

# A Novel Intelligent Multiobjective Simulated Annealing Algorithm for Designing Robust PID Controllers

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**Abstract**—This paper proposes an intelligent multiobjective simulated annealing algorithm (IMOSA) and its application to an optimal proportional integral derivative (PID) controller design problem. A well-designed PID-type controller should satisfy the following objectives: 1) disturbance attenuation; 2) robust stability; and 3) accurate setpoint tracking. The optimal PID controller design problem is a large-scale multiobjective optimization problem characterized by the following: 1) nonlinear multimodal search space; 2) large-scale search space; 3) three tight constraints; 4) multiple objectives; and 5) expensive objective function evaluations. In contrast to existing multiobjective algorithms of simulated annealing, the high performance in IMOSA arises mainly from a novel multiobjective generation mechanism using a Pareto-based scoring function without using heuristics. The multiobjective generation mechanism operates on a high-score nondominated solution using a systematic reasoning method based on an orthogonal experimental design, which exploits its neighborhood to economically generate a set of well-distributed nondominated solutions by considering individual and overall objectives. IMOSA is evaluated by using a practical design example of a super-maneuverable fighter aircraft system. An efficient existing multiobjective algorithm, the improved strength Pareto evolutionary algorithm, is also applied to the same example for comparison. Simulation results demonstrate high performance of the IMOSA-based method in designing robust PID controllers.

**Index Terms**—Evolutionary computation, genetic algorithm (GA), multiobjective optimization, Pareto solution, proportional integral derivative (PID) controller, simulated annealing.

## I. INTRODUCTION

THIS paper proposes an intelligent multiobjective simulated annealing algorithm (IMOSA) and its application to a three-objective optimal proportional integral derivative (PID) controller design problem. The PID controllers have found extensive industrial applications for several decades [1].

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Despite the strong academic interest in modern control methods in recent decades, PID controllers are still preferred in industrial process control because they are simple, easy to implement, and easy to retune online [2]. The theoretical design of accurate and robust optimal controllers for multiple-input multiple-output systems has recently received significant attention. Well-designed controllers should satisfy the following objectives: 1) disturbance attenuation; 2) robust stability; and 3) accurate setpoint tracking [1]–[3]. Because the three competing objectives cannot be measured by using the same units, optimal control design is a multiobjective optimization problem [3], [4].

The multiobjective optimal control design problem is generally cast as a single-objective problem by constructing a utility function describing the relative importance of each objective [4]–[10]. A genetic algorithm (GA) is commonly used to solve the single-objective optimization problem of PID controller design [1], [3], [5]–[8]. Chen *et al.* [1] adopted GA to design mixed  $H_2/H_\infty$  optimal PID controllers but applied it to a single-input single-output system. Krohling and Rey [5] investigated the same problem using GA with a different performance index, i.e., time-weighted square error for a short settling time. Chen and Cheng [6] adopted GA to design structure-specified  $H_\infty$  optimal controllers for practical applications, but their procedure requires prior domain knowledge, i.e., the Routh–Hurwitz criterion for reducing the domain size of each design parameter. Kitsios *et al.* [7] adopted a GA-based method blended with multiobjective characteristics to improve the performance of the method [6]. Kitsios and Pimenides [8] adopted GA to design a structure-specified robust-multivariable controller. Ho *et al.* [9] adopted a single-objective orthogonal simulated annealing algorithm to design mixed  $H_2/H_\infty$  optimal PID controllers efficiently. Takahashi *et al.* [3], in employing group properties of the Pareto set, developed a multiobjective GA approach to the mixed  $H_2/H_\infty$  optimal control design using the solutions of linear matrix inequalities or bilinear matrix inequalities as an initial population of the GA.

Ulungu *et al.* [11] proposed a tool of multiobjective simulated annealing, adopting a new acceptance strategy using a weighted-sum objective function to construct its accepted probability. To obtain a set of nondominated solutions, the multiobjective simulated annealing must modify its objective weight for every run. A uniform spread of weights cannot be guaranteed to provide a good spread of nondominated solutions. The Pareto simulated annealing adopts an automatic step to tune

the weight of the accepted probability [12]. The procedure of the Pareto simulated annealing is similar to performing many runs of multiobjective simulated annealing at once. The generation mechanisms of multiobjective simulated annealing [11], Pareto simulated annealing [12], and fast simulated annealing [13] adopt a generate-and-test technique to obtain a candidate solution for the next move. However, the generate-and-test technique performs poorly when exploring an extremely large and multimodal search space in a limited amount of computation time and, therefore, is not acceptable for many intractable engineering applications [14]. The orthogonal simulated annealing adopts an intelligent generation mechanism that can efficiently generate a good candidate solution for the next move by using a systematic reasoning method based on orthogonal experimental designs [9], [15]. The orthogonal simulated annealing performs better than the fast simulated annealing [13] in solving large parameter optimization problems [9] and performs well in solving intractable engineering problems [15].

This paper adopts the proposed IMOSA without using heuristics, domain knowledge, and differentiability assumption to obtain a set of accurate Pareto solutions to the three-objective PID controller design problems. The high performance in IMOSA arises mainly from a novel multiobjective generation mechanism based on orthogonal experimental designs and an associated Pareto-based scoring function, which does not utilize heuristics or relative preferences among multiple objectives. The performance of the IMOSA-based method was compared with that of an efficient multiobjective GA, the improved strength Pareto evolutionary algorithm (SPEA2) [16], by designing a super maneuverable fighter aircraft system [6]–[9]. Simulation results indicate that the IMOSA-based method performs well in designing the three-objective robust PID controllers.

The remainder of this paper is organized as follows. Section II presents the concept of Pareto solutions and the investigated problem. Section III presents the IMOSA-based design method. Section IV shows the performance of the proposed approach using IMOSA and SPEA2 in designing a super maneuverable fighter aircraft system. Finally, Section V concludes this paper.

## II. PROBLEM DESCRIPTION

### A. Definitions of Pareto-Optimal Solution

Real-world problems require the simultaneous optimization of several conflicting objectives. A set of alternative solutions should ideally be obtained and are known as Pareto-optimal solutions [17]. These solutions are optimal in the sense that no other solutions in the search space are superior to another when considering all objectives. Without loss of generality, let  $\min. f = \{f_1, f_2, \dots, f_n\}$  be an  $n$ -objective function where  $f_i$  is to be minimized.

When the following inequalities hold between two feasible solutions  $X_1$  and  $X_2$ ,  $X_2$  is said to weakly dominate  $X_1$  [18]:

$$\forall i : f_i(X_1) \geq f_i(X_2). \quad (1)$$

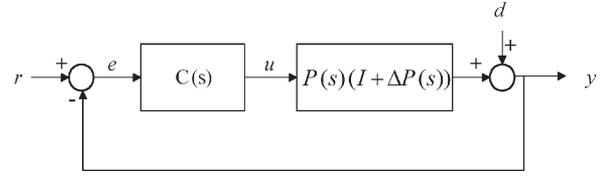


Fig. 1. Control system with plant perturbation and external disturbance.

When the following inequalities hold between  $X_1$  and  $X_2$ ,  $X_2$  is said to dominate  $X_1$ :

$$\forall i : f_i(X_1) \geq f_i(X_2) \quad \exists j : f_j(X_1) > f_j(X_2). \quad (2)$$

A feasible solution  $X^*$  is said to be a Pareto-optimal solution if and only if no feasible solution  $X$  exists where  $X$  dominates  $X^*$ . The objective vectors of the Pareto-optimal solutions in the objective space are called the Pareto-optimal front. The objective vectors of the solutions that are nondominated within a limited set of solutions are called the Pareto front. The ultimate goal of solving the investigated multiobjective optimization problem is to efficiently find an accurate set of well-spread nondominated solutions (Pareto solutions), particularly in an environment where no preference or scaling information of objective functions is available.

### B. Robust PID Controllers

Consider a multiple-input multiple-output control system with  $n_i$  inputs and  $n_o$  outputs, as shown in Fig. 1, where  $P(s)$  is the nominal plant,  $\Delta P(s)$  is the plant perturbation,  $C(s)$  is the controller,  $r(t)$  is the reference input,  $u(t)$  is the control input,  $e(t)$  is the tracking error,  $d(t)$  is the external disturbance, and  $y(t)$  is the output of the system [6]. Without loss of generality, the plant perturbation  $\Delta P(s)$  is assumed to be bounded by a known stable function matrix  $W_1(s)$

$$\bar{\sigma}(\Delta P(jw)) \leq \bar{\sigma}(W_1(jw)), \quad \forall w \in [0, \infty) \quad (3)$$

where  $\bar{\sigma}(\mathbf{A})$  denotes the maximum singular value of a matrix  $\mathbf{A}$ .

If a controller  $C(s)$  is designed such that the nominal closed loop system ( $\Delta P(s) = 0$  and  $d(t) = 0$ ) is asymptotically stable, the robust stability performance satisfies the following inequality

$$f_1 = \|W_1(s)T(s)\|_\infty < 1 \quad (4)$$

and the disturbance attenuation performance satisfies the following inequality

$$f_2 = \|W_2(s)S(s)\|_\infty < 1 \quad (5)$$

then the closed loop system is also asymptotically stable with  $\Delta P(s)$  and  $d(t)$ , where  $W_2(s)$  is a stable weighting function matrix specified by designers.  $S(s)$  and  $T(s) = I - S(s)$  are

the sensitivity and complementary sensitivity functions of the system, respectively

$$S(s) = (I + P(s)C(s))^{-1} \quad (6)$$

$$T(s) = P(s)C(s) (I + P(s)C(s))^{-1}. \quad (7)$$

Robust stability and disturbance attenuation are often insufficient in the control system design for advancing the system performance. The minimization of tracking error  $f_3$  (i.e.,  $H_2$  norm) should be considered

$$\text{minimize } f_3 = \int_0^{\infty} e^T(t)e(t)dt \quad (8)$$

where  $e(t) = r(t) - y(t)$  is the error that can be obtained from the inverse Laplace transformation of  $E(s)$  with  $\Delta P(s) = 0$  and  $d(t) = 0$

$$E(s) = (I + P(s)C(s))^{-1} R(s). \quad (9)$$

In this paper, the three objectives  $f_1$ ,  $f_2$ , and  $f_3$  are minimized simultaneously

$$\text{minimize } f = \{f_1, f_2, f_3\}. \quad (10)$$

The order of the derived optimal controller is very high when using conventional methods, making it hard to implement. To alleviate this difficulty, the mixed  $H_2/H_\infty$  optimal control problem was investigated from the suboptimal perspective. A structure-specified controller of the form [3]

$$C(s) = \frac{N_c(s)}{D_c(s)} = \frac{\mathbf{B}_u s^u + \mathbf{B}_{u-1} s^{u-1} + \cdots + \mathbf{B}_0}{s^z + a_{z-1} s^{z-1} + \cdots + a_0} \quad (11)$$

is assigned with some desired orders  $u$  and  $z$  to minimize  $f$ , where

$$\mathbf{B}_k = \begin{bmatrix} b_{k11} & \cdots & b_{k1n_i} \\ \vdots & \ddots & \vdots \\ b_{kn_{o1}} & \cdots & b_{kn_{o}n_i} \end{bmatrix} \quad (12)$$

for  $k = 0, 1, \dots, u$ . Most conventional controllers employed in industrial control systems have fundamental structures such as PID and lead/lag configurations and are special cases of the structure-specified controllers. A PID controller ( $n_i = 3$  inputs and  $n_o = 3$  outputs) has  $z = 1$ ,  $u = 2$ , and  $a_0 = 0$ , i.e.,

$$C(s) = \frac{\mathbf{B}_2 s^2 + \mathbf{B}_1 s + \mathbf{B}_0}{s}. \quad (13)$$

A PID controller has 27 design parameters. A PI controller with 18 design parameters is a special case of a PID controller where  $\mathbf{B}_2 = 0$ .

### III. IMOSA-BASED DESIGN METHOD

The high performance in IMOSA arises mainly from a novel multiobjective generation mechanism based on orthogonal experimental designs [19]–[24]. Chou *et al.* [21] first used a Taguchi-genetic single-objective approach in designing an optimal gray-fuzzy controller of a constant turning force system.

The multiobjective generation mechanism utilizes a Pareto-based scoring function to measure the performance of candidate solutions in a multiobjective search space. Section III-A briefly introduces concepts of the orthogonal experimental design. The scoring function is described in Section III-B. Section III-C presents the used orthogonal experimental design in IMOSA. The multiobjective generation mechanism and a concise example are presented in Sections III-D and E, respectively. Section III-F presents the IMOSA-based design method.

#### A. Concepts of Orthogonal Experimental Design

The effect of several factors can be investigated simultaneously by using an orthogonal experimental design with both orthogonal arrays and factor analysis. The factors are the variables (parameters), which affect response variables. A setting (or a discriminative value) of a factor is regarded as a level of the factor. A “complete factorial” experiment would perform measurements at each of all possible level combinations. However, the number of level combinations is often so large that the complete factorial approach is impracticable. Therefore, a subset of level combinations must be carefully selected, producing a “fractional factorial” experiment [19], [20]. An orthogonal experimental design utilizes the properties of fractional factorial experiments to efficiently determine the best combination of factor levels for use in design problems.

Orthogonal array is a fractional factorial array, which ensures a balanced comparison of levels of any factor. Orthogonal array is an array of numbers arranged in rows and columns where each row represents the levels of factors in each combination, and each column represents a specific factor that can be changed from each combination. The term “main effect” indicates the effect on response variables that can be traced to a design parameter. The main effect of one factor does not affect the estimation of the main effect of another factor [19]–[21].

Factor analysis using the orthogonal array’s tabulation of experimental results enables rapid estimation of the main effects, without fear of distortion of results by the effects of other factors. Factor analysis can evaluate the effects of individual factors on the evaluation function, rank the most effective factors, and determine the best level for each factor such that the evaluation function is optimized. An orthogonal experimental design uses well-planned and controlled experiments in which certain factors are systematically set and modified, and the main effect of factors on the response can then be observed. Therefore, the orthogonal experimental design is regarded as a systematic reasoning method.

#### B. Scoring Function

To develop an efficient multiobjective simulated annealing algorithm from extending the conventional simulated annealing to solve multiobjective optimization problems, the evaluation step of simulated annealing must contain an effective scoring function for evaluating candidate solutions in the multiobjective space. The scoring function of IMOSA uses a set  $E$  to evaluate the candidate solution (individual) belonging to  $E$  in the multiobjective space, which most effectively exploits

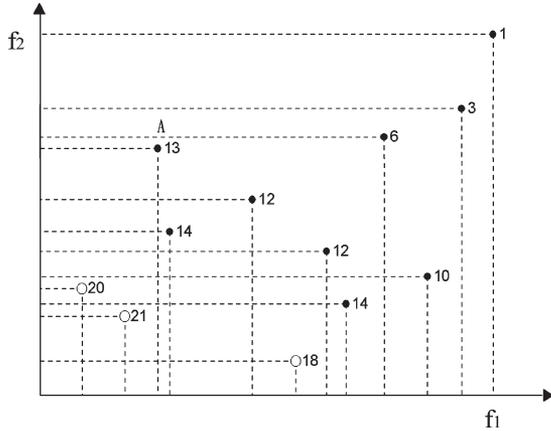


Fig. 2. Scores of 12 participant individuals in the objective space of a biobjective minimization problem. The score of the dominated individual  $A$  using the scoring function is  $F = 3 - 2 + 12 = 13$  [24].

the Pareto dominance relationship using a single performance measure. Simply, an individual has a high score if it dominates many individuals. Conversely, an individual has a low score if many individuals dominate it. The tournament-like score of an individual  $X$  can be obtained by using the following scoring function  $F$  [24]:

$$F(X) = \{p - q + c | p = |U|, q = |V|, c = |E| \\ \text{s.t. } X \prec U, V \prec X, U \subseteq E \text{ and } V \subseteq E\} \quad (14)$$

where  $\prec$  denotes domination,  $c$  is the size of  $E$ ,  $p$  is the number of individuals in  $U$  that can be dominated by  $X$ , and  $q$  is the number of individuals in  $V$  that can dominate  $X$  in the objective space.

The function  $F$ , which utilizes a pure Pareto-ranking assignment strategy, can assign discriminative scores to both non-dominated and dominated individuals. Fig. 2 shows an example of assigning scores using  $F$  for a biobjective minimization problem. For example,  $A$  dominates three individuals ( $p = 3$ ) and is dominated by two individuals ( $q = 2$ ). Therefore, the score of  $A$  is  $F = 3 - 2 + 12 = 13$ . The score assignment strategy works for both IMOSA and the intelligent multiobjective evolutionary algorithm of Ho *et al.* [24].

### C. Orthogonal Experimental Design in IMOSA

This section describes the orthogonal experimental design with three-level orthogonal arrays in IMOSA. All the design parameters are generally assigned into  $N$  groups. One group is regarded as a factor. The number of factors is thus  $N$ , and each factor has three levels. The number of total experiments is  $3^N$  for the popular “one-factor-at-once” study. To utilize an orthogonal array of  $N$  factors, obtain an integer  $M = 3^{\lceil \log_3(2N+1) \rceil}$ , build a three-level orthogonal array  $L_M(3^{(M-1)/2})$  with  $M$  rows and  $(M-1)/2$  columns, use the first  $N$  columns, and ignore the remaining  $(M-1)/2 - N$  columns. Table I shows an example of an orthogonal array  $L_9(3^4)$  with  $M = 9$  for three-objective design problems. Orthogonal arrays can reduce the number of experiments for factor analysis. The number of orthogonal experiments required to analyze all solution

factors is only  $M = O(N)$ , where  $2N + 1 \leq M \leq 6N - 3$ , which is much smaller than  $3^N$ . An algorithm for constructing orthogonal arrays can be found in [9] and [25]. After tabulating the experimental results, the summarized data were analyzed by using the factor analysis to determine the relative effects of levels of various factors, as described next.

Let  $F_j$  denote a value of a response variable (objective function  $F$ ) for the combination corresponding to the experiment  $j$ , where  $j = 1, \dots, M$ . Define the main effect of factor  $i$  with level  $k$  as  $S_{ik}$ , where  $i = 1, \dots, N$  and  $k = 1, 2, 3$

$$S_{ik} = \sum_{j=1}^M F_j \cdot \delta_j \quad (15)$$

where  $\delta_j = 1$  if the level of factor  $i$  of combination  $j$  is  $k$ , and  $\delta_j = 0$  otherwise. If the objective function is to be maximized, then level  $k$  of factor  $i$  makes the best contribution to  $F$  when  $S_{ik} = \max\{S_{i1}, S_{i2}, S_{i3}\}$ . If the objective function is to be minimized, then level  $k$  of factor  $i$  is best if  $S_{ik} = \min\{S_{i1}, S_{i2}, S_{i3}\}$ . The main effect represents the individual effect of a factor. The most effective factor has the largest main effect difference, which is given by  $\text{MED}_i = \max\{S_{i1}, S_{i2}, S_{i3}\} - \min\{S_{i1}, S_{i2}, S_{i3}\}$ ,  $i = 1, \dots, N$ . A reasoned solution consisting of all factors with the best levels can be easily derived after the best level of each factor is determined. An orthogonal experimental design is a representative quality control method and can also work to improve the move mechanism of simulated annealing [9] and crossover operator of GA [24] efficiently. Ho *et al.* [9], [24] provide concise examples of utilizing orthogonal experimental designs.

Let  $n$  be the number of objectives for the optimization problem. The multiobjective generation mechanism operates on a nondominated solution  $X$  with a high score  $F$  using orthogonal experimental designs to exploit its neighborhood and to economically generate a set of promising nondominated solutions by considering individual and overall objectives. If the overall objectives are considered, then the scoring function  $F$  to be maximized is treated as the objective function, and the reasoned solution  $Q$  is derived by using  $F_j = F(X_j)$ , where  $X_j$  is the generated combination corresponding to the experiment  $j$ ,  $j = 1, \dots, M$ . If only the individual objective function  $f_h$  is considered, then the reasoned solution  $Q_h$  is derived by using the main effect  $S_{ik}^h$  based on  $f_{hj} = f_h(X_j)$ , where  $h = 1, \dots, n$

$$S_{ik}^h = \sum_{j=1}^M f_{hj} \cdot \delta_j. \quad (16)$$

### D. Multiobjective Generation Mechanism

The multiobjective generation mechanism operates on a current solution  $X$  to generate a set of nondominated solutions and to select one good candidate solution  $\bar{Q}$  for the next move. Consider a parametric optimization problem of  $m$  parameters. Assuming that  $X = [x_1, \dots, x_m]^T$ , where  $x_i$  is a system parameter, the multiobjective generation mechanism generates two temporary solutions  $X^1 = [x_1^1, \dots, x_m^1]^T$  and

TABLE I  
EXAMPLE OF  $L_9(3^4)$  FOR THREE-OBJECTIVE DESIGN PROBLEMS

Combination experiments	Factor				Objective functions			Score
	1	2	3	4	$f_1$	$f_2$	$f_3$	$F$
1	1	1	1	1	$f_{11}$	$f_{21}$	$f_{31}$	$F_1$
2	1	2	2	2	$f_{12}$	$f_{22}$	$f_{32}$	$F_2$
3	1	3	3	3	$f_{13}$	$f_{23}$	$f_{33}$	$F_3$
4	2	1	2	3	$f_{14}$	$f_{24}$	$f_{34}$	$F_4$
5	2	2	3	1	$f_{15}$	$f_{25}$	$f_{35}$	$F_5$
6	2	3	1	2	$f_{16}$	$f_{26}$	$f_{36}$	$F_6$
7	3	1	3	2	$f_{17}$	$f_{27}$	$f_{37}$	$F_7$
8	3	2	1	3	$f_{18}$	$f_{28}$	$f_{38}$	$F_8$
9	3	3	2	1	$f_{19}$	$f_{29}$	$f_{39}$	$F_9$

$X^2 = [x_1^2, \dots, x_m^2]^T$  from perturbing  $X$ , where  $x_i^1$  and  $x_i^2$  are generated by perturbing  $x_i$  as follows:

$$x_i^1 = x_i + \bar{x}_i \quad x_i^2 = x_i - \bar{x}_i, \quad i = 1, \dots, m. \quad (17)$$

The values of  $\bar{x} = [\bar{x}_1, \dots, \bar{x}_m]^T$  are generated by the Cauchy–Lorentz probability distribution [13].

By using the same grouping scheme for  $X$ ,  $X^1$ , and  $X^2$ , all the  $m$  parameters are assigned into  $N$  nonoverlapping groups. The proper value of  $N$  is problem-dependent. A larger  $N$  increases the efficiency of the generation mechanism if the interaction effects among the groups are weak. If the existing interaction effect is not weak, then a larger group size increases the accuracy of the estimated main effect of groups. An efficient biobjective grouping criterion based on this tradeoff minimizes the interaction effects between groups and maximizes the value of  $N$ . To use all columns of an orthogonal array,  $N$  is generally specified as  $N = (3^{\lceil \log_3(2m+1) \rceil} - 1)/2$ , and the used orthogonal array is  $L_{2N+1}(3^N)$  excluding the study of intractable interaction effects. More information of determining the proper value of  $N$  can be referred to the works of Ho *et al.* [9], [24].

IMOSA utilizes an elite set  $E$  to maintain the best non-dominated individuals generated so far and to compute scores in the objective space for all individuals in  $E$ . The multiobjective generation mechanism aims at efficiently combining good parameter values from solutions  $X$ ,  $X^1$ , and  $X^2$  to generate a good next move  $\bar{Q}$ . Since there are  $n$  minimization objectives,  $f_1, \dots, f_n$ , and a combined maximization objective  $F$ ,  $n + 1$  good move directions are potentially available for IMOSA. Therefore,  $n + 1$  candidate solutions,  $Q_1, \dots, Q_n$  and  $Q$ , are first generated by using orthogonal experimental designs based on the  $n + 1$  individual objectives,  $f_1, \dots, f_n$  and  $F$ , respectively. Consequently,  $\bar{Q}$  is selected from the best one of the  $n + 1$  generated solutions and  $M$  combinations of the orthogonal experiments.

Performing a multiobjective generation mechanism on a current solution  $X$  with  $m$  parameters using a scoring function  $F$  and  $n$  objective functions  $f_1, \dots, f_n$  is described as follows.

Step 1) Generate two temporary solutions  $X^1$  and  $X^2$  using  $X$  from (17). For each of  $X$ ,  $X^1$ , and  $X^2$ , randomly assign all parameters to  $N$  groups using the same grouping scheme where each group is treated as a factor.

Step 2) Utilize the first  $N$  columns of an orthogonal array  $L_M(3^{(M-1)/2})$ , where  $M = 3^{\lceil \log_3(2N+1) \rceil}$ . Let levels 1, 2, and 3 of factor  $i$  represent the  $i$ th groups of  $X^1$ ,  $X$ , and  $X^2$ , respectively.

Step 3) Compute  $f_{hj} = f_h(X_j)$ , where  $X_j$  is the generated combination corresponding to the experiment  $j$ , where  $h = 1, \dots, n$  and  $j = 1, \dots, M$ . Add all  $X_j$ 's to the elite set  $E$ .

Step 4) Compute  $F_j = F(X_j)$  using  $f_h(e)$ , where  $e \in E$ . Let  $X_{\text{best}}$  be the best of  $X_i$  according to their scores.

Step 5) Compute the main effects  $S_{ik}$  and  $S_{ik}^h$  using  $F_j$  and  $f_{hj}$ , respectively, where  $i = 1, \dots, N$ ,  $k = 1, 2, 3$ , and  $h = 1, \dots, n$ . The reasoned solutions  $Q$  and  $\{Q_1, \dots, Q_n\}$  are formed using the combination of the best groups from the derived corresponding solutions  $X$ ,  $X^1$ , and  $X^2$ .

Step 6) Add  $Q$  and  $\{Q_1, \dots, Q_n\}$  into  $E$ .  $\bar{Q}$  is the non-dominated solution with the highest score selected from  $\{Q_1, \dots, Q_n\}$ ,  $Q$ , and  $X_{\text{best}}$ , but  $\bar{Q}$  is not equal to  $X$ .

The multiobjective generation mechanism on  $X$  can systematically generate  $M$  candidate solutions  $X_j$  uniformly sampled from the neighborhood of  $X$ , and  $n + 1$  reasoned solutions  $Q$  and  $\{Q_1, \dots, Q_n\}$ . Consequently, a good candidate solution  $\bar{Q}$  is obtained as the new current solution for the next move. Therefore, there are  $M + n + 1$  function evaluations per operation of the multiobjective generation mechanism. A good set of non-dominated solutions in  $E$  is also generated simultaneously. The multiobjective generation mechanism combines the advantages of local exploitation and global exploration, helping to generate wild-spread Pareto solutions.

1) *Local Exploitation*: Since an orthogonal array specifies a small number of representative combinations that are uniformly distributed over the whole space of all possible combinations in the neighborhood of  $X$ , the uniform sampling method of using orthogonal experimental designs is also helpful for objective functions with a high correlation among parameters. By using factor analysis, the multiobjective generation mechanism can select a number of combinations from the orthogonal experiments and reason some potentially good solutions with the best score performance as parts of the current Pareto solutions. The neighborhood of  $X$  can be adaptively specified in generating  $X^1$  and  $X^2$  using (17).

2) *Global Exploration*: The simultaneous search for the  $n + 1$  directions of objectives  $f_1, \dots, f_n$  and  $F$  helps maintain

TABLE II  
CONCISE EXAMPLE OF THE MULTIOBJECTIVE GENERATION MECHANISM USING  $L_9(3^4)$

$j$	Parameters $x_i$			$f_{1j}$	$f_{2j}$	$F$
	$x_1$	$x_2$	$x_3$			
1	3	6	9	231	957	3
2	3	5	8	242	847	6
3	3	4	7	253	737	9
4	2	6	8	132	858	9
5	2	5	7	143	748	12
6	2	4	9	151	938	6
7	1	6	7	33	759	15
8	1	5	9	41	949	9
9	1	4	8	52	839	12
$S_{i1}^1$	726	396	423			
$S_{i2}^1$	426	426	426			
$S_{i3}^1$	126	456	429			
$MED_i$	600	60	6			
Best level	3	1	1			
$Q_1$	1	6	9	31	959	9
$S_{i1}^2$	2541	2574	2844			
$S_{i2}^2$	2544	2544	2544			
$S_{i3}^2$	2547	2514	2244			
$MED_i$	6	60	600			
Best level	1	3	3			
$Q_2$	3	4	7	253	737	9
$S_{i1}$	18	27	18			
$S_{i2}$	27	27	27			
$S_{i3}$	36	27	36			
$MED_i$	18	0	18			
Best level	3	1,2,3	3			
$Q^1$	1	4	7	53	739	14
$Q^2$	1	5	7	43	749	14
$Q^3=Q$	1	6	7	33	759	15

high diversity in  $E$ . The main goals of designing novel multi-objective simulated annealing algorithms are to maintain high diversity and to pursue the Pareto front simultaneously. The multiobjective generation mechanism achieves these goals by a systematic and automatic method without tuning weights, which is necessary in the multiobjective simulated annealing [11] and Pareto simulated annealing [12]. This advantage is particularly important in an environment with no preference or scaling information of objective functions.

3) *Convergence to Pareto Front*: Simulated annealing is a point-based method, in contrast with the population-based GA. The main concern of multiobjective simulated annealing is the efficient convergence to a wild-spread Pareto front. The multiobjective generation mechanism, which operates on a current solution without the recombination of GA, can cope with the difficulty of tight constraints. The advantages of local exploitation and global exploration are helpful for an efficient convergence to a wild-spread Pareto front using a fairly small number of function evaluations. The size of the neighborhood can be larger in the early iterations to focus on global exploration and small in the late iterations for local exploitation. By incorporating the temperature of simulated annealing as a control parameter into the function that adaptively defines the neighborhood of  $X$ , IMOSA can stably converge to a satisfactory Pareto front.

### E. Concise Example

An illustrative example of the multiobjective generation mechanism using a biobjective function is given as follows:

$$\begin{aligned}
 f(x_1, x_2, x_3) &= \min. \{f_1, f_2\} \\
 f_1(x_1, x_2, x_3) &= 100x_1 - 10x_2 - x_3 \\
 f_2(x_1, x_2, x_3) &= 100x_3 + 10x_2 - x_1. \quad (18)
 \end{aligned}$$

Consider that the multiobjective generation mechanism operates on an initial solution  $X = [x_1, x_2, x_3]^T = [2, 5, 8]^T$  and that the elite set  $E$  is empty. Table II shows the result. Fig. 3 shows the sampling and reasoned solutions obtained. In Step 1), assume that  $\bar{x} = [1, 1, 1]^T$ . Therefore,  $X^1 = [3, 6, 9]^T$  and  $X^2 = [1, 4, 7]^T$  form perturbing  $X$ . In Step 2), let  $N = 3$ , and assign  $x_1$ ,  $x_2$ , and  $x_3$  to factors 1, 2, and 3, respectively. Therefore, the first three columns of  $L_9(3^4)$  are utilized. A complete factorial experiment would evaluate  $3^3 = 27$  level combinations, where the best combination  $(x_1, x_2, x_3) = (1, 6, 9)$  with  $f_1 = 31$  considering the first objective and  $(x_1, x_2, x_3) = (3, 4, 7)$  with  $f_2 = 737$  considering the second objective only.

A fractional factorial experiment uses a well-balanced subset of nine out of 27 combinations  $(X_1, \dots, X_9)$ . Table II shows the level settings for each factor. In Step 5), all the values of  $f_1(X_j)$  and  $f_2(X_j)$  corresponding to the experiment  $j$ ,

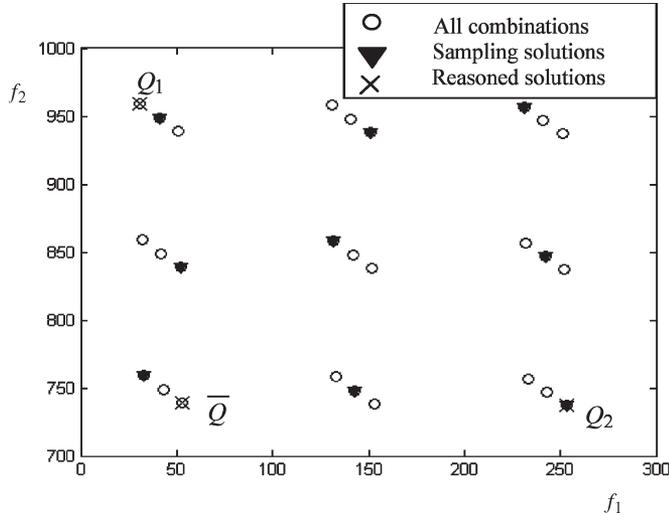


Fig. 3. Sampling and reasoned solutions obtained by one operation of the multiobjective generation mechanism.

$j = 1, \dots, 9$ , are calculated, and then  $X_1, \dots, X_9$  are added to  $E$  where  $c = 9$ . The scores  $F_j = F(X_j)$  are then calculated. For example,  $X_9 = [1, 4, 8]^T$  and  $f_1(X_9) = 52$  and  $f_2(X_9) = 839$ .  $F_9 = 4 - 1 + 9 = 12$  where  $X_9$  dominates four individuals  $X_1, X_2, X_4$ , and  $X_6$  and is dominated by  $X_7$ . The  $X_{\text{best}}$  is  $X_7$  with  $F_7 = 6 - 0 + 9 = 15$ .

The main effects  $S_{ik}$  and  $S_{ik}^h$  are calculated using  $F_j$  and  $f_{jh}$  in Step 6). For example, considering  $f_1$  only,  $S_{21}^1 = f_{11} + f_{14} + f_{17} = 396$ , and the best level of factor 1 is level 3, since  $S_{13}^1 < S_{12}^1 < S_{11}^1$ . Therefore,  $x_1 = 1$ . Finally, the best solution  $Q_1 = [1, 6, 9]^T$  with  $f_1 = 31$  can be obtained. The most effective factor is  $x_1$  with  $\text{MED}_1 = 600$ . Equation (18) confirms that  $x_1$  has the largest coefficient of 100. Similarly,  $Q_2 = [3, 4, 7]^T$  with  $f_2 = 737$  can be obtained. The best levels of factors 1 and 3 obtained by the scoring function  $F$  are 3 and 3, respectively. Therefore,  $x_1 = 1$  and  $x_3 = 7$ . Since all three levels of factor 2 have the same main effect, all intermediate combinations  $Q^1, Q^2$ , and  $Q^3$  using  $x_2 = 4, 5, 6$  are considered. Finally, the solution  $\bar{Q} = Q = Q^3 = [1, 6, 7]^T$  with  $F = 15$  is obtained.

Some observations are described next: 1) the representative nine solutions sampled from the neighborhood of the current solution  $X$  have the scores 3, 6, 6, 9, 9, 9, 12, 12, and 15, where  $F(X) = 9$ ; the generated combinations and their scores are uniformly distributed; 2) the reasoned solutions  $Q_1$  and  $Q_2$  are the best solutions considering the individual objectives  $f_1$  and  $f_2$  only. It is beneficial to obtain a well-spread Pareto front; 3) the multiobjective generation mechanism generates a very good nondominated solution  $\bar{Q}$  for the next move; and 4) a set of nondominated solutions  $\{Q^1, Q^2, Q, Q_1, \text{ and } Q_2\}$  is generated simultaneously.

#### F. Design of Controllers Using IMOSA

Designing an optimal control system is equivalent to finding an optimal solution  $X = [x_1, \dots, x_m]^T$  in a high-dimensional search space, where each point represents a vector of  $m$  de-

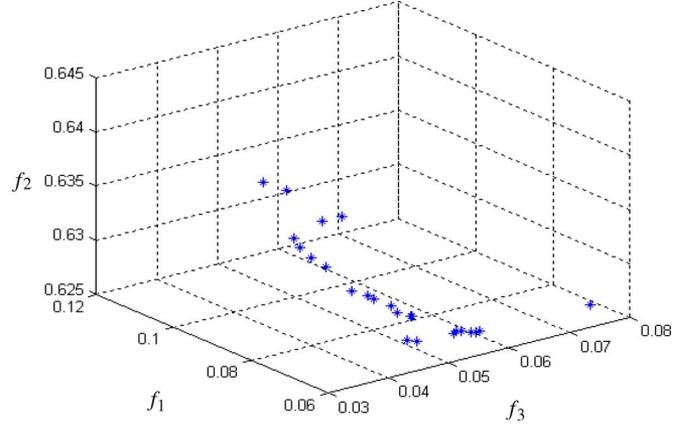


Fig. 4. Three-objective Pareto front of IMOSA by merging the results of ten independent runs.

sign parameters. For convenience and simplicity, the following equation is derived from the controller with (11):

$$X = [a_0 \cdots a_{z-1} b_{011} \cdots b_{01n_i} b_{021} \cdots b_{02n_i} \cdots b_{un_o n_i}]^T = [x_1, \dots, x_m]^T \quad (19)$$

as the controller parameter vector, where  $m = z + (u + 1) \times n_i \times n_o$ , is the number of total design parameters. Denote  $\Theta$  as the search space consisting of all admissible  $x_i, i = 1, \dots, m$ . The structure-specified mixed  $H_2/H_\infty$  optimal control design problem is equivalent to finding an optimal  $X$  from  $\Theta$  to minimize the objective function  $f$  in (10) subject to the inequality constraints (4) and (5). Chen and Cheng [6] used a prior domain knowledge, i.e., the Routh–Hurwitz criterion, to reduce the domain size of each design parameter  $x_i$ . The search space consists of all admissible  $x_i \in [-20000, 20000], i = 1, \dots, m$  [6]–[9]. To demonstrate the strong search ability of IMOSA in efficiently obtaining a good Pareto front to the investigated problem, this paper does not confine the search space  $\Theta$  using any domain knowledge.

IMOSA is based on a traditional single-objective simulated annealing algorithm for solving multiobjective optimization problems. Four choices must be made when implementing IMOSA to solve an optimization problem: 1) solution representation; 2) objective function definition; 3) design of generation mechanism; and 4) design of a cooling schedule. In this paper, the solution representation is  $X = [x_1, \dots, x_m]^T$ , and the order of  $x_i$  in  $X$  is not critical, because according to the orthogonal experimental design property, the different assignments of  $x_i$  to factors do not result in different reasoned combinations. The scoring function  $F$  acts as a good objective function. IMOSA utilizes the multiobjective generation mechanism and  $F$  to efficiently search for a candidate solution for the next move and a good set of nondominated solutions. The three parameters are usually specified in designing a cooling schedule: 1) an initial temperature  $T_0$ ; 2) a temperature update rule; and 3) a stopping criterion of the IMOSA algorithm. The design of an efficient cooling schedule for IMOSA depends on the objectives of the decision makers.

IMOSA also needs a multiobjective version of the decision step of simulated annealing. If the nondominated solution  $\bar{Q}$  dominates  $X$  and thus  $F(\bar{Q}) > F(X)$ , then  $\bar{Q}$  is always

TABLE III  
ALL OF THE PARETO SOLUTIONS OF IMOSA

no.	$f_1$	$f_2$	$f_3$	no.	$f_1$	$f_2$	$f_3$	no.	$f_1$	$f_2$	$f_3$
1	0.0665	0.6265	0.0473	9	0.0623	0.6296	0.0453	17	0.0865	0.6383	0.0400
2	0.0703	0.6306	0.0431	10	0.0793	0.6321	0.0419	18	0.0914	0.6384	0.0394
3	0.0650	0.6296	0.0447	11	0.0823	0.6325	0.0415	19	0.0652	0.6265	0.0546
4	0.0690	0.6305	0.0434	12	0.0851	0.6331	0.0414	20	0.0648	0.6265	0.0551
5	0.0657	0.6303	0.0440	13	0.0863	0.6337	0.0412	21	0.0624	0.6267	0.0552
6	0.0739	0.6306	0.0428	14	0.0781	0.6367	0.0405	22	0.0625	0.6266	0.0561
7	0.0650	0.6265	0.0479	15	0.0618	0.6272	0.0520	23	0.0632	0.6265	0.0571
8	0.0625	0.6296	0.0452	16	0.0731	0.6379	0.0406	24	0.0625	0.6265	0.0750

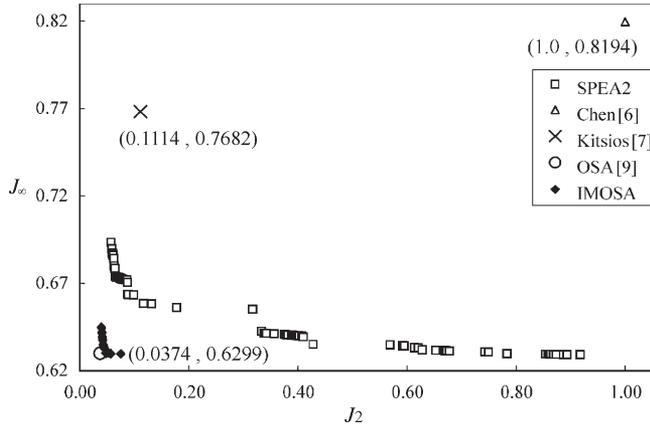


Fig. 5. Pareto fronts of IMOSA and SPEA2 by merging results of ten independent runs and the single-objective solutions obtained by Chen and Cheng [6], Kitsios *et al.* [7], and Ho *et al.* (OSA) [9]. Notably, the  $H_\infty$  optimal controller [6] has  $J_2 < 1.0$ , where the exact value of  $J_2$  is not available.

accepted. If  $\bar{Q}$  does not dominate  $X$  but  $F(\bar{Q}) \geq F(X)$ , then  $\bar{Q}$  is also accepted, because  $\bar{Q}$  compared with  $X$  dominates more individuals in  $E$ . If  $F(\bar{Q}) < F(X)$  (i.e.,  $\bar{Q}$  does not dominate  $X$ ), some  $f_i$  exists such that  $f_i(X) < f_i(\bar{Q})$ . Therefore, the acceptance criterion is defined to accept  $\bar{Q}$  with probability  $P(\bar{Q})$  as follows:

$$P(\bar{Q}) = \begin{cases} 1, & \text{if } F(\bar{Q}) \geq F(X) \\ \min. \left( \exp\left(\frac{f_1(X) - f_1(\bar{Q})}{T}\right), \dots, \exp\left(\frac{f_n(X) - f_n(\bar{Q})}{T}\right) \right), & \text{if } F(\bar{Q}) < F(X). \end{cases} \quad (20)$$

A variable Count denotes the number of trials for the same solution  $X$ , a constant  $N_{\max}$  denotes the maximal number of trials for the same solution  $X$ , and a constant  $N_{E \max}$  denotes the maximal number of solutions in elite set  $E$ . The bound  $W_1(s)$ , weighting function matrix  $W_2(s)$ , and the controller structure  $C(s)$  are specified for a given plant  $P(s)$ . The proposed IMOSA-based design method is described as follows.

- Step 1) Initialize an empty elite set  $E$  with capacity  $N_{E \max}$ , the temperature  $T = T_0$ , Count = 0, and cooling rate  $\alpha$ . Randomly generate a feasible solution  $X$ . Compute  $f_1(X), \dots, f_n(X)$  and  $F(X)$ .
- Step 2) Remove dominated solutions from  $E$ .
- Step 3) Perform the multiobjective generation mechanism using  $X$  to generate a candidate solution  $\bar{Q}$ .

Step 4) Accept  $\bar{Q}$  as the new solution  $X$  with the probability  $P(\bar{Q})$  in (20). If  $X \neq \bar{Q}$  (not accepted), then increase the value of Count by one; otherwise, reset Count = 0. If Count =  $N_{\max}$  (i.e.,  $X$  is not changed in  $N_{\max}$  iterations), then randomly select a nondominated individual from  $E$  as a new current solution  $X$  and reset Count = 0.

Step 5) Set  $T = \alpha T$ .

Step 6) If a prespecified stopping condition is satisfied, then stop the algorithm. Otherwise, go to Step 2).

If a single solution, rather than a set of Pareto solutions, is needed, then  $\bar{Q}$  is a good suggested solution. The presented IMOSA is a simple version for generalized optimization problems. Some problem-dependent learning strategies that work for conventional simulated annealing may also be good for IMOSA.

## IV. EXPERIMENTAL RESULTS

### A. Test Problem

Consider the design problem of a longitudinal control system of a super maneuverable fighter aircraft in horizontal flight at an altitude of 15 000 ft with Mach number 0.24, airspeed VT = 238.7 ft/s, attack angle  $\alpha = 25^\circ$ , and pitch angle  $\beta = 25^\circ$ . The trim value of the path angle is  $\beta - \alpha = 0^\circ$ , and the trim pitch rate is  $\gamma = 0^\circ/\text{s}$ . The longitudinal dynamics of the system can be described as

$$\begin{cases} \dot{x} = \mathbf{A}x + \mathbf{B}u \\ y = \mathbf{C}x \end{cases} \quad (21)$$

where  $\mathbf{A}$ ,  $\mathbf{B}$ , and  $\mathbf{C}$  are given as in (22), shown at the bottom of the next page.

Moreover,  $\mathbf{x} = [V_T, \alpha, \gamma, \beta]^T$ , and  $\mathbf{u} = [u_{TV}, u_{AS}, u_{SS}, u_{LE}, u_{TE}, u_T]^T$ , where  $u_{TV}$ ,  $u_{AS}$ ,  $u_{SS}$ ,  $u_{LE}$ ,  $u_{TE}$ , and  $u_T$  are the perturbations in symmetric thrust vectoring vane deflection, symmetric aileron deflection, symmetric stabilator deflection, symmetric leading edge flap deflection, symmetric trailing edge flap deflection, and throttle position, respectively. Notably, the rank of the matrix  $\mathbf{B}$  is only 3. By employing the pseudo-control technique, the system equation can be rewritten as

$$\dot{x} = \mathbf{A}x + \mathbf{B}_v v \quad (23)$$

where  $\mathbf{B}_v$  and  $v$  are given as in (24), shown at the bottom of the next page.

TABLE IV  
BEST PERFORMANCES FOR VARIOUS PI CONTROLLERS IN TERMS OF  $J = J_\infty + J_2$

Method	$J_\infty$	$J_2$	$J = J_\infty + J_2$
IMOSA	0.6300	0.0473	0.6773
SPEA2	0.6738	0.0654	0.7392
OSA [9]	0.6299	0.0374	0.6673
Kitsios [7]	0.7682	0.1114	0.8796
Chen [6]	0.8194	<1.0	NA

Assume that the reference input is  $r(t) = [0, 1 - e^{-3t}, 1 - e^{-6t}]^T$  and that the system encounters the external disturbance  $d(t) = 0.01e^{-0.2t} \cos(3162.3t) [1, 1, 1]^T$ . The bound  $W_1(s)$  of the plant perturbation  $\Delta P(s)$  is given by

$$W_1(s) = \frac{0.0125s^2 + 1.2025s + 1.25}{s^2 + 20s + 100} I_{3 \times 3}. \quad (25)$$

To attenuate disturbance, the stable weighting function  $W_2(s)$  consisting of a low-pass filter is chosen as

$$W_2(s) = \frac{0.25s + 0.025}{s^2 + 0.4s + 10\,000\,000} I_{3 \times 3}. \quad (26)$$

The three-objective Pareto front, rather than the single-objective solution to the optimal controller design problem, provides more degrees of freedom to the designers.

**B. Results**

For performance comparison, the design problem of PI controllers was solved by IMOSA, with parameters set to

$N_{E_{\max}} = 30$ ,  $T_0 = 150$ ,  $\alpha = 0.97$ , and  $N_{\max} = 30$ . The design parameters were assigned to  $N = 13$  groups, and an orthogonal array  $L_{27}(3^{13})$  was used. Each multiobjective generation mechanism evaluated 31 individuals, including  $M = 27$  orthogonal array combinations and four reasoned solutions. The stopping condition uses 10 000 evaluations of individuals. Fig. 4 shows the three-objective Pareto front of IMOSA by merging the results of ten independent runs. Table III shows all of the 24 obtained nondominated IMOSA solutions, revealing that the values of  $f_1$ ,  $f_2$ , and  $f_3$  for all nondominated solutions were less than 1.0. All these solutions satisfy constraints (4) and (5). All values of the tracing error  $f_3$  were less than 0.08, and most of them are less than 0.05. If many final nondominated solutions are needed, then an additional external set can be utilized to store all the nondominated individuals found so far. The simulation results reveal that the IMOSA-based controller is accurate.

The design problem was also solved by SPEA2 through ten independent runs. The chromosomes of SPEA2 were encoded using a binary string of 30 bits. The stopping condition is the same as that of IMOSA. The parameters  $J_\infty = (f_1^2 + f_2^2)^{1/2}$ ,

$$\begin{aligned}
 A &= \begin{bmatrix} -0.0750 & -24.0500 & 0 & -36.1600 \\ -0.0009 & -0.1959 & 0.9896 & 0 \\ -0.0002 & -0.1454 & -0.1677 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \\
 B &= \begin{bmatrix} -0.0230 & 0 & -0.0729 & 0.0393 & -0.0411 & 0.1600 \\ -0.0002 & -0.0001 & -0.0004 & 0 & -0.0003 & -0.0003 \\ -0.0067 & -0.0007 & -0.0120 & 0.0006 & 0.0007 & 0.0005 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\
 C &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}
 \end{aligned} \quad (22)$$

$$\begin{aligned}
 B_v &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \\
 v &= \begin{bmatrix} -0.0230 & 0 & -0.0729 & 0.0393 & -0.0411 & 0.1600 \\ -0.0002 & -0.0001 & -0.0004 & 0 & -0.0003 & -0.0003 \\ -0.0067 & -0.0007 & -0.0120 & -0.0006 & 0.0007 & 0.0005 \end{bmatrix} u
 \end{aligned} \quad (24)$$

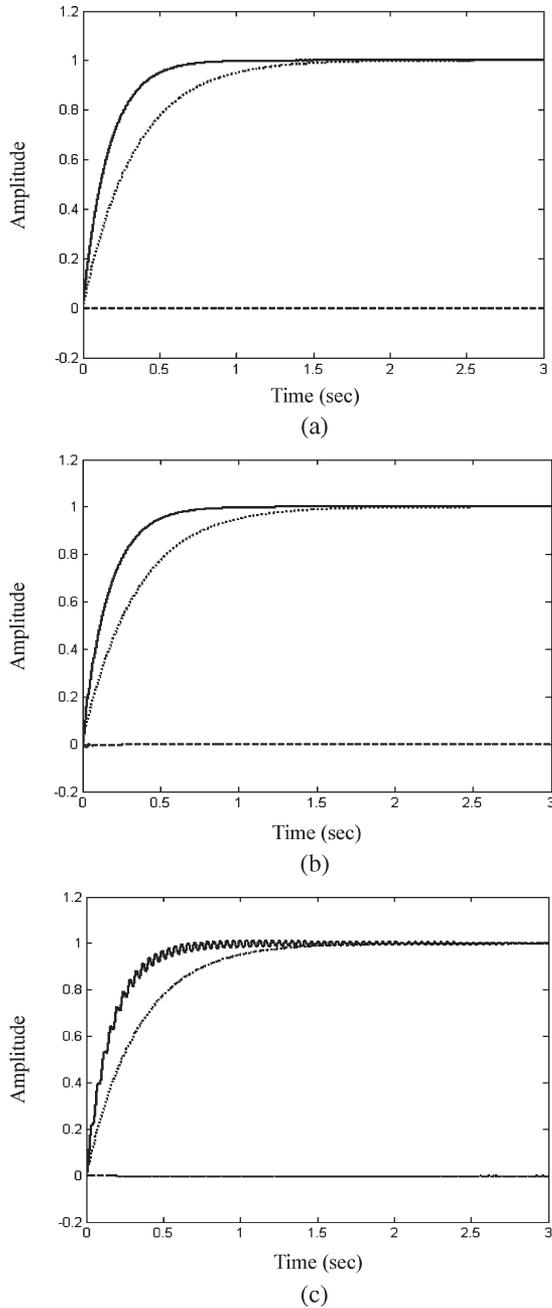


Fig. 6. System outputs  $y_1(t)$ ,  $y_2(t)$ , and  $y_3(t)$  of various PI controllers using the reference inputs  $r_1(t) = 0$ ,  $r_2(t) = 1 - e^{-3t}$ , and  $r_3(t) = 1 - e^{-6t}$ , respectively. (a) IMOSA with  $J_\infty = 0.6300$  and  $J_2 = 0.0473$ , (b) SPEA2 with  $J_\infty = 0.6738$  and  $J_2 = 0.0654$ , and (c) Chen [6] with  $J_\infty = 0.8194$ . [“- - -”:  $y_1(t)$ , “...”:  $y_2(t)$ , “—”:  $y_3(t)$ ].

$J_2 = f_3$ , and  $J = J_\infty + J_2$  were adopted to enable a convenient comparison of the performance of the proposed method with existing methods not based on the three-objective ap-

proach [6]–[9]. Fig. 5 shows the Pareto fronts of IMOSA (obtained from Table III) and SPEA2 by merging the results of ten independent runs and solutions of the single-objective methods of Chen and Cheng [6], Kitsios *et al.* [7], and the orthogonal simulated annealing algorithm [9], revealing that the nondominated solutions of IMOSA dominate all the solutions except for that of the orthogonal simulated-annealing-based method [9]. Table IV shows the best performances of various PI controllers in terms of  $J$ . The comparison results reveal that the orthogonal simulated annealing method and IMOSA perform well in designing controllers.

Table IV reveals that the robust stability and disturbance attenuation performances of the single-objective orthogonal simulated annealing method are similar to those of IMOSA. However, the value  $J_2 = 0.0473$  from the IMOSA-based method is larger than  $J_2 = 0.0374$  from the orthogonal simulated annealing method because of the concentration of computation effort on the single objective  $J_2$ .

The PI controller obtained from the IMOSA-based method with  $J = 0.6773$  is given as in (27), shown at the bottom of the page. Fig. 6 shows the system outputs of the IMOSA, SPEA2, and Chen [6] controllers with the best performance in terms of  $J$ . The system outputs of a well-designed controller with robust stability and disturbance attenuation should be the same with reference inputs. The simulation results indicate that the IMOSA-based method can provide a very good solution to the problem of designing three-objective optimal controllers with system uncertainty and disturbance.

### V. CONCLUSION

This paper has proposed an IMOSA and evaluated its performance by designing three-objective robust PID controllers for systems with uncertainties and disturbance. The optimal control design problem is formulated as an optimization problem with three minimization objectives: robust stability, disturbance attenuation, and tracking error. The IMOSA-based method without prior domain knowledge can efficiently solve the design problems of optimal control systems. A practical design example of a super-maneuverable fighter aircraft system is presented to illustrate the design procedure and to demonstrate the high performance of the proposed method. Experimental results show that the IMOSA-based method performs well in designing multiobjective optimal controllers.

IMOSA efficiently obtains a set of Pareto solutions from an initial solution, which can be used to solve various applications of multiobjective optimization problems with few interactions among parameters, particularly for obtaining a set of Pareto solutions by improving the existing single-objective solution treated as an initial solution of IMOSA. IMOSA also serves

$$C(s) = \begin{bmatrix} 3180.544 & 341.574 & -19910.276 \\ -63.017 & -258.445 & -19009.826 \\ 321.732 & 33.341 & 20000.000 \end{bmatrix} s + \begin{bmatrix} -1667.748 & -1992.274 & 1759.722 \\ 19845.673 & -19960.835 & -16556.570 \\ 17802.369 & 16240.564 & 19783.378 \end{bmatrix} \quad (27)$$

as an efficient local searcher exploiting the neighborhood of the existing single-objective solution. A future work combines IMOSA with single-objective evolutionary algorithms for intractable multiobjective optimization problems.

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