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Widesense nonblocking for multi-log\textsubscript{d} \textit{N} networks under various routing strategies


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Abstract

Chang et al. showed that the number of middle switches required for WSNB under strategies: save the unused, packing, minimum index, cyclic dynamic, and cyclic static, for the 3-stage Clos network \( C(n, m, r) \) with \( r \geq 3 \) is the same as required for SNB. In this paper, we prove the same conclusion for the multi-log\textsubscript{d} \textit{N} network. We also extend our results, except for the minimum index strategy, to a general class of networks including the 3-stage Clos network and the multi-log\textsubscript{d} \textit{N} network as special cases.

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1. Introduction

The symmetric 3-stage Clos network \( C(n, m, r) \) which has \( r \) switches of size \( n \times m \) in the first stage, \( m \) switches of size \( r \times r \) in the second(middle) stage, and \( r \) switches of size \( n \times m \) in the third stage (see Fig. 1).

The multi-log\textsubscript{d} \textit{N} network, first proposed by Lea [7], is composed of \( p \) copies of log\textsubscript{d} \textit{N} network connected in parallel (see Fig. 2). Each copy of the log\textsubscript{d} \textit{N} network, also called banyan-type networks, is constructed of \( d \times d \) switches arranged in \( n \) stages, \( N = d^n \), labeled 1, 2, \ldots, \( n \) from left to right. Each stage has \( d^{n-1}d \times d \) switches. In each copy, there is exactly one path between an arbitrary input and an arbitrary output. There are many varieties of log\textsubscript{d} \textit{N} networks, such as banyan, Omega, baseline, \ldots, but they are all equivalent in the sense that the connection property is invariant under a permutation of switches in the same stage.

A request is an (input, output) pair seeking connection. A set of requests can be routed if there exists connecting paths not intersecting each other in a node.

A multi-log\textsubscript{d} \textit{N} network is said to be strictly nonblocking (SNB) if a request can always be routed regardless of how the previous pairs are routed. It is said to be wide-sense nonblocking (WSNB) with respect to a routing strategy \( A \) if every request is routable under \( A \). It is said to be rearrangeable nonblocking (RNB) if every request can be connected provided routing paths of existing connections can be rearranged (rerouted).

For convenience of analysis, we transform a log\textsubscript{d} \textit{N} network to a digraph by converting each link, including the inputs and the outputs, to a node, while a crosspoint connecting two links in the network becomes an arc in the digraph.

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For the 3-stage Clos network, a routing strategy deals with the choice of a middle switch to route the request when many are available. Five routing strategies have been proposed in the literature (see [2] for a survey):

(i) Save the unused (STU). Do not route through an empty middle switch unless there is no choice.
(ii) Packing (P). Choose a busiest, yet available, middle switch.
(iii) Minimum index (MI). Label all middle switches from $M_1$ to $M_p$. For each request, route in the order $M_1, M_2, \ldots$, until the first available one emerges.
(iv) Cyclic dynamic (CD). If $M_k$ was used last, try $M_{k+1}, M_{k+2}, \ldots$, until the first available one emerges.
(v) Cyclic static (CS). If $M_k$ was used last, try copy $M_k, M_{k+1}, \ldots$, until the first available one emerges.

The existence of a WSNB network was first demonstrated by Beneš [1] for the symmetric 3-stage Clos network. He proved that $C(n, m, 2)$ is WSNB under packing if and only if $m \geq \lceil 3n/2 \rceil$ which is the only positive result. Smith [9] proved that $C(n, m, r)$ is not WSNB under P or MI if $m < \lceil 2n - n/r \rceil$, which was improved to $\lceil 2n - (n/(2^r - 1)) \rceil$ in Du et al. [3] and extended to all five strategies. For P, Yang and Wang [11] gave a linear programming formulation of the problem and ingeniously found the closed-form solution $m \geq \lceil 2n - n/F_{2r-1} \rceil$ where $F_{2r-1}$ is the $2r - 1$st Fibonacci number, as a necessary condition for $C(n, m, r)$ to be WSNB. Note that for $r$ large, all the above negative results show that $2n - 1$ middle switches are required for WSNB. Tsai et al. [10] culminated this line of results by giving a unifying proof for all possible strategies, not just the listed five.
For finite $r$, Du et al. [3] proved that for $r \geq 3$ $C(n, m, r)$ is WSNB for P or STU if and only if it is SNB, namely, $m \geq 2n - 1$, with a complicated proof. Chang et al. [2] simplified the proof and extended it to the other three strategies for $r \geq 2$; thus severely dashing the hope that a clever strategy can save hardware from SNB networks and yet preserve nonblockingness. We can translate the five routing strategies to the multi-log$_d N$ network by replacing “choosing a middle switch” to “choosing a copy (of log$_d N$)”. In Section 2, we prove a similar conclusion that these five strategies require the same number of copies as SNB does. In Section 3, we extend our results to a general class.

Presumably, one can ask the same question for RNB, namely, how many middle switches are required for RNB if a certain routing strategy is followed. This has not been studied in the literature, not even for $C(n, m, r)$. The reason is because RNB can also be interpreted as nonblocking if all requests are to be routed simultaneously [1,6]. Then there exist better routing strategies yielding the requirements of $n$ middle switches for the 3-stage Clos network [4] and $d^{|n/2|}$ copies for the multi-log$_d$ network [8]; showing that the cost of RNB is much less than that of SNB.

2. Main result


**Theorem 1.** Multi-log$_d N$ network is strictly nonblocking if $p \geq p(n)$, where

$$p(n) = \begin{cases} (d + 1) \times d^{\frac{n}{2} - 1} - 1 & \text{for } n \text{ even,} \\ 2 \times d^{\frac{n}{2} - 1} - 1 & \text{for } n \text{ odd.} \end{cases}$$

A request from input $x$ to output $y$, represented by $(x, y)$, has a unique path in a log$_d N$ network. Hence two intersecting paths must be routed through different copies of log$_d N$ network.

Theorem 1 was stated in [5] only as a sufficient condition. We need prove that it is also necessary.

**Theorem 2.** Multi-log$_d N$ network is strictly nonblocking only if $p \geq p(n)$.

**Proof.** For any request $\gamma = (x, y)$, assume that the path of $\gamma$ consists of links $L_0, L_1, \ldots, L_n$. For $n$ odd, let $I_1(O_2)$ be the set of inputs(outputs), except $x(y)$, which can reach $L_{(n-1)/2}$, then $|I_1| = d^{(n-1)/2} - 1$ and $|O_2| = d^{(n+1)/2} - 1$. Let $O_1(I_2)$ be the set of outputs(inputs), except $y(x)$, which can reach $L_{(n+1)/2}$. Then $|O_1| = d^{(n-1)/2} - 1$ and $|I_2| = d^{(n+1)/2} - 1$. Note that $\gamma$ cannot be routed through the same copy with any request from $I_1$ to $O_2$ or $I_2$ to $O_1$. Suppose $p = p(n) - 1$ while $|I_1| \text{ \ requests from } I_1 \text{ to } O_2 \setminus O_1 \text{ and } |O_1| \text{ \ requests from } O_1 \text{ to } O_2 \setminus I_1$ have already been connected in different copies. In this case, they can occupy $|I_1| + |O_1| = p(n) - 1 = p \ \text{ copies,} \ \text{with no copy left for } \gamma$. For $n$ even, let $I_1(O_2)$ be the set of inputs(outputs), except $x(y)$, which can reach $L_{n/2-1}$, then $|I_1| = d^{n/2-1} - 1$ and $|O_2| = d^{n/2+1} - 1$. Let $O_1(I_2)$ be the set of outputs(inputs), except $y(x)$, which can reach $L_{n/2+1}$. Then $|O_1| = d^{n/2-1} - 1$ and $|I_2| = d^{n/2+1} - 1$. Let $I_3(O_3)$ be the set of inputs(outputs), except $x(y)$, which can reach $L_{n/2}$. Then $|I_3| = |O_3| = d^{n/2} - 1$. Note that $\gamma$ cannot be routed through the same copy with any request from $I_1$ to $O_2$, $I_2$ to $O_1$, or $I_3$ to $O_3$. Suppose $p = p(n) - 1$ while $|I_1| \text{ \ requests from } I_1 \text{ to } O_2 \setminus O_3$, $|O_1| \text{ \ requests from } O_1 \text{ to } I_2 \setminus I_3$, and $|I_3 \setminus I_1| \text{ \ requests from } I_3 \setminus I_1 \text{ to } O_3 \setminus O_1$ have already been connected in different copies. In this case, they can occupy $|I_1| + |O_1| + |I_3 \setminus I_1| = |I_1| + |O_1| + |O_3 \setminus O_1| = p(n) - 1 = p \ \text{ copies, with no copy left for } \gamma$. Hence $p$ must be greater than or equal to $p(n)$. □

We call such a set of $p(n) - 1 \text{ \ requests blocking } \gamma$ the maximal blocking configuration (MBC), denote by $M(n, \gamma)$.

Note that if a network is SNB, then it is also WSNB. i.e. multi-log$_d N$ is WSNB if $p \geq p(n)$. Therefore, we only need to prove necessity in the following proofs. In all these proofs, we assume that the network carries no traffic at the beginning.

We consider strategy CD first.

**Theorem 3.** Multi-log$_d N$ network is WSNB under CD if and only if $p \geq p(n)$.

**Proof.** Suppose $p < p(n)$. Consider a sequence of $p + 1 \text{ \ requests with } p \ \text{ requests from } M(n, \gamma)$ followed by the request $\gamma$. By the property of strategy CD, these $p$ requests will be routed in $p$ copies. Then we cannot route $\gamma$ any more. Hence $p$ must be greater than or equal to $p(n)$. □
Theorem 4. Multi-log$_d$ N network is WSNB under CS if and only if $p \geq p(n)$.

Proof. Suppose $p < p(n)$. For a request $\gamma$ and any $p$ requests of $M(n, \gamma)$, say $\gamma_1, \gamma_2, \ldots, \gamma_p$, route $\gamma_1$ in copy 1, then route $\gamma$ in copy 2 (because $\gamma_1$ blocks $\gamma$ in copy 1). Then disconnect $\gamma$ and route $\gamma_2$ in copy 2. Then route $\gamma$ in copy 3. Again disconnect it and route $\gamma_3$ in copy 3. Doing this iteratively until $\gamma_p$ is routed in copy $p$. Then $\gamma$ cannot be routed any more. Hence $p$ must be greater than or equal to $p(n)$. □

For strategies P or STU, we introduce a lemma.

Lemma 5. For any request $\gamma$ and $M(n, \gamma)$, there exists a request $\gamma'$ which does not block $\gamma$ or any request in $M(n, \gamma)$ in the log$_d$ N network.

Proof. Use the graph model of the baseline network as an example. Without loss of generality, let $\gamma = (0, 0)$. For all requests $(i, j)$ in $M(n, \gamma)$, we obtain $i < N/d$ and $j < N/d$. Hence $\gamma' = (N - 1, N - 1)$ will satisfy our claim. □

Theorem 6. Multi-log$_d$ N network is WSNB under P or STU if and only if $p \geq p(n)$.

Proof. Suppose to the contrary, $p < p(n)$. For any request $\gamma$ and any $p$ requests of $M(n, \gamma)$, say $\gamma_1, \gamma_2, \ldots, \gamma_p$, we route $\gamma_1$ in copy 1 first. Then route $\gamma$ in copy 2 and route $\gamma'$ in copy 2 (because copy 1 are as busy as copy 2, we can choose copy 2). Now, we disconnect $\gamma$ and route $\gamma_2$ in copy 2. Then disconnect $\gamma'$. Similarly, we route $\gamma$ in copy 3 and $\gamma'$ in copy 3, then disconnect $\gamma$ and route $\gamma_3$ in copy 2. Finally, we route $\gamma_p$ in copy $p$. Then $\gamma$ cannot be routed any more. Hence $p$ must be greater than or equal to $p(n)$. □

MI is more complicated. We first introduce a result in [2].

Theorem 7. The 3-stage Clos network $C(n, m, r)$ for $r \geq 2$ is WSNB under MI if and only if $m \geq 2n - 1$.

In the following theorem, only the baseline architecture will be considered. However, the theorem is also true for other equivalent log$_d$ N network.

Theorem 8. Multi-log$_d$ N network is WSNB under MI if and only if $p \geq p(n)$.

Proof. We discuss two cases:

(i) $n$ is odd. Select two subset $I_1$ and $I_2$ of inputs and two subset $O_1$ and $O_2$ of outputs. Set $I_1 = O_1 = \{0, 1, 2, \ldots, d^{\lfloor(n-1)/2\rfloor} - 1\}$, $I_2 = O_2 = \{d^{\lfloor(n-1)/2\rfloor}, \ldots, 2 \times d^{\lfloor(n-1)/2\rfloor} - 1\}$. See Fig. 4. By the configuration of baseline network, every request from $I_1$ to $O_1$ or $O_2$ must intersect node 0 in stage $(n - 1)/2$ and every request from $I_2$ to $O_1$ or $O_2$ must intersect node 1 in stage $(n - 1)/2$. Therefore, for $i = 1$ or 2, all requests from $I_i$ to $O_1$ or $O_2$ must use different copies. Similarly, every request from $I_1$ or $I_2$ to $O_1$ must intersect node 0 in stage $(n + 1)/2$ and every request from $I_1$ or $I_2$ to $O_2$ must intersect node $d^{\lfloor(n-1)/2\rfloor}$ in stage $(n + 1)/2$. Therefore, for $i = 1$ or 2, all requests from $I_1$ to $O_i$ must use different copies. Now, we match this to a 3-stage Clos network $C(n^{\lfloor(n-1)/2\rfloor}, 1, 2)$, where $I_i$ is the $i$th input switch, $O_i$ is the $i$th output switch, for $i = 1$ or 2, and the complete bipartite graph induced by nodes 0 and 1 of stage $(n - 1)/2$ and nodes 0 and $d^{\lfloor(n-1)/2\rfloor}$ of stage $(n + 1)/2$ is the middle switch. Then a request $(i, j)$ on $C(n^{\lfloor(n-1)/2\rfloor}, p, 2)$ routed through the $k$th middle switch under MI corresponds to a request $(i, j)$ in the multi-log$_d$ N using copy $k$. Therefore, by Theorem 7, the network is not WSNB if $p < 2 \cdot (d^{\lfloor(n-1)/2\rfloor} - 1) = 2 \times d^{\lfloor(n-1)/2\rfloor} - 1 = p(n)$.

(ii) $n$ is even. Select four subset $I_1$, $I_1'$, $I_2$ and $I_2'$ of inputs and four subset $O_1$, $O_1'$, $O_2$, and $O_2'$ of outputs. Set $I_1 = O_1 = \{0, 1, 2, \ldots, d^{\lfloor n/2 - 1\rfloor} - 1\}$, $I_1' = O_1' = \{d^{\lfloor n/2 - 1\rfloor}, \ldots, d^{\lfloor n/2\rfloor} - 1\}$, $I_2 = O_2 = \{d^{\lfloor n/2\rfloor}, \ldots, (d + 1)d^{\lfloor n/2 - 1\rfloor} - 1\}$, and $I_2' = O_2' = \{(d + 1)d^{\lfloor n/2 - 1\rfloor}, \ldots, 2 \times d^{\lfloor n/2\rfloor} - 1\}$. See Fig. 5. Then every request from $I_1$ to $O_1$ or $O_2$ must
A vertical-copy network

Theorem 9. A vertical-copy network \( V \) is WSNB under the CS routing if and only if \( V \) is SNB.

3. Some generalizations

We extend our results to a class of networks including the 3-stage Clos networks, the \( \log_d N \) and the \( \log_d(N, k, m) \) networks as special cases.

A vertical-copy network \( V \) consists of an input stage of \( r_1(n_1 \times m) \)-crossbars, an output stage of \( r_2(m \times n_2) \)-crossbars and a middle stage of \( m \) copies of a network \( v \) with \( r_1 \) inputs and \( r_2 \) outputs. There exists exactly one link between each input(output) crossbar and each copy of \( v \). When \( v \) is the \( r_1 \times r_2 \) crossbar, \( V \) is a 3-stage Clos network. When \( n_1 = n_2 = 1 \) and \( v \) is the \( \log_d N \) network, \( V \) is a multi-\( \log_d N \) network. When \( n_1 = n_2 = 1 \) and \( v \) is the \( k \)-extra-stage \( \log_d N \) network, then \( V \) is the \( \log_d(N, k, m) \) network. In particular, if \( k = n - 1 \), then \( V \) is the Cantor network.

Suppose that the necessary and sufficient condition for \( v \) to be SNB is known. Consider \( p = p(n) - 1 \). For any request \( \gamma \), there must be a state \( s \) such that \( \gamma \) is blocked in each of the \( p(n) - 1 \) copies \( v_1, v_2, \ldots, v_{p(n)-1} \) Let \( R_i \) be the set of all requests routing through \( v_j \) in \( s \) and \( M(v, \gamma) = \{ R_i \mid i = 1, 2, \ldots, p(n) - 1 \} \). i.e., \( V \) is SNB if and only if the number of copies is larger than \( |M(v, \gamma)| \). Let “Route \( R_i \) in \( v_j \)” mean “Route all requests in \( R_i \) in \( v_j \) consecutively”.

Fig. 4. The left figure is an induced graph of the graph model of a multi-\( \log_d N \) network, for \( n \) odd. And the right figure is its correspondence to a 3-stage Clos network.
Suppose there exists a request \( \gamma \), we route \( R_1 \) in \( v_1 \), then route \( \gamma \) in \( v_2 \) (\( \gamma \) is blocked in \( v_1 \)). Then disconnect \( \gamma \) and route \( R_2 \) in \( v_2 \). Then route \( \gamma \) in \( v_3 \). Again disconnect it and route \( R_3 \) in \( v_3 \). Doing this iteratively until \( R_p \) is routed in \( v_p \). Then \( \gamma \) cannot be routed in any copy. Hence \( p \) must be greater than or equal to \( p(n) \). \( \square \)

For CD, we use another argument.

**Theorem 10.** A vertical-copy network \( V \) is WSNB under the CD routing if and only if \( V \) is SNB.

**Proof.** First, we claim every request \( \gamma \) can be routed in \( v_k \) for a given \( k \). Route \( \gamma \) in \( v_i \). If \( i \neq k \), then disconnect \( \gamma \) and route it again in \( v_{i+1} \). Similarly, if \( i + 1 \neq k \), then disconnect \( \gamma \) and route it again in \( v_{i+2} \) until \( \gamma \) is routed in \( v_k \). Note that if \( i = p \), then we let \( i + 1 \) be 1. Therefore, if \( p < p(n) \), then we can route \( R_i \) in \( v_i \) for \( i = 1 \) to \( p \) as we want. Then \( \gamma \) cannot be routed in any copy. Hence \( p \) must be greater than or equal to \( p(n) \). \( \square \)

For STU, if there exists a request \( \gamma'_i \) which does not block \( \{ \gamma \} \cup R_i \) for all \( i \), Theorem 6 remains true if \( M(n, \gamma) \) is replaced by \( M(v, \gamma) \) and \( \gamma'_i \) is replaced by \( R_i \). But we use a different argument for P.

**Theorem 11.** Suppose there exists a request \( \gamma'_i \) which does not block \( \{ \gamma \} \cup R_i \) for all \( i \). A vertical-copy network \( V \) is WSNB under the P routing if and only if \( V \) is SNB.

**Proof.** It suffices to prove the “only if” part. Suppose there are only \( p = p(n) - 1 \) copies \( v_1, v_2, \ldots, v_p \) in \( V \). For the request \( \gamma = (0, 0) \), without loss of generality, suppose \( R_i = \{ \gamma_{i,j} \mid j = 1, \ldots, \lambda_i \} \) and \( \lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_p \). Let \( |v_i| \) denote the number of connections in \( v_i \). For a given \( k \), let \( s(k, B) \) be a state satisfying the following conditions:

(i) \( |v_k| < \lambda_k \),  
(ii) Connections in \( v_i \) are those from \( R_i \),  
(iii) \( |v_i| = |v_k| + 1 \) or \( |v_i| = \lambda_i \) if \( i \in B \equiv \{ i \mid |v_i| > |v_k| \} \)

Let \( S(k) \) denote the state that \( v_i \) contains \( R_i \) for all \( 1 \leq i \leq k \). We make two claims:

**Claim A.** We can add another connection \( \delta \) of \( R_k \) in \( v_k \) in state \( s(k, B) \).

**Claim B.** \( S(k) \) can be realized.
Then the results follow from Theorems 9 and 10. For P and STU, it is easily verified that \( s^*(k, \mathbb{B}) \) differs from \( s(k, \mathbb{B}) \) by having \( v_I \) containing \( R_i \) for all \( 1 \leq i < k - 1 \), by applying induction to claim B (with \( k = k - 1 \)). In state \( s^*(k, \mathbb{B}) \), \( \gamma \) must be routed in \( v_k \). Now delete all connections in \( s^*(k, \mathbb{B}) \setminus s(k, \mathbb{B}) \) so that \( |v_k| \geq |v_i| \) for all \( i \). Then \( \gamma' \) can be routed in \( v_k \). Delete \( \gamma \) and route \( \delta \) in \( v_k \). Delete \( \gamma' \) and Claim A is proved. Also, we can keep on adding all remaining connections of \( R_k \) to \( v_k \) to prove Claim B.

Setting \( k = p \) in Claim B, then \( \gamma \) cannot be routed in any of the \( p \) copies. Hence at least \( p(n) \) copies are needed. □

**Example 1.** For simplicity, we will represent a state by its \(|\nu|\)-sequence. To help clarify the state, let \(|\nu|\) denote the fact that \( \nu \) is in \( v_I \), \(|\nu|\) the fact that \( \nu \) is \( |\nu| \) the fact that both are. Suppose \( p = 3 \) and we want to reach the state \( S(3) = (\lambda_1, \lambda_2, \lambda_3) = (2, 3, 4) \). The the \(|\nu|\)-sequence of our construction in Theorem 11 would be:

\[
\begin{align*}
(0, 0, 0) \Rightarrow (1, 0, 0) \Rightarrow (2, 0, 0) \Rightarrow (2, 1^*, 0) \Rightarrow (1, 1^*, 0) \Rightarrow (1, 2^*, 0) \Rightarrow (1, 1', 0) \Rightarrow \\
(1, 2', 0) \Rightarrow (1, 1, 0) \Rightarrow (2, 1, 0) \Rightarrow (2, 2^*, 0) \Rightarrow (2, 2^*, 0) \Rightarrow (2, 2', 0) \Rightarrow (2, 2', 0) \Rightarrow \\
(2, 2, 0) \Rightarrow (2, 3, 0) \Rightarrow (2, 3, 1^*) \Rightarrow (1, 1, 1^*) \Rightarrow (1, 2^*, 0) \Rightarrow (1, 1', 0) \Rightarrow (1, 1', 0) \Rightarrow \\
(1, 1, 1) \Rightarrow (2, 1, 1) \Rightarrow (2, 2^*, 1) \Rightarrow (2, 3^*, 1) \Rightarrow (2, 2', 1) \Rightarrow (2, 2', 1) \Rightarrow \\
(2, 3, 1) \Rightarrow (2, 3, 2^*) \Rightarrow (2, 2, 2^*) \Rightarrow (2, 2, 3^*) \Rightarrow (2, 2, 3^*) \Rightarrow (2, 2, 3') \Rightarrow (2, 2, 2^*) \Rightarrow \\
(2, 3, 2) \Rightarrow (2, 3, 3^*) \Rightarrow (2, 3, 4^*) \Rightarrow (2, 3, 3') \Rightarrow (2, 3, 4') \Rightarrow (2, 3, 3) \Rightarrow (2, 3, 4)
\end{align*}
\]

Therefore, we obtain the state \( S(3) \).

**Corollary 12.** \( \log_d(N, k, m) \) is WSNB under any of CS, CD, STU, and P if and only if it is SNB, i.e., [5],

\[
m > \begin{cases} 
k + 3 \cdot 2^{n-k-1} - 2 & \text{for } n - k \text{ even}, \\
k + 2^{n-k+1} - 2 & \text{for } n - k \text{ odd}.
\end{cases}
\]

**Proof.** Note that \( \log_d(N, k, m) \) is a vertical copy network. Then the results for CS and CD follow from Theorems 9 and 10. For P and STU, it is easily verified that \( \gamma' = (N - 1, N - 1) \) does not block any request in \( \{\gamma\} \cup R_i \) for all \( i \). Then the results follow from Theorems 11. □

What packing is a good routing strategy has been a folklore for a long time and documented in literature [1]. One motivation for that folklore is that \( C(n, m, 2) \) is WSNB under P if and only if \( m \geq [3n/2] \) [1], while it is SNB if and only if \( m \geq 2n - 1 \). The seemingly discrepancy between the \( m \geq [3n/2] \) result and Theorem 11 is explained by the fact that \( \gamma' \) does not exist in \( C(n, m, 2) \) since \( M(V, \gamma) \) occupies both input switches (see Fig. 6).
For \( r \geq 3 \), it was proved [2] that \( C(n, m, r) \) is WSNB under P if and only if it is SNB. Thus the saving of \( C(n, m, 2) \) under P seems to be a fluke rather than a testimony of its goodness. In this paper, again we showed that in the worst-case scenario, P does not help. Instead, MI is the only routing strategy which is still not ruled out to be useful.

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