers by Blevins [20], Paidoussis and Issid [22], Yoshizawa, et al. [23], Lundgren et al. [25].

2.3. Small deflection

A small elements cut from the shaft is shown in Fig. 3.

Coordinate transformation yields:

\[ \bar{M}_e = -EI_x \frac{\partial^3 Y}{\partial s_f^2} \bar{t}_1 + EI_y \frac{\partial^3 Y}{\partial s_f^2} \bar{t}_2 + GJ \frac{\partial \theta}{\partial s_f} \bar{t}_3. \]

The following fundamental equation holds

\[ \sum \bar{M} = \bar{\dot{H}}. \]
Let \( (\dot{\mathbf{H}})_{os} \) and \( (\dot{\mathbf{H}})_{of} \) be the rates of change of the angular momentum about \( O_s \) and \( O_f \) respectively. If the shaft deflection is small, the following equation can be obtained:

\[
(\dot{\mathbf{H}})_{os} = - I_{ss} \frac{\partial^3 Y}{\partial t^3 \partial s} \tau_1 + I_{ys} \frac{\partial^3 X}{\partial t^3 \partial s} \tau_2 + I_{zs} \frac{\partial^3 \theta}{\partial t^3 \partial s} \tau_3
\]

where \( I_{ss}, I_{ys}, I_{zs} \) denote the principal centroidal moments of inertia and

\[
I_{ss} = \rho_s I_{ss} \delta s, \quad I_{ys} = \rho_s I_{ys} \delta s, \quad I_{zs} = \rho_s I_{zs} \delta s.
\]

Let

\[
(\dot{\mathbf{H}})_{os} = \rho_s \dot{h}_{os} \delta s
\]

then

\[
\dot{h}_{os} = - I_{ss} \frac{\partial^3 Y}{\partial t^3 \partial s} \tau_1 + I_{ys} \frac{\partial^3 X}{\partial t^3 \partial s} \tau_2 + I_{zs} \frac{\partial^3 \theta}{\partial t^3 \partial s} \tau_3. \tag{9}
\]

Similarly,

\[
\dot{h}_{sf} = - I_{sf} \frac{\partial^3 Y}{\partial t^3 \partial s_f} \tau_1 + I_{sf} \frac{\partial^3 X}{\partial t^3 \partial s_f} \tau_2 + I_{sf} \frac{\partial^3 \theta}{\partial t^3 \partial s_f} \tau_3. \tag{10}
\]

and the tension is:

\[
N = E A_s \frac{\partial w}{\partial s}. \tag{11}
\]

### 2.4. Equations of motion of BTA drill shaft with stationary cutting tool

In BTA drilling processes, the shaft is of uniform annular cross section, the points \( C_s, C_f, S_s \) are of the same location, and the points \( O_s, O_f \) become identical. So we can take:

\[
I_{ss} = I_{ys} = I_s; \quad I_{sf} = I_{sf} = I_f
\]

\[
S_f = s; \quad \tau_f = \tau
\]

and the flow velocity \( U \) is opposite to the direction defined earlier, so the rate of change of momentum \( \dot{L}_f \) of BTA drill becomes:

\[
\frac{d\dot{L}_f}{dt} = \rho_f A_f \delta s \left( \frac{\partial}{\partial t} - U \frac{\partial}{\partial s} \right) \tau(s, t).
\]

The force equation is:

\[
\frac{\partial F}{\partial s} + \frac{\partial \overline{F}}{\partial \delta s} + \overline{q}_s + \overline{q}_f = 0 \tag{12}
\]

where

\[
\overline{q}_s = - \rho_s A_s \frac{\partial^2 \tau}{\partial t^2} + \rho_s A_s \overline{g} + \overline{q}_{es}
\]

\[
\overline{q}_f = - \rho_f A_f \left( \frac{\partial}{\partial t} + U \frac{\partial}{\partial s_f} \right) \overline{\tau}_f + \rho_f A_f \overline{g} + \overline{q}_{ef}.
\]

The moment equation is:

\[
\overline{\tau}_3 \times \overline{F} + \overline{\tau}_3 \times \overline{P} + \frac{\partial \overline{M}}{\partial s} + \overline{\Lambda}_s + \overline{\Lambda}_f = 0 \tag{13}
\]

where

\[
\overline{\Lambda}_s = - \rho_s \overline{h}_{os} + \overline{\Lambda}_{es}
\]

\[
\overline{\Lambda}_f = - \rho_f \overline{h}_{sf} + \overline{\Lambda}_{ef}.
\]

The following relations can be established from Fig. 4:

\[
\overline{F} = N \overline{\tau}_3 + \overline{\tau}_3 \times \left( \frac{\partial \overline{M}_G}{\partial s} + \overline{\Lambda}_s + \overline{\Lambda}_f \right). \tag{14}
\]
Substituting Eqn (14) into Eqn (12), the following general force equation can be obtained

\[
- \frac{\partial^2}{\partial s^2} \left( E_l s \frac{\partial^2 \mathbf{r}}{\partial s^2} \right) + \frac{\partial}{\partial s} \left( E_l s k_n^2 \frac{\partial \mathbf{r}}{\partial s} \right) + \frac{\partial}{\partial s} \left( GJ s \frac{\partial^2 \mathbf{r}}{\partial s^2} \times \frac{\partial^2 \mathbf{r}}{\partial s^2} \right) \\
+ \frac{1}{\partial s} \left[ \left( \rho s I_s + \rho_f I_f \right) \frac{\partial^2 X}{\partial t^2} \frac{\partial s}{\partial s} \right] \mathbf{t}_1 + \frac{1}{\partial s} \left[ \left( \rho s I_s + \rho_f I_f \right) \frac{\partial^2 Y}{\partial t^2} \frac{\partial s}{\partial s} \right] \mathbf{t}_2 \\
+ \frac{\partial}{\partial s} \left[ (N - PA_f) \frac{\partial \mathbf{r}}{\partial s} \right] - \rho s A_f \frac{\partial^2 \mathbf{r}}{\partial t^2} - \rho_f A_f \left( \frac{\partial}{\partial t} + U \frac{\partial}{\partial s} \right) \mathbf{t} \\
+ \left( \rho s A_s + \rho_f A_f \right) \mathbf{g} + \bar{\mathbf{q}}_{es} + \bar{\mathbf{q}}_{ef} + \frac{\partial}{\partial s} \left[ \mathbf{t}_3 \times \left( \mathbf{\bar{k}}_{es} + \mathbf{\bar{k}}_{ef} \right) \right] = 0. \tag{15}
\]

If we do not consider the twist \( \tau \), the gravity \( \mathbf{g} \), the external force, the external moment, the pressure \( P \) of the fluid flow, it is the same as the equation in Refs [18] and [19]. From \( \mathbf{t}_3 \) in Eqn (15) the following equation is obtained

\[
2 \frac{\partial}{\partial s} \left( E_l s \left( k_n^2 + \frac{1}{2} k_n^2 \right) \right) + \mathbf{t}_3 \cdot \frac{\partial}{\partial s} \left( GJ s \frac{\partial^2 \mathbf{r}}{\partial s^2} \times \frac{\partial^2 \mathbf{r}}{\partial s^2} \right) \\
+ \frac{\partial}{\partial s} \left[ (N - PA_f) - \rho f A_f \frac{\partial U}{\partial s} - (\rho s A_s + \rho_f A_f) \frac{\partial^2 \mathbf{r}}{\partial t^2} \right] \\
+ \left( \rho s A_s + \rho_f A_f \right) \mathbf{g} + \mathbf{t}_3 \cdot \mathbf{\bar{k}}_{es} + \mathbf{t}_3 \cdot \mathbf{\bar{k}}_{ef} = 0. \tag{17}
\]

where \( k_n \) is curvature in the normal direction \( k_n = \left| \frac{\partial \mathbf{r}}{\partial s} \right| = \left| \frac{\partial^2 \mathbf{r}}{\partial s^2} \right| \).
If E, I, U are constant and the twist τ, gravity \( \bar{g} \), external force, external moment are neglected, it is the same as the equation in Refs [25] and [26].

2.5. Boundary conditions

The boundary conditions are constructed as that shown in Fig. 5. The condition at location \( s = l \) can be expressed by two models.

**Model 1.** The boundary condition contains the mass \( m_1 \), the moment of inertia of mass \( J_1 \), the forces, the torque, the springs and the dampers.

**Model 2.** The boundary condition at \( s = l \) is free while the mass \( m_1 \), the moment of inertia of mass \( J_1 \), the forces, the torque, the springs and the dampers are replaced by concentrated forces and torques.

3. THE SOLUTIONS OF EQUATIONS OF MOTION

The solutions of equations of motion of the BTA drill shaft will be given for the following general cases:

\[
\ddot{q}_{es} = - c \frac{\partial \bar{f}}{\partial t} + f_1(s, t) \hat{i} + f_2(s, t) \hat{j} + f_3(s, t) \hat{k}
\]

where \(- c \frac{\partial \bar{f}}{\partial t}\) denotes viscous damping force and \(f_1, f_2, f_3\) are the external forces per unit length.

\[
\bar{N}_{es} = 0
\]

\[
\bar{N}_{ef} = - c_t \frac{\partial}{\partial t} f_4(s, t) \hat{k}
\]

where \(- c_t \frac{\partial f_4}{\partial t}\) denotes viscous damping torque and \(f_4\) is the external torque per unit length.

3.1. Lateral motion of the shaft

The equation of motion for lateral motion can be derived from Eqn (16):

\[
-EI_s \frac{\partial^4 X}{\partial s^4} + (\rho_s A_s + \rho_f A_f) \frac{\partial^4 X}{\partial t^2 \partial s^2} + \frac{\partial}{\partial s} \left[ (N - PA_f) \frac{\partial X}{\partial s} \right] \\
- \rho_s A_s \frac{\partial^2 X}{\partial t^2} - \rho_f A_f \left( \frac{\partial}{\partial t} + U \frac{\partial}{\partial s} \right) X - (\rho_s A_s + \rho_f A_f) g \\
- c \frac{\partial X}{\partial t} + f_1(s, t) = 0.
\]
B.C.'s

\[ s = 0, \quad X = 0, \quad \frac{\partial X}{\partial s} = 0 \]

\[ s = l, \quad EI_s \frac{\partial^2 X}{\partial s^2} = 0 \]

\[ s = l, \quad EI_s \frac{\partial^3 X}{\partial s^3} = -m_1 \frac{\partial^2 X}{\partial t^2} - c_1 \frac{\partial X}{\partial t} - k_1 X - N_1 \frac{\partial X}{\partial s}. \]

If \( m_1, c_1 \) in boundary condition are neglected it is similar to the equation in Ref. [22]. I.C.s

\[ t = 0, \quad X = g_{s1}(s) \]

\[ t = 0, \quad \frac{dX}{dt} = g_{s2}(s). \]

Assume the solution of Eqn (22) is of following form:

\[ X(s, t) = X_1(s, t) + X_2(s). \]

We first find \( X_2(s) \) by applying model 1 of the boundary conditions. Suppose

\[ X_2 = X_2h + X_2p \]

where \( X_2h \) is the homogeneous solution and \( X_2p \) is the particular solution. We have

\[ X_{2p} = -\frac{s^2(\rho_s A_s + \rho_f A_f)g}{2(-N_1 + P_i A_f + \rho_f A_f U^2)}. \]

Because \( N_1 < 0 \) which is a compressive force we have

\[ b^*_v = \frac{-N_1 + P_i A_f + \rho_f A_f U^2}{E I_s} \]

\[ X_{2h} = d_1 \sin b_v s + d_2 \cos b_v s - d_1 b_v s - d_2 \]

where \( d_1, d_2 \) can be obtained from the following two linear equations:

\[ \frac{\partial^2 X_2(l)}{\partial s^2} = -d_1 b_v^2 \sin b_v l - d_2 b_v^2 \cos b_v l - \frac{(\rho_s A_s + \rho_f A_f)g}{-N_1 + P_i A_f + \rho_f A_f U^2} = 0 \]

\[ -E I_s(-d_1 b_v^2 \cos b_v l + d_2 b_v^2 \sin b_v l) = \]

\[ -k_1 \left[d_1(\sin b_v l - b_v l) + d_2(\cos b_v l - 1) \right] - \frac{l^2(\rho_s A_s + \rho_f A_f)g}{2(-N_1 + P_i A_f + \rho_f A_f U^2)} \]

\[ -N_1 \left[d_1(\cos b_v l) - d_2 b_v \sin b_v l \right] \]

\[ -\frac{l(\rho_s A_s + \rho_f A_f)g}{-N_1 + P_i A_f + \rho_f A_f U^2}. \]

It is convenient to introduce the dimensionless quantities for \( X_1 \):

\[ \bar{X}_1 = \frac{X_1}{l}, \quad \bar{s} = \frac{s}{l} \]

\[ \bar{t} = \frac{1}{l^2} \left( \frac{E I_s}{\rho_s A_s + \rho_f A_f} \right)^{1/2} \]

\[ \bar{I}_p = \frac{\rho_s I_s + \rho_f I_f}{l^2(\rho_s A_s + \rho_f A_f)} \]

\[ U = UI \left( \frac{\rho_f A_f}{E I_s} \right)^{1/2} \]
\[ \tilde{\rho}_f = \frac{\rho_f A_f}{\rho_s A_s + \rho_f A_f}, \quad \tilde{c} = c \cdot \frac{l^2}{[EI_s(\rho_s A_s + \rho_f A_f)]^{1/2}} \]

\[ \tilde{m}_i = \frac{m_i}{l(\rho_s A_s + \rho_f A_f)} \quad \tilde{c}_i = c_i \cdot \frac{l}{[EI_s(\rho_s A_s + \rho_f A_f)]^{1/2}} \]

\[ \tilde{k}_i = k_1 \frac{l^3}{EI_s}, \quad \tilde{N}_i = N_i \frac{l^2}{EI_s}, \quad \tilde{P}_i = P_i A_f \frac{l^2}{EI_s} \]

\[ \tilde{f}_i = f_i \frac{l^3}{EI_s}, \quad \tilde{g}_{3i} = \frac{g_{3i}}{l} \]

\[ \tilde{q}_{3i} = g_{3i} l \left( \frac{\rho_s A_s + \rho_f A_f}{EI_s} \right)^{1/2}, \quad \omega = \Omega l^2 \left( \frac{\rho_s A_s + \rho_f A_f}{EI_s} \right)^{1/2} \]

and the equation is rearranged as:

\[
\frac{\partial^4 \tilde{X}_1}{\partial \tilde{s}^4} - I_p \frac{\partial^4 \tilde{X}_1}{\partial \tilde{t}^2 \partial \tilde{s}^2} + N_i \delta(\tilde{s} - 1) \frac{\partial^3 \tilde{X}_1}{\partial \tilde{s} \partial \tilde{t}^{2}}
\]

\[ - (\tilde{N}_i - \tilde{P}_i) \frac{\partial^2 \tilde{X}_1}{\partial s^2} + \frac{\partial^2 \tilde{X}_1}{\partial t^2} + 2U \tilde{\rho}_{f/2} \frac{\partial^2 \tilde{X}_1}{\partial \tilde{t} \partial \tilde{s}} \]

\[ + \frac{C^2}{l^2} \frac{\partial^2 \tilde{X}_1}{\partial \tilde{s}^2} + \tilde{c} \frac{\partial \tilde{X}_1}{\partial \tilde{t}} + \tilde{m}_i \delta(\tilde{s} - 1) \frac{\partial^2 \tilde{X}_1}{\partial \tilde{t}^2} \]

\[ + \tilde{c}_i \delta(\tilde{s} - 1) \frac{\partial \tilde{X}_1}{\partial \tilde{t}} + \tilde{k}_i \delta(\tilde{s} - 1) \tilde{X}_1 = \tilde{f}_i(\tilde{s}, \tilde{t}) \]  \hspace{1cm} (23)

with B.C.'s

\[ \tilde{s} = 0, \quad \tilde{X}_1 = 0, \quad \frac{\partial \tilde{X}_1}{\partial \tilde{s}} = 0 \]

\[ \tilde{s} = 1, \quad \frac{\partial^2 \tilde{X}_1}{\partial \tilde{s}^2} = 0, \quad \frac{\partial^3 \tilde{X}_1}{\partial \tilde{s}^3} = 0. \]

The *Galerkin method* is used to find $X_1$. Suppose that

\[ \tilde{X}_1(\tilde{s}, \tilde{t}) = \sum_{m=1}^{M} \phi_m(\tilde{s}) \tilde{q}_m(\tilde{t}) \]  \hspace{1cm} (24)

where $\phi_m(\tilde{s})$ satisfy all the boundary conditions of the system.

We take

\[ \phi_m(\tilde{s}) = \cosh \beta_m \tilde{s} - \cos \beta_m \tilde{s} - \sigma_m (\sinh \beta_m \tilde{s} - \sin \beta_m \tilde{s}) \]  \hspace{1cm} (25)

where $\beta_m$ satisfies the equation

\[ \cos \beta_m \cosh \beta_m + 1 = 0 \]  \hspace{1cm} (26)

and

\[ \sigma_m = \frac{\sinh \beta_m - \sin \beta_m}{\cosh \beta_m + \cos \beta_m}. \]

By the Galerkin method, the following equation in matrix form can be obtained:

\[ A \ddot{q} + B \dot{q} + C q = f(\tilde{t}) \]  \hspace{1cm} (27)

where $A$, $B$, $C$ are $M \times M$ matrices and $q$, $f$, $\Phi$ are $M \times 1$ matrices. We can write:

\[ X_1(\tilde{s}, \tilde{t}) = \Phi^T(\tilde{s}) \tilde{q}(\tilde{t}) \]  \hspace{1cm} (28)
where \( q(\ell) \) is of the following form [31]:
\[
q(\ell) = W_{\ell p} e^{\ell J} \int_{0}^{\ell} e^{-\ell J} W_{\ell r}^{-1} A^{-1} f(\tau) d\tau + W_{\ell p} e^{\ell J} W^{-1} \begin{bmatrix} q(0) \\ \dot{q}(0) \end{bmatrix}
\]
(29)

where \( J \) is the Jordan matrix, \( W = [W_{\ell p} W_{\ell t}]^T, W^{-1} = [W_{\ell t}^{-1} W_{\ell r}^{-1}] \), \( W \) = eigencolumn matrix.

The matrices \( q(0), \dot{q}(0) \) are found by using I.C. and the orthogonality of \( \phi_m(\ell) \) as follows:
\[
q_m(0) = \int_{0}^{1} \phi_m(\ell) \phi_{x3}(\ell) d\ell
\]
\[
\dot{q}_m(0) = \int_{0}^{1} \phi_m(\ell) \phi_{x4}(\ell) d\ell.
\]

Finally, we obtain
\[
X_1(s, t) = X_1(\ell, t) = L \Phi(\ell) q(\ell)
\]
\[
X(s, t) = X_1(s, t) + X_2(s).
\]

This completes the solution of equations for lateral shaft motion. \( Y(s, t) \) can be obtained in a similar way by neglecting gravity.

3.2. Longitudinal motion of the shaft

The equation of motion for longitudinal motion can be derived from Eqn (17):
\[
E_{Ax} \frac{\partial^2 w}{\partial s^2} - P_d A_f - (\rho_s A_s + \rho_f A_f) \frac{\partial^2 w}{\partial \ell^2} - c_c \frac{\partial w}{\partial \ell} + f_3(s, t) = 0
\]
(30)

where \( P_d = \frac{\partial p}{\partial s} = -\frac{8\pi l}{h^2} [32]. \)

B.C.'s:
\[
s = 0, \quad w = 0
\]
\[
s = 1, \quad E_{As} \frac{\partial w}{\partial s} = -m_l \frac{\partial^2 w}{\partial \ell^2} - k_3 w - c_3 \frac{\partial w}{\partial \ell} + N_l
\]

I.C.'s
\[
t = 0, \quad w(s, 0) = g_1(s)
\]
\[
t = 0, \quad \dot{w}(s, 0) = g_2(s).
\]

Assume the solution of Eqn (30) is of following form:
\[
w(s, t) = w_1(s, t) + w_2(s).
\]

We first find the static solution \( w_2(s) \) by solving the following reduced equation:
\[
E_{Ax} \frac{\partial^2 w_2}{\partial s^2} - P_d A_f = 0.
\]

It leads to
\[
w_2(s) = \frac{P_d A_f}{E_{Ax}} \left( \frac{1}{2} s^2 - L_s + \frac{k_3 l^2}{2(E_{Ax} + k_3 l^2)} s \right) + \frac{N_l}{E_{Ax} + k_3 l^2} s.
\]
(31)

\( w_1(s, t) \) is governed by the following reduced equation:
\[
E_{Ax} \frac{\partial^2 w_1}{\partial s^2} - (\rho_s A_s + \rho_f A_f) \frac{\partial^2 w_1}{\partial \ell^2} - c_c \frac{\partial w_1}{\partial \ell} + f_3(s, t) = 0
\]

B.C.'s
\[
s = 0, \quad w_1 = 0
\]
\[
s = 1, \quad E_{Ax} \frac{\partial w_1}{\partial s} = -m_l \frac{\partial^2 w_1}{\partial \ell^2} - c_3 \frac{\partial w_1}{\partial \ell} - k_3 w_1.
\]
I.C.’s

\[ t = 0, \quad w_1(s, 0) = g_3(s) = g_1(s) - w_2(s) \]

\[ t = 0, \quad \dot{w}_1(s, 0) = g_4(s) = g_2(s) \]

It is convenient to introduce the following dimensionless quantities

\[ b = \sqrt{\frac{E}{\rho_s l}} \frac{1}{\rho_f A_f} \]

\[ \bar{t} = \frac{t}{l} \left( \frac{E A_s}{\rho_s A_s + \rho_f A_f} \right)^{1/2} \]

\[ \bar{c} = c \frac{l}{[E A_s (\rho_s A_s + \rho_f A_f)]^{1/2}} \]

\[ \bar{c}_3 = c_3 \frac{l}{[E A_s (\rho_s A_s + \rho_f A_f)]^{1/2}} \]

\[ \bar{f}_3 = f_3 \frac{l}{E A_s}, \quad \bar{g}_3 = \frac{g_3}{l}, \quad \bar{g}_4 = \frac{g_4}{b} \]

\[ \omega = \Omega l \left( \frac{\rho_s A_s + \rho_f A_f}{E A_s} \right)^{1/2} \]

and the equation is normalized as

\[ -\frac{\partial^2 \bar{w}_1}{\partial \bar{s}^2} + \frac{\partial^2 \bar{w}_1}{\partial \bar{t}^2} + \bar{c} \frac{\partial^2 \bar{w}_1}{\partial \bar{t}^2} + \bar{m}_i \frac{\partial^2 \bar{w}_1}{\partial \bar{t}^2} \delta(\bar{s} - 1) \]

\[ + \bar{c}_3 \frac{\partial \bar{w}_1}{\partial \bar{t}} \delta(\bar{s} - 1) + \bar{k}_3 \bar{w}_1 \delta(\bar{s} - 1) = \bar{f}_3(\bar{s}, \bar{t}) \tag{32} \]

with B.C.s

\[ \bar{s} = 0, \quad \bar{w}_1 = 0 \]

\[ \bar{s} = 1, \quad \frac{\partial \bar{w}_1}{\partial \bar{s}} = 0. \]

Again the \textit{Galerkin method} is used to find \( w_1 \). Suppose that

\[ \bar{w}_1(\bar{s}, \bar{t}) = \sum_{m=1}^{\infty} \phi_m(\bar{s}) g_m(\bar{t}) \tag{33} \]

where \( \phi_m(\bar{s}) \) satisfy all the boundary condition of the system.

We take

\[ \phi_m(\bar{s}) = \sin \bar{\chi}_m \bar{s}, \quad \text{where} \quad \bar{\chi}_m = \frac{(2m - 1)\pi}{2}, \quad m = 1, 2, \ldots \]

The \( \phi_m(\bar{s}) \) are orthogonal over the span of the cantilever.

By following operation similar to that in Section 3.1, we can obtain

\[ A \dot{q} + B q + C q = f(\bar{t}) \tag{34} \]

where \( A, B, C \) are \( M \times M \) matrices and \( q, f, \Phi \) are \( m \times 1 \) matrices.

Using

\[ q_i(0) = \frac{1}{m_i} \left[ \int_0^1 \phi_i(\bar{s}) g_3(\bar{s}) d\bar{s} \right] \]

\[ \frac{dq_i(0)}{dr} = \frac{1}{m_i} \left[ \int_0^1 \phi_i(\bar{s}) g_4(\bar{s}) d\bar{s} \right] \]
Table 1. The natural frequencies of lateral vibration of the shaft of BTA drill
(hanged horizontally with no fluid)

<table>
<thead>
<tr>
<th>Mode</th>
<th>Theoretical values of Eqn (35) (Hz)</th>
<th>Theoretical values of Euler beam (Hz)</th>
<th>Experimental values (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>36.522</td>
<td>36.538</td>
<td>38.752</td>
</tr>
<tr>
<td>2</td>
<td>100.629</td>
<td>100.718</td>
<td>103.469</td>
</tr>
<tr>
<td>3</td>
<td>197.170</td>
<td>197.447</td>
<td>199.699</td>
</tr>
<tr>
<td>4</td>
<td>325.541</td>
<td>326.390</td>
<td>326.396</td>
</tr>
<tr>
<td>5</td>
<td>485.673</td>
<td>487.571</td>
<td>492.157</td>
</tr>
<tr>
<td>6</td>
<td>677.275</td>
<td>680.987</td>
<td>693.670</td>
</tr>
<tr>
<td>7</td>
<td>900.084</td>
<td>906.640</td>
<td>914.559</td>
</tr>
<tr>
<td>8</td>
<td>1153.745</td>
<td>1164.528</td>
<td>1178.076</td>
</tr>
</tbody>
</table>

where $m_{ll} = \frac{1}{2}$, the $q(t)$ can be found by the same method is Section 3.1. Finally, we obtain

$$w_1(s, t) = \tilde{w}_1(\tilde{s}, \tilde{t})l = l\Phi(\tilde{s})q(\tilde{t})$$

$$w(s, t) = \tilde{w}_1(s, t) + w_2(s).$$

This completes the solution of equations for longitudinal shaft motion.

4. SYSTEM SIMPLIFICATION

The foregoing equations of motion were established in a general sense. They could be simplified to cope with the practical engineering features.

For example, the tool shaft is of steel material so that the deformation is small and the effect of rotatory inertia could be neglected. This leads to an Euler Beam. In order to make system simplification the following two preliminary system analyses are performed.

4.1. System eigenproperties—solid shaft

The equation of motion in lateral direction can be derived from Eqn (22) as follows:

$$EI_s \frac{\partial^4 X}{\partial s^4} + \rho_s A_s \frac{\partial^2 X}{\partial t^2} + C \frac{\partial X}{\partial t} - \rho_s I_s \frac{\partial^4 X}{\partial s^4 \partial t^2} = 0. \tag{35}$$

This equation can be further simplified to become an Euler beam equation:

$$EI_s \frac{\partial^4 X}{\partial s^4} + \rho_s A_s \frac{\partial^2 X}{\partial t^2} + C \frac{\partial X}{\partial t} = 0. \tag{36}$$

The following boundary conditions are used to investigate the shaft eigenproperties.

$$s = 0, \quad \frac{\partial^2 X}{\partial s^2} = \frac{\partial^3 X}{\partial s^3} = 0$$

$$s = l, \quad \frac{\partial^2 X}{\partial s^2} = \frac{\partial^3 X}{\partial s^3} = 0.$$

The natural frequencies of modes 1–8 of the two theories and of experiment are listed in Table 1 which reveals that theoretical values of Euler Beam are closer to the experimental values than those predicted by Eqn (35).

4.2. System eigenproperties—solid shaft with static fluid

The purpose of this analysis is to examine the system under the influence of the static fluid.

The equation of motion can be rewritten from Eqn (22) for lateral shaft motion as follows:

$$EI_s \frac{\partial^4 X}{\partial s^4} + (\rho_s A_s + \rho_f A_f) \frac{\partial^2 X}{\partial t^2} + C \frac{\partial X}{\partial t} - (\rho_s I_s + \rho_f I_f) \frac{\partial^4 X}{\partial s^4 \partial t^2} = 0. \tag{37}$$
Table 2. The natural frequencies of lateral vibration of the shaft of BTA drill (hanged horizontally with static fluid)

<table>
<thead>
<tr>
<th>Mode</th>
<th>Theoretical values of Eqn (37) (Hz)</th>
<th>Theoretical values of Euler beam (Hz)</th>
<th>Experimental values (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>34.935</td>
<td>34.989</td>
<td>34.877</td>
</tr>
<tr>
<td>2</td>
<td>96.258</td>
<td>96.338</td>
<td>96.881</td>
</tr>
<tr>
<td>3</td>
<td>188.612</td>
<td>188.861</td>
<td>189.887</td>
</tr>
<tr>
<td>4</td>
<td>311.432</td>
<td>312.195</td>
<td>313.895</td>
</tr>
<tr>
<td>5</td>
<td>464.658</td>
<td>466.368</td>
<td>472.781</td>
</tr>
<tr>
<td>6</td>
<td>648.028</td>
<td>651.373</td>
<td>654.917</td>
</tr>
<tr>
<td>7</td>
<td>861.305</td>
<td>867.213</td>
<td>868.056</td>
</tr>
<tr>
<td>8</td>
<td>1104.167</td>
<td>1113.887</td>
<td>1116.072</td>
</tr>
</tbody>
</table>

Fig. 6. The setup of experimental equipments.

The equation of motion of an Euler Beam is:

\[
EI_s \frac{d^4X}{dx^4} + (\rho_s A_s + \rho_f A_f) \frac{d^2X}{dt^2} + C \frac{dX}{dt} = 0.
\]  

Both system equations are subjected to the same boundary conditions as that in Section 4.1.

Comparing the theoretical values of natural frequencies of modes 1–8 with those of experiment in Table 2, it is seen again that the theoretical values of an Euler Beam are closer to the experimental values than those predicted by Eqn (37).

Based on the results of the above two series of preliminary analyses, the proposed equations of motion can be simplified and the shaft of the BTA drill be taken as an Euler Beam. In the following studies only the simplified equations of motion will be used.

5. THE EXPERIMENTAL ARRANGEMENTS

The experimental arrangements are shown in Fig. 6, in which a heavy duty lathe is equipped with self-designed fixtures to hold the drill on both ends. The experiments which involve generally modal testing techniques [33, 34] are divided into two parts:
The shaft behavior of BTA deep hole drilling tool

Fig. 7. The arrangements of experiment (shaft without fluid, both ends are clamped).

(1) The shaft of the BTA drill is installed on the machine without introduction of oil.
(2) The shaft of the BTA drill is installed on the machine with both ends fixed and oil is supplied.

Details of the experimental arrangements are as follows:

1. Lathe:
   SAN SHING SK26120 HEAVY DUTY PRECISION LATHE.
   (Distance between chuck center and tailstock center: 3m)
2. Hammer:
   Hammer: PCB 086B05 SN5163 (range: 0-5000 lb)
   Amplifier: PCB model 480D06 power unit.
3. Accelerometer:
   Accelerometer: TEAC 601Z (weight: 0.3 g)
   Amplifier: TEAC SA620.
4. Data acquisition system:
   MicroLink: four independent channels, stand-alone type with 64k samples per channel on-board memories.
5. The deep hole drill:
   a. drill head:
      Type: SANDVIK 420.6-0014D 18.91 70
      Mass: 0.030205 kg
      Mass moment of inertia $J$: $1.420 \times 10^{-6}$ kg·m$^2$.
   b. drill tube:
      Type: SANDVIK 420.5-800-2
      Length: 1.6 m
      Internal diameter: 11.5 mm
      External diameter: 17.0 mm
      Material: JIS SNCM 21
      Density $\rho$: 7860 kg/m$^3$
      Young’s modulus $E$: $2.06 \times 10^{11}$ Pa
      Shear modulus $G$: $8.1 \times 10^{10}$ Pa
6. Fluid:
   Type: R68
   Density $\rho_f$: 866 kg/m$^3$
   Absolute viscosity $\mu$: 0.383 kg/m·s.

6. THE SHAFT BEHAVIOR IN INSTALLED CIRCUMSTANCES

The shaft is installed on the lathe by the fixtures and the shaft behavior in installed circumstances is investigated in this section. The arrangements of this experiment are shown in Fig. 7.

The equation of lateral motion is:

$$E I_s \frac{\partial^4 X}{\partial s^4} + \rho_s A_s \frac{\partial^2 X}{\partial t^2} + C \frac{\partial X}{\partial t} = 0 \quad (39)$$

with B.C.'s

$$s = 0, \quad X = \frac{\partial X}{\partial s} = 0$$

$$s = l, \quad \frac{\partial^2 X}{\partial s^2} = \frac{\partial^3 X}{\partial s^3} = 0.$$
Table 3. The natural frequencies of lateral vibration of the shaft of BTA drill (both ends clamped horizontally with no fluid)

<table>
<thead>
<tr>
<th>Mode</th>
<th>Theoretical values (Hz)</th>
<th>Experimental values (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>44.488</td>
<td>44.178</td>
</tr>
<tr>
<td>2</td>
<td>122.634</td>
<td>115.958</td>
</tr>
<tr>
<td>3</td>
<td>240.411</td>
<td>230.559</td>
</tr>
<tr>
<td>4</td>
<td>397.412</td>
<td>368.149</td>
</tr>
</tbody>
</table>

Let

\[ X(s, t) = \sum \phi(s) q(t) \]

The mode shape function \( \phi_j(s) \) can be obtained from the boundary conditions:

\[ \phi_j(s) - \cosh \beta_j s - \cos \beta_j s - \xi_j (\sinh \beta_j s - \sin \beta_j s) = 0 \]  

and

\[ \xi_j = \frac{\cosh \beta_j l - \cos \beta_j l}{\sinh \beta_j l - \sin \beta_j l} \]

\[ \cosh \beta_j l \cdot \cos \beta_j l = 0. \]

Giving an impulse to the shaft yields

\[ E I_s \frac{\partial^4 X}{\partial s^4} + \rho_s A_s \frac{\partial^2 X}{\partial t^2} + C \frac{\partial X}{\partial t} = h(t). \]

By doing the Fourier Transform of the above equation and solving for the closed form, the solution yields

\[ H_j(w) = \frac{1}{1 - \frac{w^2}{\beta_j^4 E I_s} - \frac{w}{\beta_j^4 E I_s} c_j}. \]

The theoretical natural frequency is equal to

\[ f_n = \frac{1}{2\pi} \sqrt{\frac{\beta_j^2 E I_s}{\rho_s A_s}}. \]

The frequency response function is

\[ H(s_x, s_y, w) = \sum_{j=1}^{M} \phi_j(s_x) \phi_j(s_y) H_j(w). \]

a. Natural frequency

In this experiment, the tool head is fixed with two bolts and an oil seal ring is present which makes the practical boundary condition complex and not ideal. The comparisons between the theoretical and the experimental values of natural frequencies of modes 1–4 are listed in Table 3 in which the agreement can be seen, but the discrepancies are somewhat larger than that in Tables 1 and 2.

b. Mode shape

The mode shape function of the shaft is

\[ \phi_j(s) = \cosh \beta_j s - \cos \beta_j s - \frac{\cosh \beta_j l - \cos \beta_j l}{\sinh \beta_j l - \sin \beta_j l} (\sinh \beta_j s - \sin \beta_j s). \]
The shaft behavior of BTA deep hole drilling tool

Fig. 8. The shape of mode 1 for lateral vibration (shaft without fluid, fixed–fixed).

Fig. 9. The shape of mode 2 for lateral vibration (shaft without fluid, fixed–fixed).

Fig. 10. The shape of mode 3 for lateral vibration (shaft with fluid flow, fixed–fixed).

The mode shape of modes 1–3 according to the above equation are compared with the experimental result in Figs 8–10. We find all the theoretical mode shapes are in agreement with the experimental counterparts.

Although some discrepancies due to the practical clamping and sealing are seen, the experiments conducted in this section have confirmed the general agreement between the
practical fixture effects and the ideal boundary conditions. This is very encouraging for future research since mathematical tools are obtained for this conventionally empirical area.

7. THE SHAFT BEHAVIOR OF THE BTA DRILL WITH CUTTING FLUID

In this section, the behavior of the shaft conveying fluid is investigated. The arrangements of this experiment are shown in Fig. 11.

The fluid pressure on the tool head $P_t$ is read from a pressure gauge to be:

$$ P_t = 3.92 \times 10^6 \text{ N/m}^2. $$

Due to the limit of the available hydraulic system the flow rate cannot be changed. We have

- Flow rate = $1.2 \times 10^{-4} \text{ m}^3/\text{s}$
- Flow velocity = 1.155 m/s.

The equation of motion is:

$$ EI_s \frac{\partial^4 X}{\partial s^4} - \frac{\partial}{\partial s} (N - PA_f) \frac{\partial X}{\partial s} + \rho_s A_s \frac{\partial^2 X}{\partial t^2} + \rho_f A_f \left( \frac{\partial}{\partial s} - U \frac{\partial}{\partial t} \right)^2 X + C \frac{\partial X}{\partial t} = 0 \quad (43) $$

with B.C.'s

$$ s = 0, \quad X = \frac{\partial X}{\partial s} = 0 $$
$$ s = l, \quad X = \frac{\partial X}{\partial s} = 0. $$

In this experiment, the axial force $N$ is zero. Rearranging the above equation yields

$$ EI_s \frac{\partial^4 X}{\partial s^4} + (P_t A_f + \rho_f A_f U^2) \frac{\partial^2 X}{\partial s^2} - 2\rho_f A_f U \frac{\partial^2 X}{\partial s \partial t} + (\rho_s A_s + \rho_f A_f) \frac{\partial^2 X}{\partial s^2} + C \frac{\partial X}{\partial t} = 0. \quad (44) $$

Let

$$ X(s, t) = \Sigma \phi_j(s) q(t) $$

the mode shape function $\phi_j(s)$ is

$$ \phi_j(s) = \cosh \beta_j s - \cos \beta_j s - \sigma_j \sinh \beta_j s - \sin \beta_j s $$.  

(45)
The shaft behavior of BTA deep hole drilling tool

Table 4. The natural frequencies of lateral vibration of the shaft of BTA drill (both ends clamped horizontally, conveying fluid)

<table>
<thead>
<tr>
<th>Mode</th>
<th>Theoretical values (Hz)</th>
<th>Experimental values (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>43.424</td>
<td>43.015</td>
</tr>
<tr>
<td>2</td>
<td>119.689</td>
<td>112.382</td>
</tr>
<tr>
<td>3</td>
<td>234.634</td>
<td>220.889</td>
</tr>
</tbody>
</table>

and

\[ \alpha_j = \frac{\cosh \beta_j l - \cos \beta_j l}{\sinh \beta_j l - \sin \beta_j l} \]

\[ \cosh \beta_j l \cdot \cos \beta_j l = 1. \]

Giving an impulse to the shaft, the equation of motion becomes

\[ EI_s \frac{\partial^4 X}{\partial s^4} + \left( P_t A_f + \rho_f A_f U^2 \right) \frac{\partial^2 X}{\partial s^2} - 2 \rho_f A_f U \frac{\partial^2 X}{\partial s \partial t} + \left( \rho_s A_s + \rho_f A_f \right) \frac{\partial^2 X}{\partial t^2} + C \frac{\partial X}{\partial t} = h(t). \]

By doing the Fourier Transform of the above equation and solving for the closed form solution, we can obtain

\[ H_j(w) = \frac{1}{\left( 1 - \frac{w^2}{EI_s \beta_j^4 \phi + \left( P_t A_f + \rho_f A_f U^2 \right) \beta_j^2 \phi^*} \right)} \left( \frac{w}{c_j \phi - 2 \rho_f A_f U \beta_j \phi'} \right). \]

The theoretical natural frequency is equal to

\[ f_n = \frac{1}{2\pi} \sqrt{\frac{EI_s \beta_j^4 \phi + \left( P_t A_f + \rho_f A_f U^2 \right) \beta_j^2 \phi^*}{\left( \rho_s A_s + \rho_f A_f \right) \phi}}. \]

The frequency response function is

\[ H(s_t, s_r, w) = \sum_{j=1}^{M} \phi_j(s_t) \phi_j(s_r) H_j(w). \]

a. Natural frequency

Comparing the theoretical values of natural frequencies of modes 1–3 with those of experiment in Table 4, we can find the theoretical values of natural frequencies being close to the experimental values.

b. Mode shape

The mode shape function of the shaft is

\[ \phi_j(s) = \cosh \beta_j s - \cos \beta_j s - \frac{\cosh \beta_j l - \cos \beta_j l}{\sinh \beta_j l - \sin \beta_j l} (\sinh \beta_j s - \beta_j s). \]

The theoretical and experimental mode shapes are compared in Figs 12–14. It is seen that the agreements are satisfactory, but the fluid has caused even larger discrepancies.
Fig. 12. The shape of mode 1 for lateral vibration (shaft with fluid flow, fixed-fixed).

Fig. 13. The shape of mode 2 for lateral vibration (shaft with fluid flow, fixed-fixed).

Fig. 14. The shape of mode 3 for lateral vibration (shaft with fluid flow, fixed-fixed).

8. CONCLUSION

The shaft behavior of deep hole drilling is important in the cutting process but rarely studied. This paper studied the tool shaft of deep hole drilling by treating the shaft as a pipe and using the Bernoulli–Eulerian theory. The general equations of motion for the BTA drill shaft are rigorously derived which can be reduced to different specific equations of former
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works. The constructed general equations govern the lateral, longitudinal and torsional motion of a tool shaft conveying pressurized fluid.

The solutions for lateral and longitudinal motion are obtained by combined analytical and Galerkin methods.

A preliminary comparison between theoretical and experimental results shows that the rotatory inertia effect is negligible and the term $\rho I \ddot{\theta}$ in the shaft dynamics can be deleted. The shaft can be treated as an Euler Beam.

Two series of experiments are performed to verify the lateral shaft dynamics for an installed tool with and without cutting fluid. Although the boundary conditions of fixture damping are not ideal, the comparisons of natural frequencies and shaft mode shapes between theoretical and experimental results are satisfactory. Since the natural frequencies are lower than those of usual cutting tools, and the rotating workpiece might worsen these values, it can be predicted that the shaft dynamics are not negligible in the cutting process.

The constructed general equations of motion for a tool shaft build a foundation of knowledge about it and pave the way for future studies concerning correlation between cutting process and shaft behavior.

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