Elastic-plastic fracture toughness of PC/ABS blend based on CTOD and J-integral methods

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(Received 1 December 1995)

The elastic-plastic fracture toughness of a PC/ABS blend has been investigated simultaneously by the conventional J-integral and the crack tip opening displacement (CTOD) methods for specimen thickness varying from 4 to 15 mm. The critical J (Jc) and the critical CTOD (δc) are based on crack initiation. Another new method to determine Jc based on the close relationship between the J-integral and CTOD has also been developed. Those three critical Jc values, Jc_sl, Jc_NEw, and Jc_BS obtained from the ASTM E813-81, the new J method, and the equation Jc = μmγk (1-ν²) of the CTOD method, are comparable and are independent of the specimen thickness. The constraint factor m evaluated from the linear elastic fracture mechanics (LEFM) approach using the Dugdale model is about 1.5. A rotation factor μ of approximately 2 was obtained which is also independent of the specimen thickness. The crack propagation resistance, J-curve (dJ/da) or δ-curve (dδ/da), decreases with increasing specimen thickness. A close relationship between CTOD and J-integral has been demonstrated in this study. Copyright © 1996 Elsevier Science Ltd.

(Keywords: fracture toughness; J-integral; crack tip opening displacement)

INTRODUCTION
The rapid development of the critical application of engineering plastics makes it desirable to have a practical and reproducible measurement of material fracture toughness that can be used in design. Linear elastic fracture mechanics (LEFM) has been successfully applied to those relatively brittle and rigid polymers such as polystyrene (PS) and poly(methyl methacrylate) (PMMA). For ductile polymers, such as polycarbonate/acylonitrile-butadiene-styrene (PC/ABS) polyblend, the problem due to the extensive plasticity at the crack tip precludes the application of LEFM. For LEFM, the thicker specimen required for the plane-strain condition is sometimes impractical experimentally. These shortcomings of the LEFM approach led to efforts to seek other techniques that are suitable for materials with extensive plastic yielding. Two major approaches have been developed: crack tip opening displacement (CTOD) and J-integral. The CTOD fracture parameter provides a relatively simple method by extending the fracture mechanics concepts from the plane-strain linear elastic fracture behaviour to the elastic-plastic fracture behaviour. The plane-strain linear elastic fracture toughness (Kc, as defined in ASTM E399) can be obtained only at relatively lower temperatures and large specimen size. The Jc (as defined in ASTM E813) toughness is applicable to these polymers with stable and ductile tearing behaviour. Thus, there is a transition region between the Kc and Jc toughness parameters. The CTOD fracture toughness parameter can cover this transition region as well as the regions where Kc and Jc are valid. The British Standard (BS) CTOD method for crack opening displacement (COD) testing (BS 5762) is the standardized method that covers all fracture behaviours between the extremes associated with the Kc and the Jc. It has been demonstrated that the onset of crack growth can be characterized by a critical value of the J-integral (Jc) or by a critical crack opening displacement (δc). The subsequent works by Clarke et al., Griffith and Yoder, and Andrews et al. supported the earlier observation on crack initiation. Later study by Shin strongly suggested that the crack growth can be characterized in terms of the J-resistance or δ-resistance when certain requirements are satisfied. There are many methods that have been used to describe the critical fracture behaviour. ASTM Standards, E813-81 and E813-87, use the multiple-specimen technique proposed by Landes and Begley. These two ASTM Standards were established originally for J-testing mainly for metallic materials but have been extended to characterize the toughened polymers and blends during last decades. However, the optimum procedures of the test have not yet been conclusively defined and standardized. Subsequently, several different approaches for J-integral have been developed by Seidler and Grellmann studied the fracture behaviour and morphologies of PC/ABS blends using a special technique, a stop block method. Mai and Cottere used the essential work method to characterize the fracture...
toughness for many tough polymers. Zhou et al.25 used the single-specimen normalization method to derive the $J-R$ curves and to characterize the toughness of polymeric materials for which the direct measurement of the crack growth length is not required. In our recent studies,35–36 an unconventional approach to the $J$-integral has been developed based on the crack tip hysteresis properties of polymeric materials. This newly developed hysteresis energy method does not have any drawback of the single-specimen method25 and is complementary with the existing ASTM E813 standards. The crack tip opening displacement (CTOD) method was also established to characterize the fracture behaviour mainly for metallic materials31–36. In this paper, we extend this CTOD method to characterize the fracture behaviour of the PC/ABS blend by varying specimen thickness from 4 to 15 mm. Both CTOD and conventional $J$-integral methods will be carried out simultaneously for comparative purpose.

**CTOD AND $J$-INTEGRAL FRACTURE PARAMETERS**

**Crack tip opening displacement (CTOD)**

The BS5762 COD test provides the method for analysing the load-clip gauge displacement to obtain the critical CTOD value. The CTOD can be calculated from the following equation,

$$\delta = \delta_e + \delta_p = \frac{K^2(1-\nu^2)}{2\sigma_E E} + \frac{V_p a_p (W-a)}{r_p W + 0.6a + z}$$

(1)

where $\nu$ is the Poisson’s ratio, $V_p$ is the plastic component of the mouth-opening displacement, $W$ is the ligament length, $a$ is the crack length, $z$ is the knife edge thickness, $r_p$ is the rotation factor, $\sigma_E$ is the yield stress, $K$ is the stress intensity factor, and $E$ is the Young’s modulus. Equation (1) separates the CTOD ($\delta$) into elastic and plastic components $\delta_e$ and $\delta_p$. The BS5762 COD test suggests a value of 0.4 for the rotation factor $r_p$ in equation (1). However, a more precise value for the rotation factor can be calculated if the plastic components of load-line displacement and mouth-opening displacement ($\delta_p$ and $\delta_m$, respectively) are known.

$$r_p = \frac{1}{W-a} \left\{ \frac{V_p}{q_p} \left[ 1 - \left( \frac{q_p}{16W} \right) \right] - (a+z) \right\}$$

(2)

This equation is based on an elastic and a plastic component of CTOD and implies the existence of a rotation point below the crack tip. The correlated $J$-integral value can be estimated from an equation derived by Sumpter and Turner by the following equation, where $U_p$ is the area under the load/mouth opening displacement curve. The plastic area $U_p$ can be estimated by the following equation,

$$U_p \cong P_L V_p$$

(4)

where $P_L$ is the limit load.

**The $J$-integral**

Rice3 developed the path-independent energy line integral, the $J$-integral, which is an energy-based parameter to characterize the stress–strain field near a crack tip surrounded by small-scale yielding. The $J$-integral is defined by the following equation,

$$J = \int \left( W dy - T \frac{\partial u}{\partial x} dx \right)$$

(5)

where $T$ is the surface traction, $W$ is the strain energy density, $u$ is the displacement vector, and $x, y$ are the axis-coordinates. Rice3 and Begley and Landes5,13 have shown that the $J$-integral can be interpreted as the potential energy change with crack growth which is expressed as follows,

$$J = \frac{dU}{d\alpha}$$

(6)

where $B$ is the thickness of the loaded body, and $a$ is the crack length. $U$ is the total potential energy which can be obtained by measuring the area under the load–displacement curve. Sumpter and Turner later expanded the $J$-integral equation as the following equation9,

$$J = J_e + J_p$$

(7)

$J_e$ and $J_p$ are the elastic and plastic components of the total $J$ value which can be represented by,

$$J_e = \frac{\eta_e U_e}{B(W-a)}$$

(8)

$$J_p = \frac{\eta_p U_p}{B(W-a)}$$

(9)

$U_e$ and $U_p$ are the elastic and plastic components of the total energy. Both $\eta_e$ and $\eta_p$ are their corresponding elastic and plastic work factors. $b$ is the ligament length and $W$ is the specimen width. For a three-point bend single-edge notched specimen with $a/W > 0.15$, $\eta_e$ is equal to 2. When the specimen has a span $S$ of $4W$ ($S = 4W$) and $0.4 < a/W < 0.6$, $\eta_e$ is equal to 2. Therefore, equation (7) can be reduced to,

$$J = \frac{2U}{B(W-a)}$$

(10)

ASTM E813 recommends that equation (10) can be used to calculate the $J$ value for a SENB specimen.

**The $J_c$ validity requirements**

For the fracture to be characterized as $J_c$, a specimen must meet certain size requirements in order to achieve a plane-strain stress state along the crack front. To achieve this stress state, all specimen dimensions must exceed some multiple of $J_c/\sigma_y$. According to ASTM E813 method, a valid $J_c$ value may be obtained, whenever,

$$B, (W-a), W > 25 \left( \frac{J_c}{\sigma_y} \right)$$

(11)

Paris et al.40 developed the tearing modulus concept to describe the stability of a ductile crack in terms of elastic–plastic fracture mechanics. This fracture instability occurs if the elastic shortening of the system exceeds the corresponding plastic lengthening for crack extension. A non-dimensional parameter, tearing modulus
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For the $J-\Delta a$ data to be regarded as a material property independent of specimen size, the criterion $\omega > 10$ must be met, where $\omega$ is defined as,

$$\omega = \frac{(W-a)}{Jc} \left( \frac{dJ}{da} \right)$$

THE CORRELATION OF VARIOUS FRACTURE PARAMETERS

The $J$ versus $K$

The equation used to estimate $K$ from $J$ are,

$$K^2 = JE$$

for plane stress

$$K^2 = \frac{JE}{(1-\nu^2)}$$

for plane strain

These correlations are only strictly valid for linear elastic conditions where $J$ is equivalent to the energy release rate $G$. In this region, with the proper restrictions on specimen size, it is proper to use the following expression,

$$K^\prime = \frac{JcE}{(1-\nu^2)}$$

for plane strain

The $J$ values used herein are not the results of the conventional $J_c$ testing because the $J$ values are based on the maximum load regardless of prior plasticity or crack extension, and the correlations with $K$ are strictly valid only for linear elasticity.

The $CTOD$ versus $K$

The equation used to estimate $K$ from CTOD is,

$$K^2 = mE\sigma_c\delta$$

The parameter $m$ is a constraint factor that varies from 1 to 2 based on the thickness constraint.

The relationships between $J$ and CTOD

From equations (15) and (17), the CTOD can be correlated with $J$ under small-scale condition yielding the following equation,

$$J = m\sigma_c\delta(1-\nu^2)$$

where $m$ is a dimensionless constant that releases $J$ to CTOD and yield stress; $m = 1$ for plane stress and $m = 2$ for plane strain. The value of $m$ for large-scale yielding should be between 1 and 2. The plastic term in equation (3) is similar to the equation for the plastic CTOD,

$$\delta_p = \frac{[V_{p_1}(W-a)]}{[r_pB_p(W-a)^2]}$$

From equations (3) and (19), one can obtain a simple equation for the ratio of the plastic $J$ to the plastic CTOD,

$$\frac{J_p}{\delta_p} = \frac{2U_p}{[V_pB(W-a)^2]}$$

Equation (20) has been incorporated into a computer program by plotting $J_p/\delta_p$ as a function of mouth-opening displacement.

EXPERIMENTAL

Material and test specimens

The PC/ABS blend (Shinblend A783) was obtained from Shing-Kong Synthetic Fibercorp. of Taiwan. The tensile yield strength and Young’s modulus were measured by using the standard injection moulded specimens (1/8 inch) with an extensometer. The Poisson’s ratio of the PC/ABS is assumed to be 0.35. Test specimens are the three point bending bars with dimensions of width ($W$) 20 mm, length ($L$) 90 mm, and thickness ($B$), varying from 4 to 15 mm. The specimens with a single-edged notch of initial crack length, $a$, of 10 mm ($a/W = 0.5$) were prepared by injection moulding using an Arburg injection moulding machine (Figure 1). The initial precrack was then followed by sharpening with a fresh razor blade. All the notched specimens were annealed at 60°C for 2–3 h to release possible residual stress prior to the standard bending tests.

Fracture mechanics tests

The CTOD and $J$ methods were simultaneously carried out according to the BS5762 COD test and the ASTM E813 methods as shown in Figure 1. The CTOD and $J$ tests were performed in displacement control on a 5 kN load cell universal tensile test machine (Instron model 4201). The displacement rate in all tests was 2 mm min$^{-1}$. The load $P$, the mouth-opening displacement $\delta$, and the load-line displacement $q$ were obtained simultaneously during the test by a computer. A multiple-specimen technique was employed and the specimens were loaded to various displacements to allow different amounts of stable crack growth. The deformed specimens were then frozen in liquid nitrogen and broken open by a TMI impactor. The crack growth length of broken specimen, $\Delta a$, was measured by using a travelling optical microscope. The input energy of each test specimen was obtained by measuring the area under the load–displacement curve.

RESULTS AND DISCUSSION

Critical CTOD $\delta_c$ obtained from the BS5762 COD method

A typical plot of load vs clip-gauge displacement with
corresponding crack growth length $\Delta a$ is shown in Figure 2. The plastic components of clip-gauge displacements $V_p$ are then determined from the corresponding $\Delta a$ shown in Figure 2. The $V_p$ values are used to calculate the COD values by equation (1) and the results are summarized in Table 1. Figure 3 shows the plot of crack opening displacement $\delta$ vs crack growth length according to the BS5762 COD method. The interception of the linear regression line of the $\delta$-resistance curve with the $Y$-axis ($\Delta a = 0$) is defined as the critical fracture toughness $k_c$. Table 2 shows that the critical $k_c$ values are essentially independent of the specimen thickness. The values of the resistance $\delta$-curve, $d\delta/da$, are also nearly independent of the specimen thickness which are also listed in Table 2.

**ASTM E813-81 method**

The $J$ value for each specimen is calculated by using

$$J = \frac{2U}{Bb}$$

### Table 1: Summarized COD and $J$ data for a typical PC/ABS blend with $B = 10$ mm

<table>
<thead>
<tr>
<th>$V_g$ (mm)</th>
<th>$V_p$ (mm)</th>
<th>$P$ (kN)</th>
<th>$q$ (mm)</th>
<th>$U$ (J)</th>
<th>$a$ (mm)</th>
<th>COD (mm)</th>
<th>$J$ (kJ m$^{-2}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.30</td>
<td>0.150</td>
<td>0.375</td>
<td>1.44</td>
<td>0.230</td>
<td>0.08</td>
<td>0.095</td>
<td>4.60</td>
</tr>
<tr>
<td>1.40</td>
<td>0.190</td>
<td>0.392</td>
<td>1.56</td>
<td>0.258</td>
<td>0.11</td>
<td>0.109</td>
<td>5.16</td>
</tr>
<tr>
<td>1.50</td>
<td>0.231</td>
<td>0.410</td>
<td>1.67</td>
<td>0.406</td>
<td>0.25</td>
<td>0.127</td>
<td>8.12</td>
</tr>
<tr>
<td>1.60</td>
<td>0.289</td>
<td>0.424</td>
<td>1.78</td>
<td>0.466</td>
<td>0.33</td>
<td>0.145</td>
<td>9.32</td>
</tr>
<tr>
<td>1.70</td>
<td>0.346</td>
<td>0.436</td>
<td>1.90</td>
<td>0.551</td>
<td>0.48</td>
<td>0.183</td>
<td>11.02</td>
</tr>
<tr>
<td>1.90</td>
<td>0.475</td>
<td>0.456</td>
<td>2.12</td>
<td>0.605</td>
<td>0.55</td>
<td>0.208</td>
<td>12.10</td>
</tr>
<tr>
<td>2.00</td>
<td>0.560</td>
<td>0.466</td>
<td>2.23</td>
<td>0.655</td>
<td>0.65</td>
<td>0.231</td>
<td>13.11</td>
</tr>
<tr>
<td>2.10</td>
<td>0.645</td>
<td>0.472</td>
<td>2.36</td>
<td>0.697</td>
<td>0.76</td>
<td>0.280</td>
<td>16.94</td>
</tr>
<tr>
<td>2.30</td>
<td>0.830</td>
<td>0.478</td>
<td>2.57</td>
<td>0.757</td>
<td>0.81</td>
<td>0.328</td>
<td>18.14</td>
</tr>
</tbody>
</table>

- $V_g$ (mm): Clip-gauge displacement
- $V_p$ (mm): Plastic component of clip-gauge displacement
- $P$ (kN): Load
- $q$ (mm): Load-line displacement
- $U$ (J): Input energy
- $a$ (mm): Crack growth length
- COD (mm): Crack opening displacement
- $J$ (kJ m$^{-2}$): $J$ value calculated from the equation $J = \frac{2U}{Bb}$
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Figure 3 Plot of the COD (δ) vs crack growth length (Δa) for B = 10 mm specimen

Table 2 Critical fracture toughness obtained from different methods

<table>
<thead>
<tr>
<th>Thickness B (mm)</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Critical 6c (Au = 0)</td>
<td>0.066</td>
<td>0.068</td>
<td>0.058</td>
<td>0.071</td>
</tr>
<tr>
<td>dδ/da</td>
<td>0.325</td>
<td>0.264</td>
<td>0.311</td>
<td>0.257</td>
</tr>
<tr>
<td>m value</td>
<td>1.53</td>
<td>1.65</td>
<td>1.40</td>
<td>1.44</td>
</tr>
<tr>
<td>Jc (Au = 0)</td>
<td>4.05</td>
<td>4.50</td>
<td>3.74</td>
<td>4.10</td>
</tr>
<tr>
<td>Critical JcNEW obtained from new J method</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Thickness B (mm)</td>
<td>4</td>
<td>6</td>
<td>8</td>
<td>10</td>
</tr>
<tr>
<td>JcNEW (Δa = 0)</td>
<td>4.21</td>
<td>4.54</td>
<td>3.86</td>
<td>4.48</td>
</tr>
<tr>
<td>ω parameter</td>
<td>29.92</td>
<td>31.79</td>
<td>35.20</td>
<td>33.69</td>
</tr>
</tbody>
</table>

δc, (Δa = 0): standard COD method
dδ/da: slope of δ-resistance curve
dJ/da: slope of J-resistance curve
Tm value = (W - a)/Jc dJ/da
ω = (W - a)/Jc dJ/da

Figure 4 Typical plot of J-integral by ASTM E813-81 method

Figure 5 Plot of the load-line displacement (q) vs clip-gauge displacement (V) for B = 10 mm specimen

intercepts with the blunting line (J = 2δc Δa) to locate the JcNEW value. The JcNEW determined from varying the specimen thickness are summarized in Table 2, where the thinner specimens (B = 4, 6 mm) have a slightly higher Jc values than the thicker specimens (B = 12.5, 15 mm). The dJ/da values obtained according to the linear regression R-curves of ASTM E813-81 are summarized in Table 2. The slope of the J resistance curve (dJ/da) increases slightly with decreasing specimen thickness.

The size criterion of specimens
ASTM E813 specifies that for a valid Jc measurement the size criterion requirements of equation (11) must be met. The size criterion parameters [R(W - a), W > 25(Jc/σf)] according to the ASTM E813 methods for a valid Jc are all satisfied as shown in Table 2. The size criteria for J-testing allow for the use of significantly smaller specimen dimensions than those required for LEFM. The tearing modulus [as equation (12)], Tm is used to describe the stability of the crack growth. Table 2 also shows that the tearing modulus Tm value of specimen geometries are nearly independent of the specimen thickness. In order for the J-Δa data to be regarded as an intrinsic material property independent of specimen size, the criterion parameter, ω > 10 [as equation (13)] must be met. In this paper, the criterion ω > 10 is met (Table 2).

New J method

In this paper, both the clip-gauge displacement (V) and the load–time displacement (q) can be simultaneously determined by a computer and the typical results (B = 10 mm) are listed in Table 1. Figure 5 shows a linear relationship of V vs q. Figure 6 also shows the linear relationship of δ vs q where the critical initial displacement qcrit is determined at the onset of the critical JcNEW determined from Figure 3. As soon as the qcrit is determined, the critical JcNEW can be obtained from the plot of J vs q at the onset of the qcrit value as shown in Figure 7. All of these determined qcrit and JcNEW values are summarized in Table 2. The qcrit and JcNEW values obtained from this new J method are fairly independent of the specimen thickness.

The m factor

A typical photograph of the loaded PC/ABS SENB
specimen is shown in Figure 8 where a sharply defined plastic zone ahead of the crack tip can be clearly observed. The plastic zone increases with the increase of the crack growth length. The length of the primary plastic zone \( r_{p2} \) was measured from an optical micrograph of the central section of the specimen and the data are summarized in Table 3. It was postulated earlier that an LEFM parameter \( \gamma' \) can be used to characterize the fracture behaviour of toughened materials by assuming the crack initiation occurring shortly after the onset of nonlinearity. The stress intensity factor, \( K \), is calculated by the following equation,

\[
K = Y \sigma_t d^{1/2}
\]

(21)

where \( Y \) is a geometry factor, and \( a = a_0 + \Delta a \). The Dugdale model predicts the plastic zone, \( r_p \), ahead of a crack tip as,

\[
r_p = 0.393 \left( \frac{K}{m \sigma_t} \right)^2
\]

(22)

where \( m \) is the plastic constraint factor. The \( K \) values are then calculated from equation (22) by using the length of the measured primary plastic zone. Three constraint factors, \( m = 1, m = 2^{1/7}, \) and \( m = 3^{1/2} \) are assumed to calculate the corresponding \( K \) values. The calculated \( K \) values are summarized in Table 3. Furthermore, the \( K \) values can be converted into the \( J \) value by using the \( J = K^2 \left( 1 - v^2 \right)/E \) equation. Three sets of the calculated \( J \) values from the corresponding \( K \) values are also summarized in Table 3. Figure 9 shows the plots of the \( J \) vs the crosshead displacement for ASTM E813-81 method and from the I.FFM approach with various constraint factors for \( B = 10 \text{ mm} \). The constraint \( m \) factor is then estimated from the \( J-q \) curve of the ASTM E813-81 method and the determined \( m \) values are summarized in Table 2. The constraint \( m \) factor is essentially independent of the specimen thickness varying from \( B = 4 \) to \( 15 \text{ mm} \). However, the \( m \) values obtained are lower than 2 (plane-strain condition) and are higher than 1 (plane-stress condition). This is due to the large-scale yielding of polymeric materials and the \( m \) value should be be between 1 and 2.

Critical \( J \) values obtained from different methods

The \( m \) values for different thickness specimens determined as described in the above section were used to calculate the corresponding critical \( J_{c, BS} \) values according to equation (18) and the results are summarized in Table 2. Figure 10 shows a linear relationship with slope = 1 of the \( J \) obtained from the ASTM E813-81 method (equation 10) and the \( J \) calculated from the COD method (equation 18). That means the \( J \) values from the ASTM and COD methods at different stages of deformation are essentially identical based on this study. Such observation emphasizes the close relationship between these two methods. There are three critical \( J \) values, \( J_{c, BS}, J_{c, NEW}, \) and \( J_{c, BS} \), can be derived from the ASTM E813-81 method, the new \( J \) method, and \( J \) calculated from equation (18) of the COD method, respectively. These three critical \( J \) values are shown in Table 2. These critical \( J \) values obtained from the above three methods are very close to each other and
are essentially independent of the specimen thickness as illustrated in Figure 11.

**J obtained from different methods**

Three different J values at various \( V_p \), \( J_{SUM} \), \( J_{ASTM} \), and \( J_{BG} \), can be calculated using the Sumpter equation (equation (3)), the ASTM E813 equation (equation (10)), and the COD calculation equation (equation (18)), respectively. Figure 12 shows three curves of calculated \( J_{ASTM} \), \( J_{BG} \), and \( J_{SUM} \) vs crack growth length. The calculated \( J_{ASTM} \) values are nearly identical to the calculated \( J_{BG} \) values (as shown in Figure 10) but are higher than the calculated \( J_{SUM} \) values when \( \Delta a \) is greater than 0.2 mm. The resistance curves of both of the ASTM E813 method \( dJ_{ASTM}/da \) and the COD method \( dJ_{BG}/da \) are comparable but are higher than \( dJ_{SUM}/da \) of the Sumpter derived J equation. The elastic and plastic components of the J value, \( J_e \) and \( J_p \), can be calculated from equation (3) at ambient plastic component of clip-gauge displacement \( V_p \). Figure 13 shows the plots of the \( J_e \) and \( J_p \) vs the crack growth length, respectively. The increment rate of the elastic component \( dJ_e/da \) is nearly constant while the plastic component \( dJ_p/da \) increases gradually with \( \Delta a \).
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Figure 11 Plots of the critical \( J_c \) vs the specimen thickness according to three different methods.

Figure 12 Plots of the calculated \( J \) vs the crack growth length \( \Delta a \) according to equations (3), (10) and (15).

Figure 13 Plots of \( J_a \) and \( J_b \) vs crack growth length \( \Delta a \).

**Further discussion: effect on crack initiation**

The effect of specimen thickness on crack growth resistance can be interpreted in terms of crack tip constraint and deformation remote from the crack tip. Figure 14 schematically illustrates the plastic hinge construction. After onset of initiation, the increase of the clip-gauge displacement \( dV \) and the crack opening displacement \( d\delta \) above their respective initiation values

**Figure 14** Schematic sketch illustrating the hinge point construction during crack growth.

\[ V_c \] and \( \delta_c \) are related by the following equation.

\[ d\delta = \frac{(r_b dV)}{(r_b d + \alpha)} \quad (23) \]

The \( r_b \) is the rotation distance and is defined as the hinge point position during crack growth, \( r_b \) is the rotation factor, \( b \) is the ligament length. Similarly, the crack opening displacement at the crack tip of the growing crack (crack-tip opening displacement (CTOD)) and the crack tip opening angle (CTOA) are related by the distance \( r \) by the following equation.

\[ CTOA = 2 \tan^{-1} \left( \frac{CTOD}{2r} \right) \quad (24) \]

As there is only elastic deforming behind the crack tip, \( r_b \) is directly proportional to \( r \). CTOA is related to \( d\delta/da \) by the following equation.

\[ CTOA = 2 \tan^{-1} \left( \frac{d\delta}{2da} \right) \quad (25) \]

**Table 4** summarizes the values of the rotation distance \( r_b \) and rotation factor \( r_a \) calculated from equation (23) by using the measured crack-opening and the clip-gauge displacement.

It can be seen from **Table 4** that the rotation distance \( r_b \) and the rotation factor \( r_a \) decrease slightly with the increase of thickness up to \( B = 15 \text{ mm} \). Since both \( \delta_c \) and \( \delta \) are functions of the rotation factor (or constraint) of the crack tip, a greater thickness will result in lower \( \delta_c \) and \( \delta \). Decreasing CTOD \( (\delta_c) \) while holding \( r \) constant will lead to a decrease in CTOA (equation (24)) and the crack opening resistance \( d\delta/da \) (equation (25)). This will result in a smaller change in the area under the load vs clip-gauge displacement curve with crack growth and hence a decrease in \( dJ/da \). Figure 15 shows that \( dJ/da \) decreases with increasing specimen thickness.

**Table 4** Critical \( r_b \) from the BS:5762 COD test method

<table>
<thead>
<tr>
<th>Thickness ( B ) (mm)</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
<th>12.5</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>( d\delta/dV )</td>
<td>0.216</td>
<td>0.195</td>
<td>0.172</td>
<td>0.207</td>
<td>0.183</td>
<td>0.171</td>
</tr>
<tr>
<td>( r_b ) (mm)</td>
<td>0.375</td>
<td>0.374</td>
<td>0.279</td>
<td>0.754</td>
<td>0.774</td>
<td>0.786</td>
</tr>
<tr>
<td>( r_a )</td>
<td>2.75</td>
<td>2.43</td>
<td>2.09</td>
<td>2.54</td>
<td>2.24</td>
<td>2.06</td>
</tr>
</tbody>
</table>

\( r_b \): Rotation distance

\( r_a \): Rotation factor

\( b \): Ligament length \( (b = W - a) \)
decreased. However, when the specimen thickness is increased above the size requirements, the CTOD will be decreasing CTOD ($\delta$) would be anticipated as $J_c$ is also decreased. When the specimen thickness is increased above the size requirements, the CTOD will be expected to remain constant and $J_c$ will achieve a minimum plateau value. Furthermore, if the specimen is essentially under plane-strain conditions, the size and shape of the plastic zone is not expected to alter with the increase of the specimen thickness, and $r$ is expected to remain constant. Therefore, the critical $J_c$ values are expected to remain constant. Three different critical fracture toughness $J_c$ values ($J_c_{sl}$, $J_c_{BS}$ and $J_c_{NEW}$) are shown in Table 2 and the results are fairly independent of the specimen thickness varying from 4 to 15 mm.

CONCLUSIONS

The fracture behaviour of PC/ABS polyblend has been investigated simultaneously by both COD and $J$-integral methods with various specimen thicknesses. The critical $\delta_c$ values from the COD method and $J_c$ from the $J$-integral method are essentially independent of the specimen thickness varying from 4 to 15 mm. Three critical $J_c$ values, $J_c_{sl}$, $J_c_{NEW}$ and $J_c_{BS}$, obtained from the ASTM E813-81 method, the new $J$ method, and equation (18) of the COD method, are very close and are also independent of the specimen thickness, three different $J$ values ($JSUM$, $JASTM$ and $JBS$) can be calculated from equations (3), (10) and (18), respectively. The calculated $JASTM$ is close to the calculated $JBS$ but is higher than the calculated $JSUM$ at higher crack growth length. A plastic zone can be identified and measured from the central section of the specimen and the size of the plastic zone $r_2$ increases with the increase of the crack growth. The constraint $m$ factor from the ASTM E813 method was determined to be greater than unity but less than 2 due to the large-scale yielding of polymeric materials. Finally, the rotation factors $r_2$ were found to be independent of the specimen thickness and are close to 2. The crack propagation resistance, $J$-curve ($dJ/da$) or $\delta$-curve ($d\delta/da$), decreases with the increase of the specimen thickness. A close relationship between COD and $J$-integral has been demonstrated in this study.

ACKNOWLEDGEMENT

The authors are grateful to the National Science Council of Republic of China for financial support.

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