Effect of Temperature on Fracture Toughness of PC/ABS Based on J-Integral and Hysteresis Energy Methods

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SYNOPSIS
The critical fracture toughness $J_c$ of the polycarbonate (PC)/acrylonitrile–butadiene–styrene (ABS) blend at different temperatures was obtained from ASTM E813-81, E813-87, and the recently developed hysteresis energy methods, respectively. The $J_c$ value increases with increase of the test temperature ranging from -60 to 70°C. The hysteresis energy method and the ASTM E813-81 method result in comparable $J_c$ values, while the ASTM E813-87 results in about 80-110% higher values. The critical initiation displacements determined from the plots of hysteresis energy and the true crack growth length vs. crosshead displacement are very close. This indicates that the critical initiation displacement determined by the hysteresis method is indeed the displacement at the onset of true crack initiation and the corresponding $J_c$ represents a physical event of crack initiation. The fracture toughness, $K_{ic}$ value, based on linear elastic fracture mechanics (LEFM), was determined by using $K_Q$ analysis (ASTM E399-78), and the obtained $K_Q$ value decreases with the increase of the test temperature. The $K_Q$ value is not the real LEFM $K_{ic}$ value because the criterion of $P_{max}/P_Q < 1.1$ has not been satisfied. However, the corresponding $J_Q$ obtained from the $K_Q$ analysis is comparable to the $J_c$ obtained from the E813-81 method at lower temperature (~45 or ~60°C), an indication of LEFM behavior at lower temperature. The various schemes and size criterion based on LEFM and the J-method are explored for the validity of $J_c$ and $K_{ic}$ values. © 1996 John Wiley & Sons, Inc.

INTRODUCTION
The rapid development of critical applications of engineering plastics makes it desirable to have practical and reproducible measurements of fracture toughness that can be used in engineering design. Most of the research work on the fracture of plastic materials has so far centered around the linear elastic fracture mechanics (LEFM) approach, employing the elastic analysis of the crack tip region. Conventional fracture criteria, such as the $K_{ic}$ or $G_{ic}$ values, derived from linear elastic fracture mechanics analysis have been successfully applied to those relatively brittle polymers such as polystyrene (PS) and poly(methyl methacrylate) (PMMA). However, linear elastic fracture mechanics is not suitable for most rubber-toughened polymeric materials because the problem of extensive plasticity at the crack tip has precluded the application of the criteria. Besides, a relatively larger specimen thickness required for LEFM to induce plane strain conditions is inconvenient experimentally. The path-independent contour integral (J-integral) approach was proposed by Rice as a two-dimension energy line integral that can be used as an analytical tool to characterize the crack tip stress and strain field under both elastic and plastic stress and strain.1 Begley and Landes applied the J-integral principle and developed a measurement of fracture toughness, $J_{lc}$, which represents the energy required to initiate crack growth.2 Since then, two key ASTM standards, E813-81 and E813-87, have been established for J-testing mainly for metallic materials.4,5 These two ASTM E813 standards have been used to characterize toughened polymers and blends during last decade.6-20 Seidler and Grellmann studied the fracture behavior and morphologies of PC/ABS blends using a special technique, a stop block method.20 It
was shown that the PC/ABS blends exhibit a very different fracture behavior depending on temperature and ABS content. Narisawa and Takemori studied several rubber-modified polymers and raised questions about the validity of the crack blunting line equation since the blunting phenomenon was not being observed.21 Huang and Williams suspected that the crack face may close due to plasticity-induced crack closure, completely obscuring any blunting of the crack tip.22,23 Huang later studied the in situ SEM crack growth on rubber-toughened nylon 6,6 and did observe the crack blunting but the growing process was not identical to that proposed for metals.23 When a precrack specimen of a toughened polymer is under load, viscoelastic and inelastic micromechanisms such as craze, cavitation, debonding, and shear yielding may occur significantly around the crack tip region. These above-mentioned micromechanisms occur during the processes of crack tip blunting (precrack) and during crack propagation.24–28 The crack tends to propagate within the plastic zone and results in a stable crack extension for the rubber-toughened polymer materials. In our recent studies to determine the fracture toughness of the elastomer-toughened polycarbonate, acrylonitrile–butadiene–styrene (ABS), and high-impact polystyrene (HIPS), an unconventional approach on the J-integral based on the above-mentioned hysteresis properties was employed.24–28 The Jlc values obtained based on this hysteresis energy method are very close to those obtained from the ASTM E813-81 method but are significantly lower than those from the ASTM E813-87 method.

CHARACTERIZING PARAMETERS OF FRACTURE MECHANICS

The Linear Fracture Parameters \( K_{lc} \) and \( G_{ic} \)

The theory of LEFM deals with crack initiation occurring at nominal stresses that are well below the uniaxial yield stress of the material. A precrack specimen with little plastic deformation can be carried out by LEFM to measure the fracture toughness, \( K_{lc} \), which characterizes the elastic field around the crack tip. For a single-edge notched bending (SENB) specimen by loading monotonically, its \( K_{lc} \) is given by the \( K_Q \) analysis:\(^{29}\)

\[
K_Q = (P_Q S/BW^{3/2}) Y(a/W) \quad (1)
\]

where \( Y(a/W) \) is a geometrical correction factor, \( P_Q \) is the 5% slope offset gross applied stress, and \( a, B, \) and \( W \) are the initial crack length, thickness, and
width of the specimen, respectively. For an SENB specimen with \( S/W = 4 \), the geometrical correction factor is given by

\[
Y(a/W) = 3 \times (a/W)^{1/2} \left[ (1.99 - (a/W)(1 - a/W)) \times [2.15 - 3.93(a/W) + 2.7(a/W)^2] \right] / \left[ 2 \times [1 + 2(a/W)](1 - a/W)^{3/2} \right]
\]

(2)

\( K_{1c} \) is related to the strain energy release rate of the fracture \( G_{1c} \) by the following equation:

\[
K_{1c}^2 = E \times G_{1c} / (1 - \nu^2)
\]

(3)

where \( E \) is the elastic modulus and \( \nu \) is the Poisson's ratio.

The \( J \)-Integral

The fracture toughness based on the \( J \)-integral can be expressed as follows:

\[
J = \int \left[ Wdy - \dot{T} \times \frac{\partial U}{\partial x} ds \right]
\]

(4)

where the \( T \) is the surface traction; \( W \), the strain energy density; \( U \), the displacement vector; and \( x \) and \( y \) are the axis coordinates. Physically, the \( J \)-integral can be expressed in terms of energy as

\[
J = -dU/Bda
\]

(5)

Table 1  Summarized \( J \) Data for a Typical PC/ABS Blend at \( T = -15^\circ C \)

<table>
<thead>
<tr>
<th>( D ) (mm)</th>
<th>Input Energy ( J ) (kJ/m²)</th>
<th>( J ) (kJ/m²)</th>
<th>Hysteresis Ratio (%)</th>
<th>Hysteresis Energy ( J ) (kJ/m²)</th>
<th>( a ) (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>0.138</td>
<td>3.750</td>
<td>2.28</td>
<td>0.0031</td>
<td>0.030</td>
</tr>
<tr>
<td>1.2</td>
<td>0.155</td>
<td>4.212</td>
<td>3.62</td>
<td>0.0056</td>
<td>0.041</td>
</tr>
<tr>
<td>1.3</td>
<td>0.202</td>
<td>5.489</td>
<td>5.46</td>
<td>0.0110</td>
<td>0.056</td>
</tr>
<tr>
<td>1.5</td>
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<td>6.630</td>
<td>7.23</td>
<td>0.0176</td>
<td>0.151</td>
</tr>
<tr>
<td>1.6</td>
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<td>7.772</td>
<td>9.72</td>
<td>0.0278</td>
<td>0.212</td>
</tr>
<tr>
<td>1.7</td>
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<td>7.908</td>
<td>9.88</td>
<td>0.0288</td>
<td>0.283</td>
</tr>
<tr>
<td>1.8</td>
<td>0.345</td>
<td>9.375</td>
<td>13.79</td>
<td>0.0476</td>
<td>0.352</td>
</tr>
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<td>2.0</td>
<td>0.422</td>
<td>11.467</td>
<td>18.39</td>
<td>0.0776</td>
<td>0.485</td>
</tr>
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<td>2.2</td>
<td>0.503</td>
<td>13.668</td>
<td>22.06</td>
<td>0.1110</td>
<td>0.591</td>
</tr>
<tr>
<td>2.4</td>
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<td>16.049</td>
<td>26.05</td>
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</tr>
<tr>
<td>2.6</td>
<td>0.666</td>
<td>17.598</td>
<td>30.29</td>
<td>0.2017</td>
<td>0.820</td>
</tr>
<tr>
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<td>19.973</td>
<td>33.76</td>
<td>0.2481</td>
<td>0.991</td>
</tr>
<tr>
<td>2.8</td>
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<td>3.0</td>
<td>0.847</td>
<td>23.016</td>
<td>43.93</td>
<td>0.3721</td>
<td>1.224</td>
</tr>
<tr>
<td>3.2</td>
<td>0.950</td>
<td>25.815</td>
<td>47.51</td>
<td>0.4513</td>
<td>1.383</td>
</tr>
</tbody>
</table>

\( D \): deformation displacement. \( J = 2U/B \times b \). \( \Delta a \): measured crack growth length.
Table II  Critical $J$ from the ASTM E813-81 and the LEFM Compliance Methods

<table>
<thead>
<tr>
<th>Temp</th>
<th>$J_{1c}$ (kJ/m²)</th>
<th>$J_0$ (kJ/m²)</th>
<th>$J_i$ (kJ/m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>70°C</td>
<td>7.33</td>
<td>5.63</td>
<td>3.45</td>
</tr>
<tr>
<td>50°C</td>
<td>6.66</td>
<td>5.24</td>
<td>5.49</td>
</tr>
<tr>
<td>25°C</td>
<td>6.72</td>
<td>5.61</td>
<td>4.61</td>
</tr>
<tr>
<td>0°C</td>
<td>5.91</td>
<td>5.04</td>
<td>5.65</td>
</tr>
<tr>
<td>-15°C</td>
<td>4.78</td>
<td>4.07</td>
<td>4.10</td>
</tr>
<tr>
<td>-30°C</td>
<td>4.54</td>
<td>3.98</td>
<td>2.17</td>
</tr>
<tr>
<td>-45°C</td>
<td>3.83</td>
<td>3.37</td>
<td>3.87</td>
</tr>
<tr>
<td>-60°C</td>
<td>3.36</td>
<td>2.96</td>
<td>3.58</td>
</tr>
</tbody>
</table>


$U_e$ and $U_p$ are the elastic and plastic components of the total energy. $\eta_e$ and $\eta_p$ are their corresponding elastic and plastic work factors. $(W - a)$ is the ligament length, and $W$, the specimen width. For a three-point bend single-edge notched specimen with $a/W > 0.15$, $\eta_e$ is equal to 2. When the specimen has a span of $4W$ ($S = 4W$) and $0.4 < a/W < 0.6$, $\eta_e$ is equal to 2. Therefore, eq. (5) can be reduced to

$$J = 2 \times \frac{U}{B} \times b$$  (9)

The crack growth resistance ($R$ curve) is obtained by plotting the $J$ values against the corresponding $\Delta a$ values.

In the ASTM E813-81 standard, the critical $J$ value for crack initiation, $J_{1c}$, is determined by intersecting the linear regression $R$ curve and the crack blunting line. The blunting line can be expressed by the following equation:

$$J = 2 \times m \times \sigma_y \times \Delta a$$  (10)

where $\Delta a$ is the crack growth length, and $m$, a constraint factor ($m = 1$ for plane stress and $m = 2$ for plane strain). Two lines parallel to the crack blunting line at an offset of $0.006b$ and $0.06b$ (mm) are drawn, respectively, as the minimum and maximum crack extension lines. In the ASTM E813-81 standard, instead of a bilinear fit lines, the $J-\Delta a$ curve is then fitted by a power law with the following equations:

$$J = C_1 \times (\Delta a)^{C_2}$$  (11)

$$\ln J = \ln C_1 + C_2 \ln \Delta a$$  (12)

The critical $J$ value, $J_{1c}$, is now at the intersection of the power law fitted line and the 0.2 mm blunting offset line of the following equation:

$$J = 2\sigma_y \times \Delta a - 0.4\sigma_y$$  (13)

**$J-\Delta a$ Curve from LEFM Compliance Method**

Analysis of the $J-\Delta a$ data using the LEFM theory by assuming negligible plasticity occurring in a specimen was previously proposed. That means that all the nonlinearity in the load–displacement curve is due solely to the crack extension. In this case, the $R$ curve can be linearized by plotting the $J$ values against the corresponding $\Delta a$ values.
situation, the $\Delta a$ can be determined theoretically from a compliance measurement at any point along the load–displacement curve. The energy calibration factor, $\Phi$, in an elastic body is given by

$$\Phi = \frac{(C/W)/(dC/da)}{(1 - a/W)/\Phi}$$

where $C$ is the compliance of the body. For the SENB specimen with $S/W = 4$, the $(1 - a/W)/\Phi$ is equal to 2. The equation becomes

$$\frac{da}{(W - a)} = 0.5 \frac{dC}{C}$$

Integration of eq. (15) gives

$$\Delta a = (W - a_0)[1 - (C_0/C)^{1/2}]$$

where $C_0$ is the compliance at $a = a_0$.

**J-Integral According to Hysteresis Energy Method**

Hysteresis defined in this method is not exactly same as the conventional definition; it is the energy difference between the input and the recovery in the cyclic loading and unloading steps which may include crack blunting and crack extension stages. The close relation between the precrack hysteresis and the corresponding ductile–brittle transition behavior of polycarbonate and polyacetal was previously reported. When a precrack specimen is under loading before the onset of crack extension (during blunting), a significant portion of the input energy is consumed and converted into a relatively larger crack tip plastic zone for the toughened polymers. These viscoelastic and inelastic energies may include

The $J_{ic}$ Validity Requirements

For fracture toughness to be characterized as $J_{ic}$, a specimen must meet certain size requirements to achieve a plane strain stress state along the crack front. To achieve this stress state, all specimen dimensions must exceed some multiple of $J_{ic}/\sigma_y$. According to ASTM E813, a valid $J_{ic}$ value may be obtained whenever

$$B, \ (W - a), \ W > 25(J_{ic}/\sigma_y)$$

and

$$dJ/da < \sigma_y$$

**Figure 7** Plots of $J$ vs. $\Delta a$ according to LEFM compliance method at different temperatures.
where the slope of power law regression line of E813-87, \( dJ/da \), is evaluated at \( \Delta a = 0.2 \) mm. Paris and co-workers developed the tearing modulus concept to describe the stability of a ductile crack in term of elastic-plastic fracture mechanics. This fracture instability occurs if the elastic shortening of the system exceeds the corresponding plastic lengthening for crack extension. A nondimensional parameter, tearing modulus \( (T_m) \), has been defined as the following equation:

\[
T_m = (dJ/da) \times (E/\sigma_y^2)
\]

Furthermore, for the \( J-\Delta a \) data to be regarded as a material property independent of specimen size, the criterion \( \omega > 10 \) must be met, where \( \omega \) is defined as

\[
\omega = (W - a)/J_{lc} \times dJ/da
\]

Finally, for the plane strain linear elastic behavior, \( J_{lc} \) becomes identical to the critical strain energy release rate, \( G_{lc} \), which is, in turn, related to the stress intensity factor, \( K_{lc} \), used in linear elastic fracture mechanics:

\[
J_{lc} = G_{lc} = (1 - \nu^2)K_{lc}^2/E
\]

**EXPERIMENTAL**

The PC/ABS blend (Shinblend A783) was obtained from Shing-Kong Synthetic Fiberic Corp. of Taiwan. The tensile yield strength and Young’s modulus were measured by using the standard injection-molded specimens (\( \frac{1}{8} \) in.) with an extensometer. The Poisson’s ratio of the PC/ABS is assumed to be 0.35. Test specimens are the three-point bending bars with dimensions of \( B = 8 \) mm, \( W = 20 \) mm, and \( L \)

<table>
<thead>
<tr>
<th>Temp</th>
<th>( 70^\circ C )</th>
<th>( 50^\circ C )</th>
<th>( 24^\circ C )</th>
<th>( 0^\circ C )</th>
<th>( -15^\circ C )</th>
<th>( -30^\circ C )</th>
<th>( -45^\circ C )</th>
<th>( -60^\circ C )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_1 )</td>
<td>1689</td>
<td>573</td>
<td>279</td>
<td>498</td>
<td>408</td>
<td>2043</td>
<td>1088</td>
<td>1099</td>
</tr>
<tr>
<td>( C_2 )</td>
<td>0.629</td>
<td>0.496</td>
<td>0.399</td>
<td>0.476</td>
<td>0.456</td>
<td>0.683</td>
<td>0.599</td>
<td>0.602</td>
</tr>
<tr>
<td>( J_{lc} ) (kJ/m²)</td>
<td>12.15</td>
<td>11.88</td>
<td>11.32</td>
<td>11.2</td>
<td>10.01</td>
<td>7.30</td>
<td>7.92</td>
<td>7.73</td>
</tr>
<tr>
<td>( J_0 ) (kJ/m²)</td>
<td>8.83</td>
<td>8.35</td>
<td>9.42</td>
<td>8.79</td>
<td>7.73</td>
<td>7.30</td>
<td>5.27</td>
<td>5.33</td>
</tr>
</tbody>
</table>

\( J_{lc} \): standard ASTM E813-87 method. \( J_0 \): modified version of E813-87 by using 0.1 mm offset line.
RESULTS AND DISCUSSION

$J_{\text{lc}}$: Determination by ASTM Standards and the Modified Versions

Figure 1 shows the plot of tensile yield strength vs. temperature of the PC/ABS blend, where the yield strength decreases with the increase of temperature. The load-displacement curves of the notched specimens of the PC/ABS blend at different temperatures are shown in Figure 3. The specimen at lower temperature has a considerably greater maximum load ($P_{\text{max}}$) than that at higher temperature due mainly to the higher modulus and higher yield stress. The $J$ value for each specimen is calculated from eq. (9), and the corresponding crack growth, $\Delta a$, is measured from the fracture surface of the broken specimen. Detailed data of the specimens measured at $T = -15^\circ\text{C}$ are summarized in Table I. Figure 4 shows the plots of the acceptable $J$ vs. $\Delta a$ by linear regression $R$ curves according to the ASTM E813-81 method from a few selected temperatures. The linear regression $R$ curves intercept with the corresponding blunting lines [eq. (10)] to locate the $J_{\text{tr}}$ values. Another critical $J$ value ($J_0$) is determined at the
Table IV Critical Displacement and Critical $J$ from the Hysteresis Energy Method

<table>
<thead>
<tr>
<th>Temp</th>
<th>$D_c$ (mm)</th>
<th>$J_{1c}$ (kJ/m²)</th>
<th>$D_a$ (mm)</th>
<th>$J_{ap}$ (kJ/m²)</th>
<th>$D_{ca}$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>70°C</td>
<td>1.76</td>
<td>7.61</td>
<td>1.67</td>
<td>6.98</td>
<td>1.62</td>
</tr>
<tr>
<td>50°C</td>
<td>1.80</td>
<td>8.38</td>
<td>1.59</td>
<td>6.96</td>
<td>1.62</td>
</tr>
<tr>
<td>25°C</td>
<td>1.67</td>
<td>7.55</td>
<td>1.51</td>
<td>6.33</td>
<td>1.51</td>
</tr>
<tr>
<td>0°C</td>
<td>1.59</td>
<td>7.54</td>
<td>1.42</td>
<td>6.16</td>
<td>1.38</td>
</tr>
<tr>
<td>-15°C</td>
<td>1.52</td>
<td>7.07</td>
<td>1.33</td>
<td>5.55</td>
<td>1.30</td>
</tr>
<tr>
<td>-30°C</td>
<td>1.41</td>
<td>6.10</td>
<td>1.26</td>
<td>5.33</td>
<td>1.18</td>
</tr>
<tr>
<td>-45°C</td>
<td>1.41</td>
<td>6.58</td>
<td>1.18</td>
<td>5.17</td>
<td>--</td>
</tr>
<tr>
<td>-60°C</td>
<td>1.21</td>
<td>4.38</td>
<td>--</td>
<td>5.13</td>
<td>--</td>
</tr>
</tbody>
</table>

$D_c$: critical initial displacement. $k$: from the plot of hysteresis energy vs. displacement by using the linear propagation line. $p$: from the plot of hysteresis energy vs. displacement by using the power law propagation line. $a$: from the plot of the measured crack growth length vs. displacement.

Figure 7 shows the plots of the $\Delta a$ calculated from eq. (16) vs. corresponding $J$ values at different temperatures. The calculated $\Delta a$ value from eq. (16) is significantly higher than the actually measured $\Delta a$ value from the fractured surface. The cause of the observed difference is due to the neglect of the plasticity under testing. Figure 8 shows the plots of $dJ/da$ from the ASTM E813-81, E813-87, and the LEFM compliance methods vs. temperature. The obtained $dJ/da$ values from the LEFM compliance method are substantially lower than those obtained from the ASTM E813-81 except at lower temperatures ($T = -45°C$ and $T = -60°C$). The fracture behavior of the rubber-toughened polymer is gradually approaching to meet the LEFM conditions at low temperatures, near or below the $T_g$ of the rubber.

Figure 13 Plot of the truly crack growth length ($\Delta a$) vs. displacement at $T = -15°C$.
method attributes all the nonlinearity to the elastic crack extension and neglects the occurrence of the inelasticity or the plasticity.

In ASTM E813-87 method, the $J_{lc}$ is located at the intercept between the power law fit line and the 0.2 mm offset line as shown in Figure 9 at $T = -15^\circ$C. The values of $C_1$ and $C_2$ of the power law regression line, $J = C_1 \times \Delta a^{C_2}$, within 0.15 and 1.5 mm exclusion lines, and the corresponding $J_{lc}$ values at different temperatures ranging from -60 to 70°C are summarized in Table III. The $J_{lc}$ values obtained from the E813-87 method are about 80-110% higher than those from the corresponding E813-81 method (Table II). The critical $J_{lc}$ definition has been and still is a confusing and controversial issue, whether it is treated as crack initiation (E813-81) or is simply an engineering definition for design purpose (E813-87). Only very limited comparative $J_{lc}$ data between these two ASTM standards (E813-81 and E813-87) on polymeric materials have been previously reported. Huang recently reported that the $J_{lc}$ from the E813-87 method for the rubber-toughened nylon 6,6 is significantly higher than that from the E813-81 method (38 vs. 15 kJ/m$^2$). We also found that the $J_{lc}$ values obtained from the E813-87 method for the elastomer-modified polycarbonate and high-impact polystyrene are about 20-40% higher than those from the E813-81 method. However, if the 0.2 mm offset line specified in E813-87 is now reset at 0.1 mm and the rest of the procedures remain

---

**Table V The Size Criterion Requirements for Valid $J_{lc}$ Value**

<table>
<thead>
<tr>
<th>Temp</th>
<th>70°C</th>
<th>50°C</th>
<th>25°C</th>
<th>0°C</th>
<th>-15°C</th>
<th>-30°C</th>
<th>-45°C</th>
<th>-60°C</th>
</tr>
</thead>
<tbody>
<tr>
<td>$dJ/da$</td>
<td>15.22</td>
<td>14.51</td>
<td>15.97</td>
<td>16.33</td>
<td>15.73</td>
<td>13.09</td>
<td>15.56</td>
<td>15.79</td>
</tr>
<tr>
<td>$25(J_{lc}/\sigma_y)$</td>
<td>5.47</td>
<td>4.28</td>
<td>3.68</td>
<td>2.80</td>
<td>2.18</td>
<td>2.03</td>
<td>1.67</td>
<td>1.41</td>
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<tr>
<td>$T_n$</td>
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<td>15.19</td>
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<td>10.94</td>
<td>9.37</td>
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<td>$\omega$</td>
<td>20.76</td>
<td>21.78</td>
<td>23.76</td>
<td>27.63</td>
<td>32.90</td>
<td>28.70</td>
<td>40.62</td>
<td>46.90</td>
</tr>
<tr>
<td>$\omega &gt; 10$</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Plane strain</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>

$T_n = E/\sigma_y^2 \times dJ/da, \omega = (W - \alpha)/J_{lc} \times dJ/da$.

---

**Table VI The $K_Q$ Analysis Based on ASTM E399-78 Method**

<table>
<thead>
<tr>
<th>Temp (°C)</th>
<th>$P_Q$ (kN)</th>
<th>$P_{max}$ (kN)</th>
<th>$P_{max}/P_Q$</th>
<th>$K_Q$ (Mpa $\times a_0^{1/2}$)</th>
<th>$J_Q$ (kJ/m$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>70</td>
<td>0.2282</td>
<td>0.3418</td>
<td>1.50</td>
<td>2.15</td>
<td>2.24</td>
</tr>
<tr>
<td>50</td>
<td>0.2512</td>
<td>0.3747</td>
<td>1.49</td>
<td>2.36</td>
<td>2.64</td>
</tr>
<tr>
<td>25</td>
<td>0.2608</td>
<td>0.3902</td>
<td>1.49</td>
<td>2.45</td>
<td>2.66</td>
</tr>
<tr>
<td>0</td>
<td>0.2786</td>
<td>0.4405</td>
<td>1.45</td>
<td>2.61</td>
<td>2.92</td>
</tr>
<tr>
<td>-15</td>
<td>0.2939</td>
<td>0.4265</td>
<td>1.45</td>
<td>2.76</td>
<td>3.11</td>
</tr>
<tr>
<td>-30</td>
<td>0.3020</td>
<td>0.4261</td>
<td>1.41</td>
<td>2.84</td>
<td>3.18</td>
</tr>
<tr>
<td>-45</td>
<td>0.3187</td>
<td>0.4452</td>
<td>1.39</td>
<td>2.99</td>
<td>3.32</td>
</tr>
<tr>
<td>-60</td>
<td>0.3280</td>
<td>0.4615</td>
<td>1.37</td>
<td>3.08</td>
<td>3.31</td>
</tr>
</tbody>
</table>

$P_Q$: the load at 5% slope offset line. $P_{max}$: the maximum load. $K_Q = K_Q = (P_QS/BW^{0.5}) \times (\alpha/W)$. $J_Q = J_Q = (1 - \rho^2) K_Q^2/E$, the $K_Q$ corresponding $J$ value. 
unchanged as shown in Figure 9, the critical \( J_0 \) values obtained from the E813-87 modified method are now only slightly higher than those obtained from the E813-81 method (Table II). Similar results were also obtained from elastomer-modified polycarbonate,\(^{24}\) high-impact polystyrene,\(^ {26}\) and acrylonitrile-butadiene-styrene.\(^ {25}\) After all, the 0.2 mm offset line suggested in the ASTM E813-87 standard is only an arbitrarily selected value to define the critical fracture toughness \( (J_{IC}) \). Figure 10 shows the plots of the \( J_{IC} \) from the E813-87 method and the \( J_0 \) from the E813-87 modified method vs. temperature; both \( J_{IC} \) and \( J_0 \) increase with increase of the temperature.

\( J_{IC} \) Determination from the Hysteresis Method

The hysteresis ratio and the corresponding hysteresis energy of each specimen at different displacements are summarized in Table I for \( T = -15^\circ C \). Figure 11 shows the plot of the hysteresis energy vs. crosshead displacement at \( T = -15^\circ C \). The critical initiation displacement is located at the intersection between the blunting line and the linear crack propagation line, namely, \( D_\alpha \) (the validity crosshead displacement data window used with its corresponding crack growth length, \( \Delta a \), ranging from 0.10 to 0.80 mm) or the second-order power law crack propagation line, namely, \( D_{cp} \) (the validity crosshead displacement data window is used with its corresponding crack growth length, \( \Delta a \), ranging from 0.10 to 1.50 mm). As soon as the \( D_\alpha \) and \( D_{cp} \) are located, their corresponding \( J_{IC} \) values (namely, \( J_{IC} \) and \( J_{ICP} \)) are then determined from the plot of the \( J \) vs. crosshead displacement curve as shown in Figure 12, respectively. Because the measurement of crack growth length \( (\Delta a) \) is no longer necessary by this hysteresis energy method, it is relatively easier than the conventional ASTM E813 methods. The \( J_{IC} \) values, \( (J_{IC} \) and \( J_{ICP} \)) obtained from the hysteresis energy method are very close to those from the E813-81 method (Table II), but lower (60–80%) than those from the E813-87 method (Table III). The critical \( J_{ICP} \) values are relatively closer to the \( J_{IC} \) values obtained from the E813-81 method than are the \( J_{IC} \) values. Therefore, the critical initiation displacement \( (D_{cp}) \) is probably better related to the true crack initiation than is \( D_\alpha \). Our previous articles used \( D_\alpha \) instead of \( D_{cp} \) in determining \( J_{IC} \).\(^ {24-27} \) The obtained \( D_\alpha, D_{cp}, J_{ICP}, \) and \( J_{ICP} \) values are summarized in Table IV. The critical \( J \) value determined by this unconventional hysteresis energy method has its physical meaning as the onset of crack extension rather than that based on the theoretically predicted blunting line as in the ASTM E813-81 method or that based on an arbitrarily chosen engineering definition as in the ASTM E813-87 method. A plot of the experimentally measured crack growth length \( (\Delta a) \) vs. crosshead displacement is shown in Figure 13 at \( T = -15^\circ C \). The critical initiation displacement is now located at the intersection of these two linear regression lines. The critical initiation displacements, \( D_{\alpha} \)'s, obtained from these two methods, the hysteresis energy and the crack growth length vs. displacement, are fairly close (Table IV). This coincident result implies that the critical initiation displacement determined from the hysteresis method is indeed the true onset of crack extension. Figure 14 shows the plots of the hysteresis energy vs. corresponding crack growth length, \( \Delta a \), from a few selected temperatures. The slope from the curve increases with the increase of temperature, which indicates that additional energy is required to extend the same crack growth length at higher temperature than at lower temperature in the forms of more craze and more localized shear yielding during crack growth. Therefore, more energy in terms of hysteresis energy is required for crack growth at higher temperature.

The Size Criterion of Specimens

The \( dJ/da \) values obtained according to the linear regression \( R \) curves of ASTM E813-81 and the power law regression curve of ASTM E813-87 at \( \Delta a = 0.2 \) mm are summarized in Figure 8 and Table V. The \( dJ/da \) value obtained from the E813-87 method increases gradually with increase of temperature while the \( dJ/da \) value from the E813-81 method is fairly temperature-independent. Overall, the \( dJ/da \) values from the E813-87 method are about 30–60% higher.
than those from the E813-81 method. Tearing modulus, $T_m$, is used to describe the stability of the crack growth. Table V shows that the tearing modulus $T_m$ value increases with the increase of temperature which indicates greater crack propagation resistance at higher temperature. For the $J$--$\Delta a$ data to be regarded as an intrinsic material property independent of specimen size, the critical parameter of $\omega > 10$ must be met. In this study, all specimen dimensions employed meet the ASTM size criterion, $\omega > 10$, and the obtained $\omega$ parameter increases with decrease of temperature as shown in Table V. Other size requirements such as those of eq. (17) have also been satisfied. The size criteria produce a plastic plane-strain stress condition at the crack front and allow for the use of significantly smaller specimen dimensions than those required for LEFM testing. Table V summarizes the different size criterion values and essentially all these criteria meet requirements of the validity $J_{ic}$ test in this study.

The $K_Q$ Analysis

A typical load--displacement curve for a precrack bending specimen at different temperatures is shown in Figure 3. The load $P_Q$ is obtained from the 5% secant offset of the load--displacement curve (ASTM E399-78) which is used to calculate $K_Q$. Now, ASTM D5045 for plastics has replaced ASTM E399 for metallic materials, but ASTM E399 was the one discussed in this article. If the amount of the plastic development is small at the running crack tip and the rate of the crack growth above $P_Q$ is rapid, the corresponding $K_Q$ is considered as a validity $K_{ic}$. For the criterion of a validity $K_Q$ as a $K_{ic}$, the following relationships must be met:

\[ B, \ a, \ (W - a) > 2.5(K_Q/\sigma_y)^2 \]  
\[ P_{\text{max}}/P_Q < 1.10 \]  

where $P_{\text{max}}$ is the maximum load on the load--displacement curve and $\sigma_y$ is the yield strength. Table VI summarizes the $K_Q$ and the corresponding $J_Q$ values, which are determined by using eq. (21). Since all the $P_{\text{max}}/P_Q$ values obtained do not meet the criterion of $P_{\text{max}}/P_Q < 1.1$ at temperatures ranging from $-60$ to $70^\circ$C, the corresponding $K_Q$ values obtained cannot be considered as true $K_{ic}$ values of LEFM. Figure 15 shows the plots of the $K_Q$, $J_Q$ (from the ASTM E399-78) and $J_{ic}$ (from the E813-81) vs. temperature. Both $K_Q$ and $J_Q$ decrease with the increase of the temperature, while the corresponding $J_{ic}$ increases with increase of temperature. At low temperature ($-60^\circ$C), $J_{ic}$ and $J_Q$ are nearly identical, an indication of LEFM behavior.

CONCLUSION

The $J_{ic}$ values obtained from the hysteresis energy method using the critical initiation displacement $D_{ic}$’s are comparable to those obtained from the E813-81 method but are about 70–90% lower than those obtained from the E813-87 method. All these critical $J$ values obtained from ASTM E813-81, E813-87, and hysteresis energy methods increase with increase of temperature. From the ASTM E399-78 method, the $K_Q$ values are not the real LEFM $K_{ic}$ values because the $P_{\text{max}}/P_Q < 1.1$ criterion is not met. However, the corresponding $J_Q$ values are comparable to those obtained from the E813-81 method at lower temperatures, an indication of LEFM behavior. The specimen geometry of this study essentially meets all the size criterion, and, therefore, the $J_{ic}$’s obtained are considered to be valid. The hysteresis energy method inherently adjusts for the occurrence of the crack blunting and thus avoids the controversy of the blunting line issue. Besides, it is simple without the requirement of the tedious crack growth length measurements.

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REFERENCES


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