is the propagation constant, and $\eta$ is a random variable corresponding to shadowing which is lognormally distributed with a mean of 0 dB and a standard deviation of $\sigma$, dB. For a large number of users, the PDF of MAI is usually assumed to approximate a Gaussian process with a variance equal to the sum of variances of individual interfering users. In a reverse link of a DS/CDMA system, other users transmit with orthogonal PN codes and their signals fade independently. The desired user (reference user) is assumed to be user 1. The effect of AWGN, MAI, fading and shadowing is incorporated into acquisition analysis simply by increasing the variance of interference [4]. For an asynchronous DS/CDMA system, the variances of detecting and reference MFs are given by

$$\sigma_D^2 = \frac{N_0 G T_c}{4} + \frac{G T_c^2}{6} \sum_{k=2}^{K} P_k$$

$$+ P_1 T_c \int_{0}^{D} \Phi(\tau) |W R_k^*(\tau-\xi) + MG^2_k(\tau-\xi)| d\tau + \sigma_s^2$$

(5)

$$\sigma_H^2 = \frac{N_0 G T_c}{4} + \frac{G T_c^2}{6} \sum_{k=2}^{K} P_k + P_1 MT_c T \int_{0}^{D} \Phi(\tau) d\tau + \sigma_s^2$$

(6)

where $G = TT_c$ is the processing gain, $T_c$ is the chip duration, $P_1$ is the signal power of desired user, $D$ is the multipath delay spread, $M$ is MF length, $\Phi(\tau)$ is the average channel output power, $\tau-\xi$ is delay difference between the received and local PN codes in detecting MF, $W$ is covariance between individual samples of transmitted signal within MF, $R_k(\tau)$ is the autocorrelation function of the transmitted PN code, and $G_k(\tau)$ is a function representing self-noise. It is assumed that $R_k(\tau)$ is zero and $G_k(\tau)$ is unity owing to orthogonality of the reference code and the transmitted code. For a perfect power control, $P_1 = P(k = 2, \ldots, K)$. The PDF of the squeezed sum of the MF correlator outputs is distributed with two degrees of freedom. The acquisition scheme has two modes of operation: a search mode and a verification mode. The detection and false alarm probabilities of search mode are given by

$$P_{D1} = \int_{\beta}^{\infty} \frac{1}{2 \sigma_D^2} \exp\left(-\frac{x}{2 \sigma_D^2}\right) dx = \exp\left(-\frac{3 \sigma_D^2}{2 \sigma_D^2}\right)$$

(7)

$$P_{F1} = \int_{0}^{\beta} \frac{1}{2 \sigma_D^2} \exp\left(-\frac{x}{2 \sigma_D^2}\right) = \exp\left(-\frac{1}{2}\right)$$

(8)

where $t = 2 \sigma_D^2$ is estimated in reference MF, $\beta$ is a gain factor of reference branch, and $\sigma_s^2 = \sigma_f^2$ is variance of $H(t)$ cell in detecting MF. The acquisition process is modelled as a discrete-time Markov process. The mean acquisition time is derived by the same procedure used by Ibrahim and Aghvami [2] in the circular state diagram.

**Results and discussion:** Assuming that both reverse and forward links suffer from identical shadowing, a mobile user estimates signal strength by measuring the pilot signal, and controls its transmission power. For numerical examples, the phase adjustment parameter $\Delta = 0.5$, the penalty factor caused by a false alarm $10^{-6}$ (chips), PN code length $2^{15}$ (chips), chip rate $2$ Mchip/s, $\beta = 10^{-1}$, delay spread $D = 37$ (frequency-selective fading), processing gain $G = 128$, MF length $M = 512$, $p = 4$, and the exponential multipath intensity profile [2] for $F(\tau)$ was assumed. The thresholds of search and verification modes were chosen to minimise mean acquisition time in each situation. The number of verification mode tests and the number of successful verification tests were chosen as four and two, respectively. The gain factor $\beta$ was chosen to ensure minimum false alarm probability and maximum detection probability in each SNR-chip. In Fig. 1, mean acquisition time against number of users is shown for an MF-RF acquisition scheme with power control error as a parameter. The numerical example is shown for an SNR-chip $-10$ dB and standard deviation of shadowing $\sigma_s = 8$ dB (a typical value for an urban environment). The analytic result is confirmed by Monte-Carlo simulation. For a perfect power control, the logarithmic standard deviation $\sigma_f = 0$ dB. Imperfect power control increases mean acquisition time substantially when the standard deviation of the received power is $>1$ dB. The other acquisition schemes (an MF scheme [3], a parallel MF scheme [4], a parallel MF-RF scheme [5], and a hybrid acquisition scheme) have also been analysed for imperfect power control by analytic approaches and simulations. As a result of the analysis, we reached the same conclusion as in the MF-RF case, that power control error $>1$ dB increases mean acquisition time very significantly. Therefore, the power control must be accurate within $1$ dB, and be fast enough to compensate for the Rayleigh fading effect as well as for the shadowing effect.

The considerations in this Letter can be applied to the reverse link design of a DS/CDMA system for packet-type services.

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References


Estimation of the Gilbert model parameters using the simulated annealing method

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Indexing terms: Simulated annealing, Error statistics

An estimation method based on the simulated annealing algorithm is proposed for computing the Gilbert model parameters from sample error sequences. Experimental results indicate that the simulated annealing method yields estimates which more closely match the experimental error statistics.
Introduction: Transmission bit errors encountered in most digital mobile radio channels are found to cluster in bursts. There have been numerous parameterised models proposed to characterise such channels with memory [1]. In this Letter, we have placed the emphasis on Gilbert’s two-state Markov chain model [2]. As shown in Fig. 1, there are no errors in state G (good state), while errors occur in state B (bad state) with the probability \( P \). To simulate the channels’ bursty nature, the state transition probabilities \( P \) and \( p \) are small, such that the probabilities \( Q \) and \( q \) of persisting in \( G \) and \( B \) states will be high. The principal difficulty encountered in estimating the Gilbert model parameters \( \{ P, p \} \) is that the parameters are not directly observable, so methods of deducing them from easily measured error statistics must be considered. This task can be carried out either by exponential curve fitting [2] or by using the gradient iterative method [3]. The former method is subjective and unreliable, but it can provide good initial values for further analysis. As regards the gradient iterative method, it only allows downhill moves along the cost function, and hence can easily trap the final solution into a local optimum.

Error sequence characterisation: The dynamics of sample error sequences are described by the error gap distribution \( P(0|1) \) which gives the probability that at least \( m \) successive error-free bits will be encountered next, given that an error bit has just occurred. In many applications [2], it suffices to postulate that the error gap distribution can be well approximated by the sum of two exponentials i.e.

\[
P(0|1) = \alpha_1 e^{-\alpha_2} + \alpha_2 e^{-\alpha_3}
\]

under the constraint that \( \alpha_1 + \alpha_2 = 1 \). Proceeding in this way, the original descriptive modelling problem can be formulated as one of optimal identification of two pairs of parameters \( \{ \alpha_1, \beta_1 \} \) and \( \{ \alpha_2, \beta_2 \} \). This can be accomplished by minimising the sum of quadratic errors between the measured error gap distribution and its modelled fit. Accordingly, we define a cost function for the overall error gap distributions as follows:

\[
E = \frac{1}{m} \sum_{n=1}^{M} \left[ y(n) - \log_{10} \left( \sum_{i=1}^{2} \alpha_i \beta_i^n \right) \right]^2
\]

where \( M \) is the longest interval between two consecutive errors and \( y(n) \) is the measured value, expressed logarithmically, of the error gap distribution. As we shall see later, the minimisation of cost can be achieved using the simulated annealing technique. Given that the optimal values of \( \{ \alpha_1, \beta_1, \beta_2 \} \) have been determined, the model parameters \( \{ P, p \} \) can be calculated as follows [2]:

\[
P = \frac{(1 - \beta_1)(1 - \beta_2)}{(1 - h)}
\]

\[
h = \beta_1 \beta_2 - \alpha_2 (\beta_1 - \beta_2)
\]

\[
p = \alpha_1 (\beta_1 - \beta_2) + (1 - \beta_1)(\beta_2 - h)/(1 - h)
\]

Proposed estimation scheme: The estimation of \( \{ \alpha_1, \beta_1 \} \) can be viewed as being constructed in two steps: first, to save computation time, an exponential curve fitting technique was performed to locate the region \( U \) that is likely to contain the optimal solution. Then, the simulated annealing technique is applied to determine the global optimal solution which minimises the cost function. Basically, the simulated annealing method consists of two nested loops: the inner one proceeds until the equilibrium condition is satisfied, while the outer loop is terminated at a very low temperature. A detailed description is outlined below.

(i) Set an initial temperature \( T(0) = 10\sigma \), where the standard deviation \( \sigma \) of the costs is empirically determined to be 0.5. Also, an initial state \( u_0 = (\alpha_1, \beta_1, \beta_2) \) is chosen from \( U \) with a cost function \( Q(u) \).

(ii) A new state which belongs to \( U \) is generated using a random perturbation mechanism. Let \( A^C \) denote the cost difference between new and old states and \( rand(0,1) \) denotes a random number uniformly distributed between 0 and 1. The new state is accepted if either \( A^C < 0 \) or \( e^{-A^C/T} > rand(0,1) \) is satisfied, otherwise the new state is rejected.

(iii) In the inner loop, for each temperature \( T \), if the following equilibrium condition has been reached at the \( n \)-th iteration:

\[
\sum_{i=1}^{n} \exp[-L(n_i - C(u_i))/T] < \zeta
\]

then the algorithm stops and goes to step (iv), else repeats steps (ii)-(iii). The value of \( \zeta \) is empirically chosen to be 1.0 and \( n_i = \Sigma_{i=1}^{n} C(u_i) \), \( n_i \) is the number of accepted states.

(iv) The cooling schedule is performed according to

\[
T(k) = T(0)/(k+1), \quad k \text{ is the index of the outer loop.}
\]

(v) If \( T \) is sufficiently small (in the order of 10-4), then the program ends the outer loops and finds the optimal solution, else returns to step (ii).

Experimental results: The effectiveness of the proposed scheme for channel modelling was substantiated experimentally using two data files, referred to as data1 and data2, with bit error rates of 1.3 and 0.5%. They are generated, respectively, from Gilbert’s model using different pairs of model parameters \( \{ P, h, p \} = \{ 0.003, 0.84, 0.034 \} \) and \( \{ P, h, p \} = \{ 0.002, 0.945, 0.02 \} \), each with a length of 100,000 bits. Table 1 presents the comparative scores associated with the simulated annealing scheme and the gradient iterative method. Study of these scores suggests that the simulated annealing technique is preferable to the gradient method for use in estimating the Gilbert model parameters. As shown in Fig. 2, for the case of data 1, the resultant modelled fit provides an approximation of the experimental error gap distribution to a reasonable

---

**Table 1:** Comparative results of gradient method and simulated annealing method

<table>
<thead>
<tr>
<th>Method</th>
<th>Gradient</th>
<th>Annealing</th>
<th>Gradient</th>
<th>Annealing</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P )</td>
<td>0.00461</td>
<td>0.00378</td>
<td>0.00104</td>
<td>0.00093</td>
</tr>
<tr>
<td>( h )</td>
<td>0.88851</td>
<td>0.84500</td>
<td>0.88899</td>
<td>0.90879</td>
</tr>
<tr>
<td>( p )</td>
<td>0.03667</td>
<td>0.03895</td>
<td>0.02320</td>
<td>0.01570</td>
</tr>
<tr>
<td>Initial cost</td>
<td>1.90131</td>
<td>0.73350</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Final cost</td>
<td>0.04320</td>
<td>0.02950</td>
<td>0.0174</td>
<td>0.00599</td>
</tr>
</tbody>
</table>

---

**Fig. 2** Experimental error gap distribution and its Gilbert’s Markov modelled fit

---
degree of accuracy. To elaborate further, the learning curves of different estimation algorithms are illustrated in Fig. 3. The results indicated that the simulated annealing method cannot only yield better estimates, but can also converge to the final solution more quickly.

![Cost function](image)

**Fig. 3** Learning curves of gradient iterative method and simulated annealing method

- - - - simulated annealing method
- - - - simulated gradient iterative method

Conclusions: This Letter explores the benefits of a simulated annealing algorithm for use in estimating the Gilbert model parameters. Experimental results indicate that the proposed method can obtain better estimates than does the gradient iterative method.

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References


Exact calculation of buffer contents variance and delay jitter in a discrete-time queue with correlated input traffic

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Indexing terms: Discrete time systems. Queueing theory. Asynchronous traffic mode.

A discrete-time queueing system with infinite buffer size and a finite number of independent heterogeneous two state on/off sources is considered. Exact closed-form expressions are derived for the variances of the buffer contents and the delay. In passing, the correctness of a conjecture recently made by Pieloor and Lewis [3] is proved.

Introduction: Discrete-time buffer systems have been studied quite extensively in the past couple of years, since they can be used to model statistical multiplexers and switching elements in ATM-based communication networks. Moments of the buffer contents and the cell delay are useful performance measures for these systems. Most of the previous analytical work in this respect concentrates on the calculation of closed-form expressions for the mean values of these quantities, see e.g. [1, 2]. In comparison, little research effort has been spent on the derivation of similar expressions for the corresponding variances. Recently, Pieloor and Lewis [3] proposed an expression for the variance of the queue population in a discrete-time queue fed by a number of independent heterogeneous on/off sources, with geometrically distributed on/off periods. Using numerical studies they showed their formula to be accurate, and hence they claimed it to be exact. However, no rigorous proof of its correctness could be provided. We present a method for the exact calculation of not only the buffer contents variance, but also the cell-delay variation in the above queueing system. Besides allowing us to prove the conjecture by Pieloor and Lewis [3], we expect that the techniques presented in this Letter can also be adapted to derive these variances in the case of more general correlated arrival processes.

Queueing model description: We consider a discrete-time single server queueing system with infinite storage capacity. Time is divided into fixed length slots, such that one cell can be transmitted from the buffer during each slot. The buffer is fed by N independent heterogeneous traffic sources. Each source stochastically alternates between on periods, during which it generates one cell per slot, and off periods, during which it generates no cells. In this Letter, (the lengths of) the on periods and the off periods of source n (1 ≤ n ≤ N) are modelled as geometric random variables with parameters αn and βn, respectively. The average load ρn of source n, i.e. the steady-state probability that source n generates a cell during an arbitrary slot is then given by ρn = (1 − βn)/(1 − αn − βn). Also, as in [4], we define the 'time scale parameter' Kn of source n as the ratio of the mean length of an on period (or an off period) in our model, to the corresponding quantity in the case of a Bernoulli arrival process, i.e. Kn ≡ (1 − αn)/(1 − αn − βn) = ρn/(1 − βn). High values of Kn indicate a high degree of correlation in the cell arrival process.

Buffer contents variance: Let e, denote the number of cells generated by source n during an arbitrary slot in the steady-state, and let s denote the 'system contents', i.e. the number of cells present in the buffer including the possible cell under transmission, at the start of the next slot. We define the joint probability generating function (PGF) of the vector (e1, ..., en, s) as

\[
P(x_1, ..., x_N, z) = E \left[ \prod_{n=1}^{N} x_n^{e_n} z^{-s_n} \right]
\]

where E[.] denotes the expected value of the argument. In a similar way as described in [2] for the homogeneous traffic case, the following functional equation can then be established for P(x1, ..., xN, z):

\[
z P(x_1, ..., x_N, z) = \left( \prod_{n=1}^{N} [\beta_n + (1 - \beta_n)x_n] \right)
\times \left\{ \frac{1 - \alpha_n - \alpha_1 x_1 z}{\beta_n + (1 - \beta_n)x_n z} \right\}^{N-1}
\times \left[ \frac{1 - \alpha_N - \alpha_N x_N z}{\beta_N + (1 - \beta_N)x_N z} \right]^{N-1}
\times (z - 1)(1 - \rho) \]

where \( \rho \) is the total input load, i.e. \( \rho = \sum_{n=1}^{N} \alpha_n \).

Unfortunately, it is not a simple matter to derive \( P(x_1, ..., x_N, z) \) from this equation. In spite of this, explicit expressions can be obtained from eqn. 2 for all the moments of the system contents distribution in a rather straightforward manner. To do this, first of all the desired moment in terms of the consecutive partial derivatives of the function \( P(x_1, ..., x_N, z) \) with respect to \( z \), for \( x_1 = 1 \), \( 1 ≤ n ≤ N \), and \( z = 1 \) must be expressed. Secondly, these partial derivatives must be evaluated using eqn. 2. The mixed partial derivatives with respect to \( x_n \) and \( z \) of the \( P \)-function, that appear during this calculation, can be eliminated from the results by expressing them in terms of others again by using the functional eqn. 2.