Dynamic multicast routing under delay constraints in WDM networks with heterogeneous light splitting capabilities

Ming-Tsung Chen a,*, Shian-Shyong Tseng a,b, B.M.T. Lin c

a Department of Computer and Information Science, National Chiao Tung University, Hsinchu 300, Taiwan, ROC
b Department of Information Science and Applications, Asia University, Taichung, Taiwan, ROC
c Department of Information and Finance Management, National Chiao Tung University, Hsinchu 300, Taiwan, ROC

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Abstract

Because optical WDM networks will be realized as network backbone in the near future, multicasting in WDM networks needs to be supported for various network applications. In this paper, we propose a new dynamic multicast routing problem under delay constraints (DMR-DC) for finding an optimal light-forest with the minimum multicast cost from these links with available wavelengths for routing a multicast request that arrives in random with a given delay bound in a WDM network with heterogeneous light splitting capabilities, where a light-forest is a set of light-trees used to set up switches to route the request. Multicast cost is defined by communication cost ratio and wavelength consumption ratio. The problem is to determine a light-forest with less wavelength consumption and less communication cost. This problem is NP-hard because it can be reduced from the minimum Steiner tree problem. In this paper, we propose an efficient three-phase (generation, refinement, and conversion) solution model to find approximate solutions in a reasonable time.

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1. Introduction

An optical network [1] is a type of high-capacity telecommunication networks that provide routing, grooming, and restoration at wavelength level. The technology of wavelength division multiplexing (WDM) networks [2], providing the capacity of optical wavelength-division multiplexing on optical fibers to form multi-communication channels at different wavelengths, establishes connectivity among optical components for optical communication. The network can provide huge communication bandwidth to meet the increasing demands for a low transmission delay. The type of optical switches employing the technique to routing data in wavelength level is referred as wavelength-routing switches. A network deploying the type of switches is referred as a wavelength-routing WDM network. When data is to be transmitted between the input port and the output port of a switch using two different wavelengths, the switch needs to have the capacity of wavelength conversion. To transmit data among (wavelength-routing) switches in a (wavelength-routing) WDM network, a light-path [3], a connection based on wavelength to carry data without optical-to-electrical conversion, would be set up in a way similar as circuit-switched networks. The cost of utilized wavelengths and the delay time of transmitting optical signals to a destination by a light-path are referred as communication cost and transmission delay of the light-path, respectively. The communication cost may be the numbers or the costs of fibers and switches used for establishing the connection.

Many new network applications (videoconferencing, video on demand system, real-time control, on-line shopping, gaming, stock exchanging, and so on) are inspiring new communication models, among which multicasting is an important one used to send data (messages) from a single source to multiple destinations. Two schemes, multiple-unitcast and multicast have been employed to route data (Zhang et al. [9]). The multiple-unitcast scheme is a virtual topology consisting of a set of light-paths from the source to all destinations, where the number of light-paths may equal the number of destinations. If there exists some link shared by
more than one light-path, each light-path would need a different wavelength for routing data. As shown in Fig. 1(a), two light-paths, \( v_1 \rightarrow v_2 \rightarrow v_3 \) and \( v_1 \rightarrow v_2 \rightarrow v_4 \), would need two different wavelengths \( \lambda_1 \) and \( \lambda_2 \) because the link between \( v_1 \) and \( v_2 \) is shared. If each light-path requires one specific wavelength, the wavelength consumption may become unaffordable. The multicast scheme is thus proposed to reduce wavelength consumption.

In the multicast scheme, a switch with the capacity of splitting the (optical) signal of input port to multiple signals of different wavelengths \( l \) is called a multicast capable (MC) node; otherwise, it is called a multicast incapable (MI) node. The split signals can be transmitted by links to other switches concurrently. Therefore, locating an MC node for routing data to several destinations would have significant wavelength saving over the multiple-unicast scheme. The light splitting capacity of a switch is used to describe the maximal number of split signals in an output port.

The light splitting capacity of an MC node (respectively, MI node) is greater than (respectively, equal to) 1. As shown in Fig. 1(b), since \( v_2 \) is an MC node, only the wavelength \( \lambda_1 \) is required for routing data to \( v_3 \) and \( v_4 \) and the wavelength \( \lambda_2 \) can be saved. As defined above, the trail of routing data could be a light-tree [4] that does not containing infeasible nodes. A node is called infeasible if the number of outbound edges is greater than its light splitting capacity. If all nodes in a network are MC nodes, one light-tree may be sufficient for routing data to all destinations; otherwise, a set of light-trees, aggregated as a light-forest, may be required for the network with sparse light splitting in which some of the nodes are MC nodes. The problems of finding light-paths in the multiple-unicast scheme and finding a light-tree or a light-forest in the multicast scheme are called the routing problem and the multicast routing problem, respectively.

For different routing problems, several algorithms [5,6,7] and one protocol [8] were proposed in traditional networks and several heuristics [9–13] were proposed in (WDM) networks. To provide high quality of service (QoS), the communication cost and the wavelength consumption are usually discussed so as to evaluate the efficiency of the routes. Furthermore, for interactive multimedia applications that need to guarantee efficient transmission, delays from the source to all destinations will be limited under a given bound. The delay bound may be determined according to the degree of urgency or priority of the data. Therefore, data transmission with delay bounds reflects the realistic demand in the future.

For the multicast routing problem in networks with sparse light splitting, Zhang et al. [9] and Sreenath et al. [10] considered the network without and with wavelength conversion, respectively. Four rerouting algorithms, reroute-to-source, reroute-to-any, member-first, and member-only were introduced in Ref. [9], were developed to find a light-forest to route a request. The MC nodes with wavelength conversion were called as virtual sources in Ref. [10]. The virtual source approach consists of two phases, networking partitioning phase and tree generation phase. It was proposed in Ref. [10] to construct a multicast tree. Minimizing communication cost, wavelength conversion cost and wavelength consumption of light-forest subject to a transmission delay bound was not discussed in Refs. [9,10]. The multicast routing problem involving in wavelength assignment is called the multicast routing and wavelength assignment problem. Jia et al. [11] and Chen [12] solved the problem by decomposing the problem into two sub-problems, multicast routing and wavelength assignment, so as to reduce the complexity. In Ref. [11], the problem for routing a request with a delay bound was solved under the assumption that every node in network has light splitting capability. Two integrated algorithms corresponding to the two sub-problems were proposed to minimize the sum of wavelength cost and communication cost. Considering both wavelength cost and conversion cost, Chen [12] proposed an integrated approximation algorithm to deal with the problem without delay bounds. For routing on a network with power splitters having full range wavelength conversion and with wavelength converters having an unlimited splitting capacity, a mixed integer programming model was proposed by Yang et al. [13] to solve the multicast routing and wavelength assignment for light-trees with delay bounds. In their paper, the objective was not only to minimize the number of used fibers and to obtain the optimal placement of power splitters but also to design the logical topology based on light-trees for multiple connection demands. The study of Ref. [13] is based on the assumption that a multicast request is routed only by a light-tree. It is possible that no light-tree can be found to satisfy the delay bound constraint and to cover all destinations in the network without enough power splitters or enough wavelength converters.

In Refs. [9–13], all MC nodes were assumed to have the capability of splitting an input signal to multiple output signals. The number of output signals is no greater than the outbound edges of the MC node so that all destinations connecting to the MC node can be routed successfully. MC nodes of this type are called an unrestricted MC (UMC) node. Due to the sophisticated architecture [1] of MC nodes, using the MC nodes with superior light splitting capacities to build network is usually more expensive than using those with inferior light splitting capacities or MI nodes. Therefore, a WDM network with heterogeneous light splitting capabilities (WDM-He network), in which the light splitting capacities of all nodes could be different, addressed in the paper can better reflect the real-world requirement.

To the best of our knowledge, only a limited number of papers have been reported on the dynamic multicast routing
problem for routing requests that will arrive in random with a delay bound in a WDM-He network with or without wavelength conversion for minimizing the total cost incorporating communication cost and wavelength consumption. To better provide a realistic objective function to reflect the cost for routing a request, we consider a linear combination of communication cost and wavelength consumption, $\alpha$(communication cost) + $\beta$(wavelength consumption). This objective function is called the multicast cost hereafter in this paper. Notice that communication cost ratio $\alpha$ and wavelength consumption ratio $\beta$ can be appropriately chosen according to the topology and the load of the network.

This paper discusses the dynamic multicast routing problem with delay constraints (DMR-DC) of finding an optimal light-forest with minimum multicast cost to route a request arrives in random with a delay bound in a WDM-He network. Since the minimum Steiner tree problem (MSTP) [14] can be reduced to the studied problem by considering the minimum communication cost for connecting the source and all destinations in the request with unlimited delay bound by setting $\alpha = 1$ and $\beta = 0$, the DMR-DC problem is NP-hard. Table 1 shows the comparisons of previous research papers according to the following five characteristics: network deployment, delay bound, wavelength assignment, wavelength conversion, and factors of objective function. However, the procedure of wavelength assignment has not been discussed in this paper. But the DMR-DC problem is difficult to solve because many issues are simultaneously taken into account: the request is associated with a delay bound, the network deploys MI nodes and heterogeneous MC nodes, and a light-forest is evaluated by multicast cost. To simplify our discussion, the network used in the rest of this paper stands for the WDM-He network and without wavelength conversion. Because the DMR-DC problem is NP-hard, it is very unlikely to optimally solve it in polynomial time. In this paper, we develop an efficient three-phase (generation, refinement, and conversion) solution model to derive approximate solutions in an acceptable time.

The remainder of this paper is organized as follows. In Section 2, we formally define the studied problem. In Section 3, a solution model consisting of three phases will be developed. Structures of each phase will be described in detail. Section 4 presents the simulation of our proposed model and discussion of the numerical results. Finally, Section 5 gives some concluding remarks. List of acronyms is given in Appendix A for better comprehension.

<table>
<thead>
<tr>
<th>Network deployment</th>
<th>Delay bound</th>
<th>Wavelength assignment</th>
<th>Wavelength conversion</th>
<th>Factors of objective function</th>
</tr>
</thead>
<tbody>
<tr>
<td>MSTP [14]</td>
<td>UMC node</td>
<td>No</td>
<td>No</td>
<td>CC ($\alpha = 1$, $\beta = 0$)</td>
</tr>
<tr>
<td>Zhang [9]</td>
<td>MI nodes + UMC node</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Sreenath et al. [10]</td>
<td>MI node + UMC node</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Jia et al. [11]</td>
<td>UMC node</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>DMR-DC</td>
<td>MI node + MC node</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
</tbody>
</table>

CC, communication cost; WC, wavelength consumption.

2. Formulation

A weighted graph $G=(V, E, \theta, c, d)$ is used to represent a WDM-He network with switch set $V = \{v_1, v_2, ..., v_n\}$ and directed optical link set $E = \{e_1, e_2, ..., e_m\}$. Function $\theta: V \rightarrow N$ defines the light splitting capacity of switches, function $c: E \rightarrow \mathbb{R}^+$ defines the communication cost of links, and function $d: E \rightarrow \mathbb{R}^+$ specifies the transmission delay over the links. In graph $G$, there are $n$ nodes and $m$ edges. Node $v_i \in V$, $1 \leq i \leq n$, is an MC node when $\theta(v_i) > 1$; otherwise, $\theta(v_i) = 1$. Moreover, the light splitting capacities of MC nodes may be different.

A request $r$ with delay bound $\Delta$ is represented as $(s, D, \Delta)$ with destination set $D = \{d_1, d_2, ..., d_q\}$ and indicates that the data needs to be routed from a certain source $s$ to all destinations $d_i$, $1 \leq i \leq q$, where $s \in V$, $D \subseteq V - \{s\}$ is a set of destinations, $|D| = q$, and the transmission delay of routing data to all destinations must be bounded by the delay bound $\Delta$. For different sources, destinations and emergence levels, the delay bounds may be different. A tighter delay bound will result in fewer routes to be chosen and make the request likely to be suspended. For most of the cases, the delay bound of a request may be determined through previous experiences concerning the specified source, destinations, and application domain.

A light-path, represented by $P(s, d_i)$, is a route for transmitting data from $s$ to a destination $d_i$ without wavelength conversions. The communication cost and the transmission delay of light-path $P(s, d_i)$ are given as:

- communication cost: $c(P(s, d_i)) = \sum_{e \in P(s, d_i)} c(e)$
- transmission delay: $d(P(s, d_i)) = \sum_{e \in P(s, d_i)} d(e)$

For two nodes $u$ and $v$, there could be several light-paths from $u$ to $v$. Among these potential light-paths, the one with the minimum communication cost is called the minimum cost light-path (MCLP) and denoted by $P^*(u, v)$. The light-path with the minimum transmission delay is called the minimum delay light-path (MDLP) and denoted by $P^*(u, v)$.

We consider the graph that is the union of the light-paths for all source-destination pairs. In this graph, there could be infeasible nodes or cycles. Here, the infeasibility of a node means that the number of outbound nodes from the node outweighs its light splitting capacity such that the destinations connecting the infeasible node cannot be routed by one wavelength, and the existence of cycles means that the route
according to the graph will consume more communication cost than the other without cycles. In order to distinguish divergence in the graphs of unifying light-paths, the graph without cycles is called as a routing-tree. It implies that a routing-tree may not be a light-tree; accordingly, a routing-tree needs to be converted into several light-trees such that the request can be successfully routed. The minimal number of light-trees converted from the routing-tree is referred as a wavelength consumption of the routing-tree. In other words, finding a routing-tree is equivalent to finding a light-forest after it is converted.

Example 1. As shown in Fig. 2(a), graph G represents a WDM network with 13 nodes (n = 13) and 19 links (m = 19). Nodes v7 and v10 are MC nodes. Each link in the graph is associated with a value-pair ‘alb’, where a and b are the communication cost and the transmission delay of the link, respectively. For a given request r = (v9, [v6, v8, v10]), 3, 3), two routing-trees T1 and T2 are shown in Fig. 2(b) and (c). T1 is a light-tree because it does not include any infeasible node. Nevertheless, T2 includes an infeasible node v5 (for out(T2,v5)=2 to represent the outbound edges of v5 in T2 and θ(v5)=1, out(T2,v5)>θ(v5)) such that it is not a light-tree.

(1) Wavelength consumption of T

For the special case that the depth of T equals 2, each ST, degenerates as a single node. To obtain a recursive definition, ω(STi) is defined as 1. When s is an MC node, the wavelengths whose value is equal to the minimal integer greater than the number of leaves divided by θ(s) are required because θ(s) output signals can be split in s; that is, ω(T) = [out(T,s)/θ(s)]. Due to ω(STi) = 1 for 1 ≤ i ≤ τ, out(T,s) = Σ1≤i≤τ,ω(STi). Therefore, ω(T) = Σ1≤i≤τ,ω(STi)/θ(s)]. When s is an MI node, each leaf needs one wavelength to be routed such that ω(T) is equivalent to the number of leaves of s; that is, ω(T) = out(T,s) = Σ1≤i≤τ,ω(STi). Equally ω(T) = Σ1≤i≤τ,ω(STi)/θ(s)] holds for MI node s. In general, ω(T) = Σ1≤i≤τ,ω(STi)/θ(s)] when T has a depth of 2.

When the depth of T is greater than 2, ω(T) must be greater than or equal to the maximum number of wavelength consumptions of sub-trees. Therefore, the value of ω(T) would be defined recursively as:

$$
\omega(T) = \begin{cases} 
1 & T \text{ having a root node only,} \\
\max \left( \frac{\sum_{1 \leq i \leq \tau} \omega(ST_i)}{\theta(s)}, \sigma(T) \right) & \text{otherwise}
\end{cases}
$$

where \( \omega(T) = \max_{1 \leq i \leq \tau} \omega(ST_i) \).

(2) The communication cost and the transmission delay of T

As discussed above, the total communication cost of a link for routing a request to all destinations in sub-trees is defined as the product of the wavelength consumption of the sub-tree and link’s communication cost; e.g. the communication cost of a link \( e_{s,t} \) to connect for STi is equal to \( \omega(ST_i) \times c(e_{s,t}) \). The total communication cost and the transmission delay for routing a request from s to each destination in the sub-tree STi are \( \omega(ST_i) \times c(e_{s,t}) + c(ST_i) \) and \( d(ST_i) + d(e_{s,t}) \), respectively. Therefore, the communication cost and the transmission delay of T are defined recursively as

$$
c(T) = \sum_{1 \leq i \leq \tau} (\omega(ST_i) \times c(e_{s,t}) + c(ST_i)),
$$

and

$$
d(T) = \max_{1 \leq i \leq \tau} (d(ST_i) + d(e_{s,t})).
$$

Example 2. Three sub-trees (t = 3) ST1, ST2 and ST3 in routing-tree T rooted at s are shown in Fig. 3. In tree T, nodes s and s1 are MC nodes with \( \theta(s) = 3 \) and \( \theta(s1) = 2 \), and the others are MI nodes. Due to \( \sigma(ST_1) = 1, \sigma(ST_2) = 1 \), \( \omega(ST_1) = \max \left( \frac{1+1+1}{\theta(s1)}, 1 \right) = 2 \), and \( \omega(ST_2) = 1, \omega(T) = \max_{1 \leq i \leq \tau} \omega(ST_i) = 2 \) at \( \omega(T) = \max \left( \frac{\sum_{1 \leq i \leq \tau} \omega(ST_i)}{\theta(s)}, \sigma(T) \right) \). It implies that a ST1 or ST2 are feasible in T. Therefore, the communication costs for ST1 and ST2 are found. The communication costs...
and the transmission delays of $ST_1$, $ST_2$, and $ST_3$ can be computed as $c(ST_1) = 1 \times 8 + 1 \times 6 + 1 \times 2 = 16$, $d(ST_1) = \max(0.5, 0.5, 1.5) = 1.5$, $c(ST_2) = 8$, $d(ST_2) = 0.68$, $c(ST_3) = 0$, and $d(ST_3) = 0$, respectively. Therefore,

$$c(T) = (\alpha c(ST_1) + c(ST_1)) + (\alpha c(ST_2) + c(ST_2)) + (\alpha c(ST_3) + c(ST_3)) = 2 \times (2 \times 8 + 1 \times 6 + 1 \times 2) + (1 \times 3 + 8) + (1 \times 2 + 0) = 33$$

and $d(T) = \max(d(ST_1) + d(e_{s,z}), d(ST_2) + d(e_{s,z}), d(ST_3) + d(e_{s,z})) = \max(1.5 + 1.4, 0.68 + 1.3, 0 + 0.4) = 2.9$ can be obtained.

A routing-tree $T$ would be a candidate if its transmission delay does not exceed the delay bound. For a given request, there may be several different candidates eligible for routing the request. The multicast cost function $f$ for calculating the multicast cost of the candidate $T$ is defined as

$$f(T) = \alpha c(T) + \beta d(T),$$

where $\alpha$ and $\beta$ are the communication cost ratio and the wavelength consumption ratio, respectively. In the above description, a candidate cannot be directly used to set up nodes for routing a request because it may contain infeasible nodes. The candidate needs to be converted into a set of light-trees. For example, for a candidate $\hat{T}$ and $(\alpha c(\hat{T}) > 1$, an equivalent light-forest $\Gamma = \{\hat{T}_1, \hat{T}_2, \ldots, \hat{T}_\omega(\hat{T})\}$ can be obtained from converting $\hat{T}$ such that $\hat{T} = \bigcup_{k=1}^{\omega(\hat{T})} \hat{T}_k$, $c(\hat{T}) = \sum_{k=1}^{\omega(\hat{T})} c(\hat{T}_k)$, $d(\hat{T}) = \max_{1 \leq k \leq \omega(\hat{T})} d(\hat{T}_k)$, $f(\hat{T}) = \sum_{k=1}^{\omega(\hat{T})} f(\hat{T}_k)$, and $\alpha(\hat{T}_k) = 1$ for $1 \leq k \leq \omega(\hat{T})$. Therefore, once a candidate is found, an equivalent light-forest will be a solution to the DMR-DC problem. An optimal candidate implies that an optimal light-forest is obtained.

**Example 3.** Suppose that the routing tree shown in Fig. 3 is a candidate. The candidate that needs two wavelengths as computed in Example 2 can be converted into two light-trees as shown in Fig. 4. In other words, the two light-trees can be merged into a graph, which is equivalent to the candidate, and the sum of the communication costs of the two light-trees also equals the communication cost of the candidate.

3. Solution model

Due to the fact that the DMR-DC problem is NP-hard, we propose a three-phase (generation, refinement, and conversion) solution model with most cost-reduction first progressive replacing heuristic (MCRFPR) to find approximate solutions in polynomial time. The model shown in Fig. 5 consists of the following three phases:

1. **Generation phase**— use Prim’s algorithm [16] to find a candidate. If no candidate can be found, then an empty tree will be reported.

2. **Refinement phase**— use MCRFPR to refine the candidate to obtain a better approximate candidate.

3. **Conversion phase**— convert the approximate candidate into an equivalent light-forest.

3.1. Generation phase

This phase consists of two steps: (1) check all transmission delays of MDLPs between the source and all destinations with regards to the delay bound; (2) unify these MDLPs to construct a candidate. In the first step, no feasible candidate would exist when there is some MDLP whose transmission delay exceeds the delay bound. Because there is only one path with the minimum transmission delay between two nodes and the first step is checked, the graph found after the second step is
a candidate, but it may not be optimal. The details are described in the Generation procedure.

Generation($\tau(s, D, \Delta)$)

\{
  1. for all $d_i$ in $D$//Transmission delay of $P^{d}_i(s, d_i)$ cannot be greater than $\Delta$
  2. if ($d(P^{d}_i(s, d_i)) > \Delta$)/$P^{d}_i(s, d_i)$ is the MDLP between $s$ and $d_i$
  3. return null//No candidate can be found to transmit the request
  4. end for-loop
  5. $\hat{T}^d = \bigcup_{d_i \in D} P^{d}_i(s, d_i)$//$\hat{T}^d$ would be a candidate
  6. return $\hat{T}^d$
\}

Example 4. For the WDM network and request $r$ shown in Fig. 2(a), four MDLPs $P^d(v_9,v_0)$, $P^d(v_9,v_2)$, $P^d(v_9,v_{10})$, and $P^d(v_9,v_8)$ are shown in Fig. 6(a). The resultant graph $\hat{T}^d$ shown in Fig. 6(b) is a candidate for routing $r$.

3.2 Refinement phase

A candidate might be found in the generation phase, but it may not be optimal for most of the cases. It is intuitive to refine the path between two nodes in the candidate to progressively decrease multicast cost without violating the delay constraint. Nevertheless, another issue concerning how to select the two nodes (node-pair) arises. Refining a path is equivalent to rerouting the data from the end to the start of the path. To estimate the multicast cost reduction of two nodes $u$ and $v$, the cost-reduction (CR) of $(u, v)$, $CR(\hat{T}^d, u, v)$ is proposed to represent the expected multicast cost reduction that can be attained by rerouting the request from $u$ to $v$. For the candidate $\hat{T}^d$ shown in Fig. 7, $x$ is the nearest common predecessor node. Suppose that $P^{c}_x(v, u)$ is the path between $v$ and $u$ in $\hat{T}^d$; that is, $P^{c}_x(v, u)$ is a concatenation of $P^{d}_x(v, x)$ and $P^{d}_x(x, u)$. There exist many different routes to replace $P^{c}_x(v, u)$ to reroute the request from $u$ to $v$. For $P^{c}_x(v, u)$, a light-path from $v$ to $u$ with the minimum communication cost, using $P^{c}_x(v, u)$ to replace $P^{c}_x(v, u)$ can potentially achieve multicast cost reduction, although not fully guaranteed. Therefore, we use $P^{c}_x(v, u)$ to define CR ($\hat{T}^d, u, v$) such that the expectation is close to reality. After rerouting the request from $u$ to $v$ by $P^{c}_x(v, u)$, we have the graph $\hat{T}^d$ by eliminating $P^{c}_x(u, x)$ from $\hat{T}^d$ and appending $P^{c}_x(u, v)$ as shown in Fig. 8.

It can be seen that the number of outgoing edges of $v$ in $\hat{T}^d$ can be increased by 1 to be out($\hat{T}^d, v$) + 1. For the case $\text{out}(\hat{T}^d, v) < \theta(v)$, $\text{out}(\hat{T}^d, v)$ cannot be greater than $\theta(v)$ such that the wavelength consumption of $\hat{T}^d$ remains unchanged. Therefore, the multicast cost reduction is the difference between the communication cost of the eliminated path $P^{c}_x(u, x)$ and the communication cost of the path $P^{c}_x(v, u)$, or $\alpha(c(P^{c}_x(u, x)) - c(P^{c}_x(v, u)))$. Otherwise, the reduction needs to subtract the cost of one extra wavelength ($\beta$) and the communication cost of the partial light-path connecting $s$ and $v$ (i.e., $c(P^{c}_x(s, v))$ for using the extra wavelength. Therefore, the CR($\hat{T}^d, u, v$) of the node-pair $(u, v)$ is defined as:

![Fig. 7. A candidate $\hat{T}^d$.](image)

![Fig. 8. A new graph $\hat{T}^d$ after rerouting $u$ to $v$.](image)
where \( x \) is the nearest common predecessor node of \( u \) and \( v \).

Example 5 For the routing-tree \( T_z \) shown in Fig. 1(c), \( P^r_{T_z}(v_8, v_9) = (v_8, v_9, v_4, v_6, v_8) \) and \( P^r_{T_z}(v_9, v_8) = (v_9, v_5, v_8, v_9) \) are MCLPs between \( v_8 \) and \( v_9 \) and light-path of \( T_z \) between \( v_8 \) and \( v_9 \). Due to \( \text{out}_{T_z}(v_9) = 2 < \theta(v_9) = 3 \), \( \text{CR}(T_z, v_8, v_9) = c(P^r_{T_z}(v_9, v_9)) - c(P^r_{T_z}(v_8, v_9)) - \beta = 19 - 14 - 0 - \beta = 5a - \beta \). 

\( \text{CR}(T_z, v_8, v_9) = 4 \) for \( a = 1 \) and \( \beta = 1 \) is the expected reduction of \( v_8 \) rerouting to \( v_9 \).

Among node-pairs, the one with the maximum cost-reduction is called the most promising node-pair. Rerouting the most promising node-pair first infers that the multicast cost reduction of the node-pair may be more outstanding. Accordingly, an iterative heuristic, MCRFPR, is proposed in this phase. Each iteration in the heuristic consists in the following three steps:

1. Choose the most promising node-pair in the candidate.
2. Reroute the two nodes in the most promising node-pair to form a new graph. Find a minimum spanning tree (MST) from the graph to obtain a new routing-tree. Check the routing-tree to be a candidate with less multicast cost.
3. Repeat the previous two steps until multicast cost cannot be reduced again.

In the second step, a new graph \( G^a, \hat{T}^a \cup P^r(u, v) \), is obtained after the most promising node-pair is rerouted. Prim’(\( G^a \)) stands for an implementation of Prim’s algorithm [16] for finding a minimum spanning tree \( \hat{T}^a \) from \( G^a \). When \( \hat{T}^a \) is a candidate and its multicast cost is smaller than the original multicast cost, \( \hat{T}^a \) will replace the original candidate. Finally, when all node-pairs have been rerouted tentatively to reduce the multicast cost of previous candidate, the ultimate candidate will be the approximate candidate. Three heuristics are proposed to select node-pairs in Step 1: all nodes reroute to parent (ARP), destinations reroute to neateast nodes (DRNN), and all nodes reroute to neateast nodes (ARNN). They may achieve different improvements on multicast cost using different execution times.

3.2.1 All nodes reroute to parent (ARP)

The major heuristic of ARP is to reroute a node to some predecessor in the light-path. Therefore, each selected node-pair \((u, v)\) must satisfy two conditions that \( u \) and \( v \) are included in the light-path \( P_{T^a}(s, d_i) \) for some \( d_i \in D \) and that the transmission delay of the new light-path between \( s \) and \( d_i \) by rerouting \( u \) to \( v \) is not greater than the delay bound. The later will be checked by the rule \( d(P_{T^a}(s, v)) + d(P^r(v, u)) + d(P_{T^a}(u, d_i)) \leq \Delta \). The detail of the heuristic is described as follows.

\[
\text{ARP}(\hat{T}^a, D) \{
\text{1. for each } d_i \in D
\text{2. for each node-pair } (u, v), u \text{ and } v \in V(P_{T^a}(s, d_i))
\text{3. if } (d(P_{T^a}(s, v)) + d(P^r(v, u)) + d(P_{T^a}(u, d_i)) \leq \Delta
\text{4. If } (\text{CR}(\hat{T}^a, u, v) > 0)
\text{5. keeping } (u, v) \text{ in array } K \text{ by sorting } \text{CR}(\hat{T}^a, u, v) \text{ in decreasing order}
\text{6. for each node-pair } (u, v) \text{ in } K
\text{7. } G^a = \hat{T}^a P^r(u, v)
\text{8. } \hat{T}^a = \text{Prim’}(G^a)
\text{9. delete all leaf nodes in } \hat{T}^a \text{ which does not belong to } D
\text{10. if}(d(\hat{T}^a) < \Delta)\text{//check the transmission delay of } \hat{T}^a
\text{11. if } (f(\hat{T}^a) < f(\hat{T}^a))
\text{12. } \hat{T}^a = \hat{T}^a
\text{13. end-for loop}
\text{14. return } \hat{T}^a
\}
\]

3.2.2 Destinations reroute to nearest nodes (DRNN)

In DRNN, each destination \( d_i \) will try to reroute its nearest node such that the new candidate with less multicast cost still satisfy the delay constraint, where the nearest node of \( d_i \), \( \delta(d_i) \), is defined to satisfy \( d(P_{T^a}(s, \delta(d_i))) + d(P^r(\delta(d_i), d_i)) \leq \Delta \) and \( \text{CR}(\hat{T}^a, d_i, \delta(d_i)) \leq \text{CR}(\hat{T}^a, d_i, u) \) for all \( u \in V - \{d_i\} \). Therefore, for each node-pair \((u, v)\) chosen in this heuristic, it is necessary that either of \( u \) and \( v \) belongs to \( D \). The number of node-pairs in this heuristic would be significantly smaller than that in the ARP heuristic, but finding the nearest node of a given node is time-consuming. The details of this heuristic are described as follows.

\[
\text{DRNN}(\hat{T}^a, D) \{
\text{1. for each } d_i \in D
\text{2. if the nearest nodes } \delta(d_i) \text{ can be found}
\text{3. keeping } (d_i, \delta(d_i)) \text{ in array } K \text{ by sorting } \text{CD}(\hat{T}^a, d_i, \delta(d_i)) \text{ in decreasing order}
\text{4. for each node-pair } (u, v) \text{ in } K
\text{5. } G^a = \hat{T}^a P^r(u, v)
\text{6. } \hat{T}^a = \text{Prim’}(G^a)
\text{7. delete all leaf nodes in } \hat{T}^a \text{ which does not belong to } D
\text{8. if}(d(\hat{T}^a) < \Delta)\text{//check the transmission delay of } \hat{T}^a
\text{9. if } (c(\hat{T}^a) < c(\hat{T}^a))
\text{10. } \hat{T}^a = \hat{T}^a
\text{11. end-for loop}
\text{12. return } \hat{T}^a
\}
\]
3.2.3. All nodes reroute to near nodes (ARNN)

The difference between ARNN and DRNN is that in ARNN each node will try to reroute to its nearest node. Because the number of selected node-pairs in this heuristic is much greater than the numbers of selected node-pairs in DRNN and ARP, the execution time of heuristic DRNN is longer than the other two. Moreover, there are many node-pairs that can be used to refine the candidate. Thorough inspection suggests a promising approach to locating the nearest optimal candidate than two other heuristics.

3.3. Conversion phase

The primary concept in this phase is to convert infeasible nodes to construct a set of light-trees, or a light-forest. Suppose that each node \( u \) has two buffers \( S(u) \) and \( \text{Child}(u) \) to keep light-trees with root \( u \) and all successors of \( u \) in \( \mathcal{P} \), where \( S(u,i) \) and \( \text{Child}(u,i) \) are the \( i \)th light-tree and the \( i \)th successor of \( u \), respectively. For the case that \( u \) is a leaf node, it must be noted that there is no light-tree stored in \( S(u) \) and \( |S(u)| = 0 \). For the case that there are \( r \) successors in \( u \), because \( u \) can split an input signal into \( \theta(u) \) output signals to other nodes, the request can be routed to \( \theta(u) \) successors by a specified wavelength. Therefore, routing \( \theta(u) \) light-trees chosen from \( S(u) \) will need one wavelength, and then connecting \( u \) and the \( \theta(u) \) light-trees will form a light-tree rooted at \( u \). Repeating the procedure, all light-trees rooted at \( u \) will be constructed. For \( |S(u)| \) light-trees rooted at \( u \), \( |S(u)| \) wavelengths are required to route the request passing through \( u \). For a candidate \( \mathcal{P} \) found in the refinement phase, \( S(s) \), the light-forest keeping \( |S(s)| \) light-trees for routing the request to all destinations, is the solution reported by our three-phase heuristic.

In order to utilize the capability of signal splitting to reduce wavelength consumption, we will propose a simple heuristic in which the successors with larger numbers of light-trees will be choosing first to use the same wavelength. The details are given in the following Conversion procedure.

Conversion (\( \mathcal{P}^\theta, s \))

1. if \( s \) is a leaf node
2. \( T = \emptyset \)
3. insert \( T \) into \( S(s) \)/keep the converted sub-trees in \( s \)
4. return
5. else
6. \( \tau = |\text{Child}(s)| \)
7. for \( i = 1 \) to \( \tau \)
8. Converting (\( \mathcal{P}^\theta, \text{Child}(s,i) \))
9. end for-loop
10. while \( \text{Child}(s) \neq \emptyset \)
11. choose the first \( \theta(s) \) successors from \( \text{Child}(s) \) in the order of
12. the numbers of light-trees
13. delete \( S(\text{Child}(s,i), 1) \) into \( S(s) \)
14. remove \( \text{Child}(s,i) \) when \( S(\text{Child}(s,i)) \) is empty for \( 1 \leq i \leq \theta(s) \)
15. end loop
16. return

Example 6 As shown in Fig. 6(c), the candidate \( \mathcal{P}^\theta \) is an approximate candidate found after the refinement phase. \( S(u) \) and \( \text{Child}(u) \) will be computed backward for \( u \in V(\mathcal{P}^\theta) = \{v_q, v_y, v_0, v_8, v_{10}\} \) in the Conversion Phase. Because \( v_q \) and \( v_{10} \) are leaf nodes, \( S(v_q) = \emptyset \), \( S(v_{10}) = \emptyset \), and \( \text{Child}(v_{10}) = \emptyset \) will be obtained first. Secondly, when \( v_0 \) is computed, \( \text{Child}(v_0) = \{v_q\}, U_{1 \leq v \leq v_q}(e_{v_0,v}) \cup S(\text{Child}(v_0,i), 1) = e_{v_0,v} \cap S(v_1,1) = \{e_{v_0,v_8}, e_{v_0,v_9}\} \), and \( S(v_0) = \{\{e_{v_0,v_8}\}, \{e_{v_0,v_9}\}\} \) can be obtained, where \( \theta(v_0) = 1 \). Then, \( \text{Child}(v_3) = \{v_q, v_{10}\} \), \( U_{1 \leq v \leq v_q}(e_{v_0,v}) \cup S(\text{Child}(v_3,i), 1) = e_{v_0,v} \cap S(v_5,1) = \{e_{v_0,v_8}, e_{v_0,v_9}\} \), and \( S(v_3) = \{\{e_{v_0,v_8}, e_{v_0,v_9}\}\} \). Finally, due to \( \theta(v_3) = 1 \), \( \text{Child}(v_3) = \{v_q, v_{10}\} \), and \( S(v_3) = \{\{e_{v_0,v_8}, e_{v_0,v_9}\}\} \), \( U_{1 \leq v \leq v_q}(e_{v_0,v}) \cup S(\text{Child}(v_3,i), 1) = e_{v_0,v} \cap S(v_5,1) = \{e_{v_0,v_8}, e_{v_0,v_9}\} \), and \( S(v_3) = \{\{e_{v_0,v_8}, e_{v_0,v_9}\}\} \). Therefore, there are two light-trees shown in Fig. 6(d) obtained in the Conversion Phase, and the equivalent light-forest of \( \mathcal{P}^\theta \) will be stored in \( S(v_0) \).

4. Simulation

Our work focuses on finding an approximate light-forest such that switches in the network can be set up to route the request to all destinations. To evaluate the performance of our solution model, we adopt the approach proposed in Waxman [17]. In this approach, there are \( n \) nodes in the network. The nodes are randomly distributed over a rectangular grid, and each node is placed on an integer coordinates. In a network, each directed link between two nodes \( u \) and \( v \) has the probability function \( P(u, v) = \lambda \exp(-p(u,v)/\gamma \delta) \), where \( p(u,v) \) is the distance between \( u \) and \( v \), \( \delta \) is the maximum distance between each two nodes, and \( 0 < \lambda, \gamma \leq 1 \). In the probability function, a larger value of \( \lambda \) produces a network with higher link densities, and a small value of \( \gamma \) increases the densities of short links relative to longer ones. In our experiments, we set \( \lambda = 0.7 \) and \( \gamma = 0.9 \), let 15% of the nodes be MC nodes with randomly generated light splitting capacities, and set the size of rectangular grid to be 50.

Eight types of networks were tested: 40 switches (\( n = 40 \)), 60 switches (\( n = 60 \)), 80 switches (\( n = 80 \)), and 100 switches (\( n = 100 \)), for which each 80 different requests were randomly generated. Each 80 requests are categorized into four groups corresponding to three destinations (\( q = 3 \)), five destinations (\( q = 5 \)), seven destinations (\( q = 7 \)), and nine destinations (\( q = 9 \)). The communication cost and the transmission delay of each link are defined as the distance of two nodes of the link on the grid and a random number from the uniform interval [0.1, 3], respectively. For each request, the source and the destinations were generated randomly. Nevertheless, the value of delay bound \( D \) needs to be reasonable for...
otherwise it is very likely that no feasible light-forest can be found. In the following experiments, \( D \) is set to be equal to \( \chi \) times the derived minimum transmission delay between the source and all destinations in each request, where \( \chi \) is a control parameter dictating the tightness between delay bound and minimum transmission delay. For example, \( \chi = 3 \) means that all delay bounds of requests were set to be \( 3 \times \) their minimum transmission delay. The experiments consist of four parts: comparisons between heuristics and MDHN for different requests without delay bounds, comparisons of heuristics and Jia’s heuristic [11] for different requests with delay bounds, comparisons between different requests with different delay bounds, and comparisons for different wavelength consumption ratios. Our codes were implemented in C++ on a computer with an Intel Pentium-M 1.5 GHz CPU and 1 GB RAM.

4.1. Comparisons between heuristics and MDHN for different requests without delay bounds

For a WDM network whose nodes are UMC nodes, a Steiner tree covering the source and all destinations can be viewed as a light-forest for routing a request without delay bound; that is, the MSTP [13] can be reduced to the studied problem with \( n \) and \( \beta = 0 \). The Minimum Distance Heuristic Network (MDHN) heuristic proposed in Ref. [15] to find approximate solutions for the MSTP can be applied to the DMR-DC problem. It is worthy to note that the routing heuristic proposed by Jia [11] is an MST-based algorithm; therefore, the comparisons between Jia’s heuristic and our heuristics would not be discussed here. All nodes in the four networks are set to be UMC nodes, and the delay bound of each request is set to be a large value such that the transmission delay of each request is not constrained.

The numerical results of using different heuristics to find the approximate solutions for routing requests in the four networks (\( n = 40 \) and 60, \( q = 80 \) and 100) are summarized in Table 2. For the columns corresponding to each request group (\( q = 3, 5, 7, \) and 9), we keep track of the average multicast cost (MC) and the sum of elapsed execution time (ET) over every 20 requests, where the unit in ET is millisecond. According to these experimental results, all heuristics (ARP, DRNN, ARNN) or aggregate heuristics (ARP + DRNN, ARP + ARNN) appear to be able to find more efficient solutions than MDHN, but the elapsed execution times in the five heuristics are much longer than MDHN. Among the five heuristics, the execution time required by DRNN is the least, and ARP + DRNN and ARP + ARNN find the most efficient light-trees. Between the two aggregate heuristics, ARP + ARNN runs faster.

### Table 2

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<th>MC</th>
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ARP + ARNN, 15% of nodes in all networks are MC nodes with randomized light-splitting capacities in the experiments. For five different values ($\chi = 3, 2, 1.5, 1.3, \text{ and } 1.1$), the experimental results are shown in Table 4. In Table 4, the elapsed execution time slightly decreases as the values of $\chi$ increase. For different values of $\chi$, ARP + ARNN takes more execution times but find better solutions than DRNN. Moreover, the execution times of ARP + ARNN are close to those of DRNN as the delay bounds become tighter.

### 4.4. Comparisons for different wavelength consumption ratios

From the previous experiments, we know that ARP + ARNN can find better approximate solutions. Therefore, we use this aggregate heuristic to study the performances under different wavelength consumption ratios ($\beta$). In the experiments, the value of $\alpha$ is set to be 1, the values of $\beta$ are chosen from the values, 0.1, 1, 2, 3, 4, 5, 10, 20, 30, 40, 50, 90, 100, and the numbers of destinations are chosen from 15, 20, and 25 ($q = 15, 20, \text{ and } 25$). All requests will be routed in networks with 100 nodes ($n = 100$). The experimental results shown in Figs. 9 and 10 indicate that when the values of $\beta$ grow the communication cost gradually increases and the number of wavelengths gradually decreases. Furthermore, the variation is distinguished for the request with more number of destinations. The wavelength consumption ratio will set to be a larger value to save the wavelength when the network is deployed less wavelength bandwidth; otherwise, the wavelength consumption ratio will set to be a smaller value.

### Table 3
Experimental results for different requests with delay bounds

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<th>ARNN</th>
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### Table 4
Experimental results for different requests with different delay bounds

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<td>49.05 10</td>
<td>54.73 10</td>
<td>51.30 10</td>
<td>49.05 10</td>
<td>51.30 10</td>
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5. Conclusions

In this paper, we have formulated and studied a new dynamic multicast routing problem under delay constraints in WDM networks with heterogeneous light splitting capacities. A three-phase solution model, consisting of generation, refinement, and conversion, is proposed to solve the problem. The most cost-reduction first progressive replacing (MCRFPR) heuristic was proposed in the refinement phase to refine the candidate found in the generation phase. Based upon three different approaches to selecting node-pairs, three heuristics were proposed to obtain a more efficient light-forest with less multicast cost.

To evaluate the performance of our solution model, several experiments have been conducted. All heuristics proposed in this paper can produce solutions with smaller multicast costs than MDHN and Jia heuristics for routing requests without or with delay bounds in WDM networks. The aggregate heuristic $\text{ARP} + \text{ARNN}$ gave better approximate light-forests but required the longest execution time. The number of node-pairs selected to refine the candidate will affect the execution time and the efficiency of light-forests. Therefore, applying a better heuristic to efficiently reduce the number of node-pairs can produce an approximate light-forest with less execution time. For the two weight ratios, they can be effectively used to construct the light-forest with less wavelength consumption or less communication cost.

Because WDM networks with wavelength conversion can provide higher flexibility for routing requests, the overhead of converting wavelength needs to be evaluated in multicast cost for finding an efficient light-forest. Nevertheless, for WDM networks providing sparse wavelength conversion, an extra constraint describing a node with/without wavelength conversion needs to be included. Therefore, the problem is more difficult. To solve this sophisticated problem, we may modify the multicast cost and all heuristics. We are now trying to refine our solution model to solve two problems, routing a request in the network with sparse wavelength conversion and routing multiple requests currently in the network without wavelength conversion.

Acknowledgements

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Appendix A. List of acronyms

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Description</th>
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<tbody>
<tr>
<td>WDM</td>
<td>Wavelength division multiplexing</td>
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<tr>
<td>MC</td>
<td>Multicast capable node</td>
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<tr>
<td>MI</td>
<td>Multicast incapable node</td>
</tr>
<tr>
<td>WDM-He</td>
<td>WDM network with heterogeneous light splitting capabilities</td>
</tr>
<tr>
<td>DMR-DC</td>
<td>Dynamic multicast routing problem with delay constraints</td>
</tr>
<tr>
<td>MST</td>
<td>Minimum spanning tree</td>
</tr>
<tr>
<td>MSTP</td>
<td>Minimum Steiner tree problem</td>
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<tr>
<td>MCLP</td>
<td>Minimum cost light-path</td>
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<tr>
<td>MDLP</td>
<td>Minimum delay light-path</td>
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<tr>
<td>MCRFPR</td>
<td>Most cost-reduction first progressive replacing heuristic</td>
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<tr>
<td>CR</td>
<td>Cost-reduction</td>
</tr>
<tr>
<td>ARP</td>
<td>All nodes reroute to parent heuristic</td>
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<tr>
<td>DRNN</td>
<td>Destinations reroute to nearest nodes heuristic</td>
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<tr>
<td>ARNN</td>
<td>All nodes reroute to nearest nodes heuristic</td>
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<tr>
<td>MDHN</td>
<td>Minimum distance heuristic network heuristic</td>
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</table>

References