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Anti-control of chaos of single time-scale brushless DC motor

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Anti-control of chaos of single time-scale brushless DC motors is studied in this paper. In order to analyse a variety of periodic and chaotic phenomena, we employ several numerical techniques such as phase portraits, bifurcation diagrams and Lyapunov exponents. Anti-control of chaos can be achieved by adding an external constant term or an external periodic term.

Keywords: brushless DC motor; chaos; anti-control

1. Introduction

Chaos is undesirable in most engineering applications. Many researchers have devoted themselves to find new ways to suppress and control chaos more efficiently. However, under certain circumstances, chaos is desirable. Chaotic phenomena are very useful in many applications such as fluid mixing (Ottino 1992), human brains (Schiff et al. 1994), heart beat regulation (Brandt & Chen 1997), etc. Therefore, making a periodic dynamical system chaotic or preserving chaos of a chaotic dynamical system is meaningful and worth investigating.

The theme of this paper is the brushless DC motor (BLDCM). The major advantage of the BLDCM is the elimination of the physical contact between the brushes and the commutators. The brushless DC motor has been widely applied in direct-drive applications, such as robotics (Asada & Youcef-Toumi 1987), aerospace (Murugesan 1981), etc. In order to analyse a variety of periodic and chaotic phenomena, we employ several numerical techniques, such as phase portraits, bifurcation diagrams and Lyapunov exponents. Herein, chaos anti-control of the BLDCM is investigated by adding controlling terms.

This paper is organized as follows. In §2, the electromechanical system and the reference numerical results of periodic and chaotic phenomena are presented. In §3, the effects of four different implementations of a controlling input aimed at achieving anti-control of chaos are investigated, i.e. the addition of a constant term or a periodic term.

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One contribution of 15 to a Theme Issue ‘Exploiting chaotic properties of dynamical systems for their control’.
2. Regular and chaotic dynamics of a brushless DC motor

In this section, the dynamic characteristics of a BLDCM (Krause 1986; Hemati & Leu 1992; Hemati 1994; Ge & Chang 2004) are investigated. First, the dynamic system model is given. Second, the state equations are transformed to a compact form. Finally, we present the reference numerical analysis of periodic and chaotic behaviour of a BLDCM.

A brushless DC motor is an electromechanical system. The physical model of the BLDCM is shown in figure 1 (Krause 1986), where $Q_{1,3}$ is the light transistor; $Q_{4,9}$ the transistor; $D$ the light diode; $L_{1,3}$ the stator winding; and $H_{1,3}$ the light sensor.

The equation of electrical dynamics can be described by Hemati (1994) and Ge & Chang (2004)

$$\frac{d}{dt} I(t) = L^{-1}(\theta) \left[ V(t) - RI(t) - \left( \frac{dL(\theta)}{d\theta} I(t) + \frac{dA_M(\theta)}{d\theta} \right) \frac{d\theta}{dt} \right], \quad (2.1)$$

where $I(t)$ is the phase current vector; $L(\theta)$ the inductance matrix; $V(t)$ the vector corresponding to the voltages across the phase windings; $R$ the winding resistance matrix; $A_M(\theta)$ the flux linkage vector due to the presence of permanent magnets; and $\theta$ the displacement variable. The equation of mechanical dynamics can be described by

$$\frac{d}{dt} \omega = \frac{1}{J} [T(I, \theta) - T_l(t)], \quad (2.2)$$
where \( \omega \) is the rotator angular velocity; \( J \) the inertia of rotator; \( T(I, \theta) \) the electromagnetic torque; and \( T_l(t) \) the external torques imposed on the rotator shaft.

Accounting for viscous damping friction, the external torques can be described by
\[
T_l(t) = b\omega + T_L,
\]
where \( b \) is viscous damping coefficient; and \( T_L \) the torque due to external load, cogging effect, Coulomb friction, etc.

So far, equations (2.1) and (2.2) explicitly depend on \( \theta \). This is not expected, since the solutions are hard to obtain. Therefore, we transform the above equations to the rotating frame via Park’s transformation and the explicit dependence on \( \theta \) can be eliminated. We can obtain
\[
\frac{d}{dt} i_q = \frac{1}{L_q} [-R i_q - n \omega (L_d i_d + k_i) + v_q], \tag{2.4}
\]
\[
\frac{d}{dt} i_d = \frac{1}{L_d} [-R i_d + n L_q \omega i_q + v_d], \tag{2.5}
\]
and the electromagnetic torque is described by
\[
T(i_q, i_d) = n [k_i i_q + (L_d - L_q) i_q i_d], \tag{2.6}
\]
where \( i_q, i_d \) is the quadrature axis and direct axis current; \( v_q, v_d \) the quadrature axis and direct axis voltage; \( L_q, L_d \) the fictitious inductance on the quadrature axis and direct axis; \( R \) the winding resistance; \( n \) the number of permanent pole pairs; and \( k_i = \sqrt{3/2} k_e \), where \( k_e \) is the permanent magnet flux constant.

We next transform the system equations to a compact form through an affine and a single time-scale transformation (Hemati 1993),
\[
x = \Phi \dot{x} + \zeta, \tag{2.7}
\]
where \( x \) is the \( m \)-dimensional state vector; \( \Phi \) the \( m \times m \) non-singular matrix constant; and \( \zeta, m \times 1 \), the constant vector.

The transformation matrix need not be a specified form; for our purposes and simplicity, we choose
\[
\Phi = \begin{bmatrix}
\sigma_1 & 0 & 0 \\
0 & \sigma_2 & 0 \\
0 & 0 & \sigma_3 
\end{bmatrix}, \quad \zeta = \begin{bmatrix}
\zeta_1 \\
\zeta_2 \\
\zeta_3 
\end{bmatrix}. \tag{2.8}
\]

Combining equations (2.7), (2.8) and (2.2)–(2.6), we obtain the equations in compact forms with a greatly reduced number of parameters,
\[
\begin{aligned}
\frac{d}{dt} \hat{x}_1 &= \hat{v}_q - \hat{x}_1 - \hat{x}_2 \hat{x}_3 + \rho \hat{x}_3 \\
\frac{d}{dt} \hat{x}_2 &= \hat{v}_d - \delta \hat{x}_2 + \hat{x}_1 \hat{x}_3 \\
\frac{d}{dt} \hat{x}_3 &= \sigma (\hat{x}_1 - \hat{x}_3) + \eta \hat{x}_1 \hat{x}_2 - \hat{T}_L
\end{aligned}, \tag{2.9}
\]
where
\[
\hat{v}_q = \frac{\tau}{\sigma_1 L_q} v_q, \quad \hat{v}_d = \frac{\tau}{\sigma_2 L_d} (v_d - R \zeta_2), \quad \hat{T}_L = \frac{\tau}{J \sigma_3} T_L.
\]
Here, we have to assert that equation (2.9) is non-dimensionalized. In the sections below, a variety of different control inputs added to equation (2.9) are also non-dimensionalized. However, if we transform them to the original forms, each control input is dimensional and has its practically physical meaning. The external terms in the first and second equations are external voltages. The external terms in the third equation are external mechanical torques.

In addition, the BLDCM is an autonomous system. It means that the period of the system is not explicitly known, so a different choice of Poincaré section would lead to a different bifurcation diagram. In the sections below, adding control inputs changes the dynamics of the system; thus, we have to modify the choice of Poincaré section. We obtain almost the same bifurcation diagram by modifying Poincaré section. The only difference is the shift in $\hat{x}_3$-axis. Therefore, we present only the original bifurcation diagram.

Finally, we present the numerical results. The selection of parameter values aims at the appearance of chaotic motion. The parameters in numerical simulation are selected as $\hat{v}_q = 0.168$, $\rho = 60$, $\hat{v}_d = 20.66$, $\delta = 0.875$, $\eta = 0.26$, $\hat{T}_L = 0.53$, and the initial condition is $\hat{x}_1(0) = \hat{x}_2(0) = \hat{x}_3(0) = 0.01$. The phase portrait, bifurcation diagram and Lyapunov exponents are shown in figure 2, by considering $\sigma$ as the reference (main) bifurcation parameter. It can be observed that the motion is period 1 for $\sigma = 4.05$, period 2 for $\sigma = 4.15$ and period 4 for $\sigma = 4.21$. For $\sigma = 4.55$, the motion is chaotic.

3. Anti-control of chaos

In order to preserve or induce chaotic phenomena of a BLDCM, two classes of control inputs are considered; the addition of either dimensionless constant terms or various dimensionless periodic terms to equation (2.9).

The first case actually represents the easiest way to possibly achieve anti-control. Indeed, adding constant terms to equation (2.9) is equivalent to simply changing the values of some characteristic quantities of the system ($\hat{v}_q$, $\hat{v}_d$ and $\hat{T}_L$, respectively) in such a way to shift its electromechanical properties towards a region of parameters space, where the desired kind of response does occur. Apart from being aimed at achieving anti-control, this is substantially the ‘naïf’ approach for the control proposed, e.g. in Blazejczyk et al. (1993) with reference to other systems, and named ‘control by system design’. Addition of a constant term is also used in Rajasekar et al. (1997) in the framework of electronic circuits.

In order to easily detect suitable values of the constant controlling terms, the analysis is made starting from a reference value of the main bifurcation parameter ($\sigma = 4.55$), for which chaos already occurs in the absence of control input. A value of the latter corresponding to well-established chaos is then used to check whether the response is robustly chaotic all over the considered range of the main bifurcation parameter $\sigma$.

In contrast to the previous case, the addition of variable periodic terms to the system equations somehow represents a more mature control approach herein aimed at achieving robust anti-control. In this case, preliminary bifurcation diagrams are obtained by varying one of the two independent control parameters governing the external inputs, with the aim of detecting a suitable heuristic combination of the relevant values capable to guarantee persistence of chaos all over the $\sigma$ range.
Three different non-harmonic periodic excitations are considered, which differ from each other by their shape. The matter of choosing the optimal shape of the excitation was investigated in depth by Lenci & Rega (2004).

\( (a) \) Anti-control of chaos by addition of a constant term

First, we add an external constant input \( \hat{v}_1 \) to the first equation of (2.9). The selection of the value of the anti-controlling parameter \( \hat{v}_1 \) depends upon the bifurcation diagram, figure 3a, in which \( \hat{v}_1 = 10 \) is selected, because of the appearance of chaos at that value. Indeed, from figure 3a, the system is chaotic when \( 9.2 < \hat{v}_1 < 11 \), and the chaotic phenomenon is robust when \( \hat{v}_1 = 10 \). Thus, it is chosen as our control input and the corresponding bifurcation diagram and Lyapunov exponents are shown in figure 3b,c. Comparing figures 3b and 2b, or figures 3c and 2c, it is possible to see how the range of chaotic phenomena is considerably increased.

Second, we add an external constant input \( \hat{v}_2 \) to the second equation of equation (2.9). The process of choice and the numerical results are shown in figure 4. It is very clear that the chaotic phenomenon is increased for \( \hat{v}_2 = 10.0 \).
Third, we add an external constant input $T_3$ to the third equation of equation (2.9). The process of choice and the numerical results are shown in figure 5. It is very clear that the chaotic phenomenon is increased for $T_3 > 7.5$.

From the above numerical results, all three kinds of constant inputs are successful for anti-control of a chaotic BLDCM. The first and second cases offer larger parameter ranges for chaos than the third case.

(b) Anti-control of chaos by addition of a periodic term

(i) Adding a periodic term of a sawtooth wave

First, we add an external sawtooth wave input $\hat{v}_1(t)$ to the first equation of equation (2.9), which is described by

$$\hat{v}_1(t) = \frac{a}{b} t - a + \sum_{k=1}^{\infty} [(-2a) \cdot u(t - \tau)],$$

(3.1)

where $a$ is the amplitude of the sawtooth wave; $\tau = 2kb$, where $2b$ is the period of the sawtooth wave; and $u(t)$ the unit step function.
Since there are two control parameters, $a$ and $b$, the choice of control input is more complicated. The process of choice and the numerical results are shown in figure 6. It is very clear that the chaotic phenomenon is increased for $a=5.0$ and $b=5.0$.

Second, we add an external sawtooth wave input $\dot{v_2}(t)$ to the second equation of equation (2.9). The process of choice and the numerical results are shown in figure 7. It is very clear that the chaotic phenomenon is increased for $a=4.0$ and $b=5.0$.

Third, we add an external sawtooth wave input $\dot{T}_3(t)$ to the third equation of equation (2.9). The process of choice and the numerical results are shown in figure 8. It is very clear that the chaotic phenomenon is increased for $a=6.0$ and $b=5.0$.

From the above numerical results, all three kinds of external tooth wave inputs are seen to be successful for anti-control of a chaotic BLDCM. The third is better than the second, and the second is better than the first. Altogether, this case is less effective than the case of adding a square wave in §3b(ii), but better than the case of a adding constant term in §3a.

Figure 4. (a) Bifurcation diagram of $\dot{x}_3$ for control parameter, $\dot{v}_2 = 0.1-11.0$. (b) Bifurcation diagram of $\dot{x}_3$ for $\dot{v}_2 = 10.0$. (c) Lyapunov exponents of for $\dot{v}_2 = 10.0$.

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Adding a periodic term of a square wave

First, we add an external square wave input $v_1(t)$ to the first equation of equation (2.9), which is described by

$$v_1(t) = a \cdot \sum_{k=1}^{\infty} [(-1)^k \cdot 2a \cdot u(t-k \cdot \tau)],$$

where again $a$ is the amplitude of the square wave; $\tau = kb$, where $2b$ is the period of the square wave; and $u(t)$ the unit step function.

Let $b=1.0$, an external square wave input is added and the corresponding bifurcation diagrams for $b=1.0$, $a=0.1–11.0$ and $a=10.0$, $b=0.1–11.0$ are rich for chaos as in §3b(i). For $a=10.0$ and $b=5.0$, the chaotic phenomenon is also rich. We add an external square wave input for $a=10.0$ and $b=5.0$, and obtain the
bifurcation diagram versus $\sigma$ shown in figure 9a, to be compared with that of figure 2b. It is very clear that the chaotic phenomenon is increased. In figure 9b, the corresponding Lyapunov exponent diagram is given.

Second, we add an external square wave input $\hat{v}_2(t)$ to the second equation of equation (2.9). The bifurcation diagrams for $b=1.0$, $a=0.1$–11.0 and $a=8.0$, $b=0.1$–11.0 are rich in chaos as in §3b(i). It is very clear that the chaotic phenomenon is increased for $a=8.0$ and $b=5.0$ in figure 10a. In figure 10b, the corresponding Lyapunov exponent diagram is given.

Third, we add an external square wave input $\hat{T}_3(t)$ to the third equation of equation (2.9). The bifurcation diagrams for $b=1.0$, $a=0.1$–11.0 and $a=10.0$, $b=0.1$–11.0 are rich in chaos as in §3b(i). It is very clear that the chaotic phenomenon is increased for $a=10.0$ and $b=5.0$ in figure 11a. In figure 11b, the corresponding Lyapunov exponent diagram is given.

From the above numerical results, all three kinds of external square wave inputs are seen to be successful for anti-control of a chaotic BLDCM. The anti-control effect of square wave inputs is better than the constant input.
(iii) Adding a periodic term of a triangular wave

First, we add an external triangular wave input $v_1(t)$ to the first equation of equation (2.9), which is described by

$$
\hat{v}_1(t) = -\frac{2a}{b} t + a + \sum_{k=1}^{\infty} \left[ 4a \left( \frac{k-1}{b} \right) u(t-\tau) \right],
$$

where again $a$ is the amplitude of the triangular wave; $\tau = kb$, where $2b$ is the period of the triangular wave; and $u(t)$ the unit step function.

The bifurcation diagrams for $b=1.0$, $a=0.1–11.0$ and $b=5.0$, $a=0.1–11.0$ are rich in chaos as in §3b(i). It is very clear that the chaotic phenomenon is increased for $a=5.0$ and $b=5.0$ in figure 12a. In figure 12b, the corresponding Lyapunov exponent diagram is given.

Second, we add an external triangular wave input $\hat{v}_2(t)$ to the second equation of equation (2.9). The bifurcation diagrams for $b=1.0$, $a=0.1–11.0$ and $a=4.0$, $b=0.1–11.0$ are rich in chaos as in §3b(i). It is very clear that the chaotic phenomenon is increased for $a=4.0$ and $b=5.0$ in figure 13a. The corresponding Lyapunov exponent diagram is given in figure 13b.

Figure 7. Bifurcation diagram of $\hat{x}_3$ for (a) $b=1.0$, $a=0.1–11.0$, (b) $a=4.0$, $b=0.1–11.0$ and (c) $a=4.0$, $b=5.0$.
Figure 8. Bifurcation diagram of $\dot{x}_3$ for (a) $b = 1.0$, $a = 0.1 - 11.0$, (b) $a = 6.0$, $b = 0.1 - 11.0$ and (c) $a = 6.0$, $b = 5.0$.

Figure 9. (a) Bifurcation diagram of $\dot{x}_3$ for $a = 10.0$, $b = 5.0$. (b) The corresponding Lyapunov exponent diagram.
Figure 10. (a) Bifurcation diagram of $\dot{x}_3$ for $a=8.0$, $b=5.0$. (b) The corresponding Lyapunov exponent diagram.

Figure 11. (a) Bifurcation diagram of $\dot{x}_3$ for $a=10.0$, $b=5.0$. (b) The corresponding Lyapunov exponent diagram.

Figure 12. (a) Bifurcation diagram of $\dot{x}_3$ for $a=5.0$, $b=5.0$. (b) The corresponding Lyapunov exponent diagram.
Third, we add an external triangular wave input $\hat{T}_3(t)$ to the third equation of equation (2.9). The bifurcation diagrams for $b = 1.0$, $a = 0.1–11.0$ and $a = 5.0$, $b = 0.1–11.0$ are rich in chaos as in §3b(i). It is very clear that the chaotic phenomenon is increased for $a = 5.0$ and $b = 5.0$ in figure 14a. The corresponding Lyapunov exponent diagram is given in figure 14b.

From the above numerical results, all three kinds of external triangular wave inputs are seen to be successful for anti-control of chaotic BLDCM. The third is better than the second, and the second is better than the first. As a whole, the anti-control effect of this case is similar to the case of adding a sawtooth wave in §3b(i).

4. Conclusions

A brushless DC motor is studied in this paper. It is an autonomous third-order electromechanical system. In order to analyse the periodic and chaotic phenomena, we employ several numerical techniques such as phase portrait, bifurcation diagram and Lyapunov exponents.
The dynamic characteristics of a BLDCM are discussed in §2. The system model is described, and some numerical results highlighting the occurrence of periodic and chaotic phenomena are presented.

In §3, four implementations of inputs are added to achieve anti-control of a chaotic BLDCM. First, a constant term is added. Each external constant input is successful, and the equation to which the constant term should be added depends on the choice of parameter, $\sigma$, in the equation. Second, a periodic term is added. Each external square wave input and sawtooth wave input are successful. For the latter, the third is better than the second, and the second is better than the first. Each external triangular wave input is also successful. The third is better than the second, and the second is better than the first.

By adding constant or periodic terms, the effects of chaos anti-control are rather satisfactory. Other types of periodic terms, or even non-periodic terms may be suggested in the future, since this method is very simple.

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