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Appendix A: The extracting processes of the guiding properties, $Z_c$, loss, and SWF for a finite length TL

To obtain the complex propagation constant $\gamma$ from the two-port scattering parameters of the through line, the complex characteristic impedance must be obtained beforehand in this dissertation. This can be achieved by respectively adding an open load and a short load at the output port of the microstrip [61]. The input impedance with open and short loads at the output port can be expressed as

$$Z_{in,open} = Z_c \cdot \coth \gamma \ell$$  \hspace{1cm} (A-1)

$$Z_{in,short} = Z_c \cdot \tanh \gamma \ell$$  \hspace{1cm} (A-2)

$Z_c$ is the complex characteristic impedance of the transmission line with complex propagation constant $\gamma$ and length $\ell$.

Let $Z_o$ be the reference impedance level, normally $50 \ \Omega$ in a typical S-parameter measurement system, the two-port scattering matrix $S$ can be related to $Z_{in,open}$ and $Z_{in,short}$ by the following expressions[61],

$$Z_{in,open} = Z_o \cdot \frac{1 + S_{11}'(\infty)}{1 - S_{11}'(\infty)}$$  \hspace{1cm} (A-3)
\[ Z_{in,short} = Z_o \cdot \frac{1 + S_{11}'(0)}{1 - S_{11}'(0)} \]  \hspace{1cm} (A-4)

where

\[ S_{11}'(\infty) = S_{11} + \frac{S_{12}S_{21}}{1 - S_{22}} \]  \hspace{1cm} (A-5)

\[ S_{11}'(0) = S_{11} - \frac{S_{12}S_{21}}{1 + S_{22}} \]  \hspace{1cm} (A-6)

\[ S_{11}, S_{12}, S_{21} \text{ and } S_{22} \text{ are the elements of the two-port } S \text{-parameters referenced to } Z_o. \text{ It should be noticed that } S_{11} = S_{22} \text{ and } S_{12} = S_{21} \text{ for symmetrical and passive circuits.} \]

Substituting equations (A-3), (A-4), (A-5) and (A-6) into equations (A-1) and (A-2), the complex characteristic impedance \( Z_c \) can be solved as follows:

\[ Z_c = Z_o \cdot \frac{1 + S_{11}'(\infty) \cdot 1 + S_{11}'(0)}{1 - S_{11}'(\infty) \cdot 1 - S_{11}'(0)} \]  \hspace{1cm} (A-7)

For symmetrical and passive circuits, the \( S_{11} = S_{22} \text{ and } S_{12} = S_{21} \) substitute into Eq. (A-5)-(A-7), then we can get the simplified formula of \( Z_c \) that shows as (3.6).

The scattering matrix \( S \) is then re-normalized to \( Z_c^* \) and \( Z_c \) for the input port-1 and the output port-2, respectively [62], where the \( Z_c^* \) is the conjugate of
the \( Z_c \), and the resultant scattering matrix
\[
\begin{bmatrix}
S_{p11} & S_{p12} \\
S_{p21} & S_{p22}
\end{bmatrix}
\] should have the following characteristics

\[
S_{p11} = 0 \quad (A-8)
\]

\[
S_{p21} = e^{-\gamma \ell} \quad (A-9)
\]

The detail derivation of Eq. (A-8) and Eq. (A-9) will be given in the Appendix B of this dissertation, assuming that the guiding transmission line under investigation supports only a bound mode with characteristic impedance \( Z_c \) and propagation constant \( \gamma \). After invoking the complex impedance re-normalization process, the complex propagation constant \( \gamma \) (\( \gamma = \alpha + j \beta \)) becomes [8]

\[
\alpha / k_o = \left( -\ln \left| S_{p21} \right| / \ell \right) / k_o \quad (A-10)
\]

\[
\beta / k_o = \sqrt{\varepsilon_{r, \text{eff}}} = \text{arg}(S_{p21}) \cdot \frac{c}{2\pi \ell} \quad (A-11)
\]

Where \( \alpha \) is the attenuation constant (unit = Np/m), \( k_o \) is the free-space wave number (\( = \frac{2\pi}{\lambda_o} \) or \( \frac{2\pi f}{c} \)), \( \left| S_{p21} \right| \) is the magnitude of \( S_{p21} \), \( \ell \) is the length of the equivalent transmission line (unit = m), \( \beta \) is the phase constant (unit = rad/m),
\( \varepsilon_{r, eff} \) is the effective relative dielectric constant of the transmission line, \( \arg(S_{p21}) \) is the total delay phase of \( S_{p21} \) (unit = rad), \( c \) is the speed of light (\( \approx 3 \times 10^8 \) m/sec) and \( f \) is the operating frequency (unit = Hz).

The guiding properties of a transmission line can be fully characterized by impedance characteristic \( (Z_c(f)) \) and dispersion characteristic \( (\gamma(f) = \alpha(f) + j \beta(f)) \). However, the relative parameters of loss per guided-wavelength \( (Loss(dB/\lambda_g)) \) and slow-wave factor \( (SWF = \lambda_0/\lambda_g = \beta/k_0) \) bear more meaning for slow-wave structures. The characteristic impedance, loss per guided-wavelength, and slow-wave factor can be obtained from Eqs. (A-7), (A-10) and (A-11). Moreover, \( Loss(dB/\lambda_g) \) and \( SWF \) can also be expressed as follows:

\[
Loss(dB/\lambda_g) = 20 \cdot \log_{10}(e^{-\alpha \lambda_g}) = 10^\ell \cdot Loss(dB/mm) \cdot \lambda_g
\]  

(A-12)

\[
SWF = \frac{\lambda_0}{\lambda_g} = \frac{\beta}{k_0} = \sqrt{\varepsilon_{r, eff}}
\]  

(A-13)

where \( \alpha \) is the attenuation constant (unit = \( 1/m \)), \( \lambda_g \) is the guided-wavelength (unit = \( m \)), and \( Loss(dB/mm) = 20 \cdot \log_{10}(e^{-0.001: \alpha}) \).
The guiding properties (\(Z_c\), SWF, and loss) are extracted directing from the S-parameter of the simulated or measured DUT transmission line (MS, CCS,...). The extracting processes can be summarized as follows:

**Step1**: properly choose the length \(\ell\) of transmission line, that \(\ell < 0.25\lambda_g\) is suggested, else the extracting parameters will be caused by large disturbance near those frequencies where the line length \(\ell = \frac{n}{2}\cdot\lambda_g\), \(n = 1, 2, 3, \text{etc.}\), and \(\lambda_g\) is the guided-wavelength of transmission line. The spurious resonance and numerical error are severely and obviously as the line length \(\ell\) near to a multiple of half-guided-wavelength. The numerical method to reduce the error will be listed in Appendix C.

**Step2**: to get its two-port scattering parameters \([S]\) reference to \(Z_o\), normally 50-\(\Omega\), by simulation or measurement.

**Step3**: at first, to decide the characteristic impedance of the transmission line, \(Z_c\), that is derived from the input impedance with open and short loads at the output port, the final expression is

\[
Z_c = \sqrt{Z_{\text{in,open}} \cdot Z_{\text{in,short}}} = Z_o \cdot \sqrt{\frac{1 + S_{11}'(\infty)}{1 - S_{11}'(\infty)} \cdot \frac{1 + S_{11}'(0)}{1 - S_{11}'(0)}}
\]

Where \(S_{11}'(\infty) = S_{11} + \frac{S_{12}S_{21}}{1-S_{22}}\) and \(S_{11}'(0) = S_{11} - \frac{S_{12}S_{21}}{1+S_{22}}\),

the \(S_{11} = S_{22}\) and \(S_{12} = S_{21}\) for symmetrical and passive circuits,

**Step4**: to perform the re-normalized process, to the scattering matrix \([S]\), that is
re-normalized to $Z_c^*$ and $Z_c$ for the port-1 and port-2, respectively.

The resultant scattering matrix $S_p$ is

$$
\begin{bmatrix}
S_{p11} & S_{p12} \\
S_{p21} & S_{p22}
\end{bmatrix}
$$

where $S_{p11} = 0$, $S_{p21} = e^{-\gamma l}$ and $\gamma = \alpha + j \beta$, $\gamma$: propagation constant, $\alpha$: attenuation constant, $\beta$: phase constant,

**Step 5:** to extract the normalized propagation constant $\gamma / k_o (\alpha / k_o, \beta / k_o)$ from the $S_{p21}$,

$$
\frac{\alpha}{k_o} = \left( -\frac{\ln |S_{p21}|}{l} \right) / k_o
$$

$$
\frac{\beta}{k_o} = \sqrt{\varepsilon_{r,\text{eff}}} = \arg(S_{p21}) / 2\pi
$$

where $|S_{p21}|$ is the insertion loss and $\arg(S_{p21})$ is the delay-phase of transmission line, at the condition, $Z_s = Z_c^*$ for the input port-1 and $Z_L = Z_c$ for the output port-2.

**Step 6:** to extract the **Loss (dB/λ_g)** and **SWF** from the $\alpha$, $\lambda_g$, and $\beta / k_o$,

$$
\text{Loss}(dB / \lambda_g) = 20 \cdot \log_{10}(e^{-\alpha \lambda_g}) = 10 \cdot \text{Loss}(dB / \text{mm}) \cdot \lambda_g
$$

$$
\text{SWF} = \frac{\lambda_0}{\lambda_g} = \frac{\beta}{k_o} = \sqrt{\varepsilon_{r,\text{eff}}}
$$
Appendix B: The derivation of the power-wave scattering parameter matrix, $S_p$, for a finite length TL with complex reference impedances $Z_1$ and $Z_2$

The propagation characteristics of the finite length transmission line can be characterized by $Z_c$ (complex characteristic impedance), $\gamma (= \alpha + j \beta$, complex propagation constant, where $\alpha$ is the attenuation constant and $\beta$ is the phase constant), and $\ell$ (length of the transmission line) using two-port transmission line circuit model, $\begin{bmatrix} A & B \\ C & D \end{bmatrix}$ matrix.

The equivalent circuit of this model is shown as follows,

![Equivalent circuit of two-port TL](image)

Fig. B-1. The equivalent circuit model of the two-port TL.

Where $I_1$, $V_1$ and $I_2$, $V_2$ are expressed as terminated current and voltage at port-1 and port-2, respectively.
Equation (B-1) formulates the relation between these 2 ports [61], [63],

\[
\begin{bmatrix}
V_1 \\
I_1
\end{bmatrix} = \begin{bmatrix} A & B \\
C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} 
\] (B-1)

Where \( \begin{bmatrix} A & B \\
C & D \end{bmatrix} \) is equal to \( \begin{bmatrix} \cosh \gamma \ell & Z_c \cdot \sinh \gamma \ell \\
\frac{1}{Z_c} \cdot \sinh \gamma \ell & \cosh \gamma \ell \end{bmatrix} \).

The input impedance \( Z_{in}(=V_1/I_1) \) can be derived from Eq. (B-1) as follows,

\[
Z_{in} = Z_c \cdot \frac{Z_2 \cdot \cosh \gamma \ell + Z_c \cdot \sinh \gamma \ell}{Z_c \cdot \cosh \gamma \ell + Z_2 \cdot \sinh \gamma \ell} \] (B-2)

Where \( Z_2 \) is equal to \( (V_2/(-I_2)) \). The input impedance \( Z_{in}(=V_1/I_1) \) is equal to \( Z_c \) as \( Z_2(=V_2/(-I_2)) \) is equal to \( Z_c \).

From Eq. (B-1) and Fig. (B-1) we have,

\[
\begin{align*}
V_1 &= AV_2 - BI_2 \\
I_1 &= CV_2 - DI_2 \\
V_1 &= -I_1Z_1 \\
V_2 &= -I_2Z_2
\end{align*} \] (B-3, B-4, B-5, B-6)
On the other hand, the definition of the incident and reflected power waves $a_i$ and $b_i$, $i=1, 2$ for port-1, port-2, indicates that [62]

\[
a_i = \frac{V_i + Z_i \cdot I_i}{2 \cdot \sqrt{|\text{Re}(Z_i)|}} = \frac{V_i + Z_i \cdot I_i}{2 \cdot \sqrt{\text{Re}(Z_i)}}
\]

(B-7)

\[
b_i = \frac{V_i - Z_i^* \cdot I_i}{2 \cdot \sqrt{|\text{Re}(Z_i)|}} = \frac{V_i - Z_i^* \cdot I_i}{2 \cdot \sqrt{\text{Re}(Z_i)}}
\]

(B-8)

Where $|\text{Re}(Z_i)|$ is the magnitude of the real part of $Z_i$, $\hat{Z}_i$ is defined as the square root of $|\text{Re}(Z_i)|$, and $Z_i^*$ is the conjugate of $Z_i$.

From Eq. (B-7) and Eq. (B-8), we can solve $V_1$ and $I_1$ in terms of $a_1$ and $b_1$ for port-1 by the following formula,

\[
V_1 = \frac{(Z_1^* \cdot a_i + Z_i \cdot b_i) \cdot \hat{Z}_i}{\text{Re}(Z_1)}
\]

(B-9)

\[
I_1 = \frac{(a_1 - b_1) \cdot \hat{Z}_i}{\text{Re}(Z_1)}
\]

(B-10)

Similarly we can solve $V_2$ and $(-I_2)$ in terms of $a_2$ and $b_2$ for port-2 by the
following formula,

\[ V_2 = \frac{(Z_2^\ast \cdot a_2 + Z_3 \cdot b_2) \cdot \hat{Z}_2}{\text{Re}(Z_2)} \]  \hspace{1cm} (B-11)

\[-I_2 = -\frac{(a_2 - b_2) \cdot \hat{Z}_2}{\text{Re}(Z_2)} \]  \hspace{1cm} (B-12)

Equation (B-9) to (B-12) can be rearranged in matrix form as follows,

\[
\begin{bmatrix}
V_1 \\
I_1
\end{bmatrix} = \begin{bmatrix}
\frac{Z_1^\ast \cdot \hat{Z}_1}{\text{Re}(Z_1)} & \frac{Z_1 \cdot \hat{Z}_1}{\text{Re}(Z_1)} \\
\hat{Z}_1 & -\hat{Z}_1
\end{bmatrix} \cdot \begin{bmatrix}
a_1 \\
b_1
\end{bmatrix} \equiv U_1 \cdot \begin{bmatrix}
a_1 \\
b_1
\end{bmatrix} \tag{B-13}
\]

\[
\begin{bmatrix}
V_2 \\
-I_2
\end{bmatrix} = \begin{bmatrix}
\frac{Z_2^\ast \cdot \hat{Z}_2}{\text{Re}(Z_2)} & \frac{Z_2 \cdot \hat{Z}_2}{\text{Re}(Z_2)} \\
\hat{Z}_2 & \hat{Z}_2
\end{bmatrix} \cdot \begin{bmatrix}
a_2 \\
b_2
\end{bmatrix} \equiv U_2 \cdot \begin{bmatrix}
a_2 \\
b_2
\end{bmatrix} \tag{B-14}
\]

Substituting Eq. (B-13) and (B-14) into Eq. (B-1), we have

\[
U_1 \cdot \begin{bmatrix}
a_1 \\
b_1
\end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \cdot U_2 \cdot \begin{bmatrix}
a_2 \\
b_2
\end{bmatrix} \tag{B-15}
\]

\[
\Rightarrow \begin{bmatrix}
a_1 \\
b_1
\end{bmatrix} = U_1^{-1} \cdot \begin{bmatrix} A & B \\ C & D \end{bmatrix} \cdot U_2 \cdot \begin{bmatrix}
a_2 \\
b_2
\end{bmatrix} \equiv \begin{bmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{bmatrix} \cdot \begin{bmatrix}
a_2 \\
b_2
\end{bmatrix} \equiv V \cdot \begin{bmatrix}
a_2 \\
b_2
\end{bmatrix} \tag{B-16}
\]
⇒ \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} V_{22} & -\det(V) \\ V_{12} & V_{12} \\ 1 & -V_{11} \\ V_{12} & V_{12} \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} S_{p11} & S_{p12} \\ S_{p21} & S_{p22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = S_p \cdot \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \quad \text{(B-17)}

Where $S_p$ is the power-wave scattering parameter matrix that is referred to the reference impedance $Z_1$ and $Z_2$ at port-1 and port-2, respectively, 
\[ \det(V) = (V_{11} \cdot V_{22} - V_{12} \cdot V_{21}) \].

Assume the system under consideration is reciprocal and passive, where $\text{Re}(Z_1)$, $\text{Re}(Z_2)$ are positive and $Z_1, Z_2$ are complex, the scattering parameters of the matrix $S_p$ can be found by the following equations,

\[ S_{p11} = \left( Z_2 \cdot \cosh \gamma \ell - \frac{Z_1^* \cdot Z_2}{Z_c} \cdot \sinh \gamma \ell + Z_c \cdot \sinh \gamma \ell - Z_1^* \cdot \cosh \gamma \ell \right) \]  
\[ \quad \frac{Z_2 \cdot \cosh \gamma \ell + \frac{Z_1 \cdot Z_2}{Z_c} \cdot \sinh \gamma \ell + Z_c \cdot \sinh \gamma \ell + Z_1 \cdot \cosh \gamma \ell} \quad \text{(B-18)} \]

\[ S_{p21} = \left( \frac{2 \cdot \hat{Z}_1 \cdot \hat{Z}_2}{Z_2 \cdot \cosh \gamma \ell + \frac{Z_1 \cdot Z_2}{Z_c} \cdot \sinh \gamma \ell + Z_c \cdot \sinh \gamma \ell + Z_1 \cdot \cosh \gamma \ell} \right) = S_{p12} \quad \text{(B-19)} \]

where $-\det(V) = 1$,  

109
\[ S_{p22} = \begin{pmatrix}
- Z_2^* \cdot \cosh \gamma \ell - \frac{Z_1 \cdot Z_2^*}{Z_c} \cdot \sinh \gamma \ell + Z_c \cdot \sinh \gamma \ell + Z_1 \cdot \cosh \gamma \ell \\
Z_2 \cdot \cosh \gamma \ell + \frac{Z_1 \cdot Z_2}{Z_c} \cdot \sinh \gamma \ell + Z_c \cdot \sinh \gamma \ell + Z_1 \cdot \cosh \gamma \ell
\end{pmatrix}
\] (B-20)

The above scattering parameters can be further simplified for the following two cases:

Case1: \( Z_1 = Z_2 = Z_c \equiv R_c + jX_c \)

\[ S_{p11} = \frac{jX_c}{R_c + jX_c} = S_{p22}, \quad S_{p21} = \left( \frac{R_c}{R_c + jX_c} \right) e^{-j\gamma \ell} = S_{p12} \] (B-21)

For its special case of positive and real characteristic impedance, \( Z_c = R_c \)

\[ S_{p11} = S_{p22} = 0, \quad S_{p21} = S_{p12} = e^{-j\gamma \ell} \] (B-22)

Case2: \( Z_2 = Z_c \) and \( Z_1 = Z_c^* \)

\[ S_{p11} = 0, \quad S_{p21} = S_{p12} = e^{-j\gamma \ell}, \quad S_{p22} = \left( \frac{jX_c}{R_c + jX_c} \right) \left( 1 - e^{-2j\gamma \ell} \right) \] (B-23)
After the characteristic impedance $Z_c$ of the transmission line is found, the new parameters $S_p$ can be obtained by changing the original 50-ohm ($Z_1 = Z_2 = 50 \text{ ohm}$) system to the new complex impedance ($Z_1^* = Z_2 = Z_c$) system. Secondly, we can directly extract the loss and slow-wave factor (SWF) parameters from the magnitude and phase of $S_{p21}$ as Eq. (A-10) to Eq. (A-13).
Appendix C: Using numerical technique to obtain accurate broadband characterization of TL

The results of the guided properties from S-parameter 3D Full-Wave analyses and measurement may exist larger error as the length $\ell$ of the Device-Under-Testing (DUT) transmission line equals to the multiple of half-wave-length. The relatively large error in the characteristic impedance values, except that due to the junction structural and/or electrical discontinuities, spurious resonance influence, and measurement uncertainties [64], [65], is mainly caused by numerical error. For example, the characteristic impedance $Z_c$ can be found symbolically from Eq. (A-7), the expression \[
\frac{1 + S'_{11\text{ inf}}}{1 - S'_{11\text{ zero}}}
\]
is numerically closely to \(\frac{2}{2} = 1\), and the expression \[
\frac{1 + S'_{11\text{ zero}}}{1 - S'_{11\text{ inf}}}
\]
is numerically closely to \(\frac{0}{0}\) at frequencies where the line length $\ell \sim \frac{n}{2} \lambda_g$, $n = 1, 2, 3, \ldots$ and so on, $\lambda_g$ is guided wavelength. Therefore a small variation in the denominator and/or numerator, such as $\pm 1\%$ error in the $S'_{11\text{ zero}}$ (ex. -0.98 to -0.99) and $S'_{11\text{ inf}}$ (ex. 0.99 to 0.98), will cause about $\pm 35\%$ relatively large variation in $Z_c$ (ex. $1.414 \cdot Z_o$ to $0.707 \cdot Z_o$) in numerical simulations. The extraction of the guided properties, $SWF (=\beta/k_o)$ and $Loss\ (dB/\lambda_g)$, will also cause some degree error at frequencies where the line length $\ell$ is near a multiple of
half-wave-length.

For the special accuracy problem encounters with $Z_c$, $SWF$ and $Loss (dB/\lambda_g)$, we present here a numerical method to reduce the error. The procedures are described as follows:

Using well-known numerical curve fitting and interpolation technique, such as cubic-spline interpolation, we inserted as many data (2-port S-parameter) as possible in the variation frequency band, where the line length $\ell$ is near a multiple of half-wave-length.

We find out the error dominating frequencies where the line length $\ell = \frac{n}{2} \lambda_g$, $n = 1, 2, 3, \ldots$ and so on. From these data where the phase delay get from the parameter $\angle S_{p21}$ (or $\angle S_{21}$) are equal or most near to a multiple of $180^\circ$ and the values of relative expression $(1+S'_{11\_zero})$, $(1-S'_{11\_inf})$ are equal or most near to zero.

We delete all interpolated data except those in the error dominating frequencies. Let $x_{on}$, $n = 1, 2, 3, \ldots$ etc., be those data with respect to the frequency condition of the line length $\ell = \frac{n}{2} \lambda_g$, $n = 1, 2, 3, \ldots$ and so on.
The waiting processed data curves, the values of the real parts and the imaginary parts of the \((1+S'_{11_{\text{zero}}})\), \((1-S'_{11_{\text{inf}}})\) near at \(x_{o1}, x_{o2}, x_{o3}\), etc., can be replaced approximately using \(n_i\) –order power series polynomial, respectively. Where the \(n_i\) –order is decided from the variation range of the guided properties \((Z_c, SWF\) and \(Loss\))\((dB/\lambda_g)\), and the power series polynomial form is similar with Taylor series expansion to \(x_{on}\), which shows as following general expression,

\[
f(x) = f(x_{on}) + a_{f1} \cdot (x - x_{on}) + a_{f2} \cdot (x - x_{on})^2 + \ldots + a_{fn} \cdot (x - x_{on})^{n_i}
\]  

(C-1)

Where the symbol \(x\) represent frequency variable of variation range near \(x_{on}\); \(f(x_j)\) is the response value of \(f(x)\) at \(x = x_j\) for \(j = 1, 2, 3, \ldots n_i\) and \(x_1 < x_2 < x_3 \ldots < x_{n_i}\); the \(a_{f1}, a_{f2}, a_{f3}, \ldots \) and \(a_{fn_i}\) are the constant coefficient of the first-order, the second-order, the third-order, \(\ldots\), and the \(n_i\) –order item, respectively. The constant coefficient \(a_{fj}\) can be solved from \(n_i\) simultaneous equations as follows:

\[
[D] \equiv \begin{bmatrix}
  f(x_1) - f(x_{on}) \\
  f(x_2) - f(x_{on}) \\
  \vdots \\
  f(x_{n_i}) - f(x_{on})
\end{bmatrix} = \begin{bmatrix}
  (x_1 - x_{on}) & (x_1 - x_{on})^2 & \ldots & (x_1 - x_{on})^{n_i} \\
  (x_2 - x_{on}) & (x_2 - x_{on})^2 & \ldots & (x_2 - x_{on})^{n_i} \\
  \vdots & \vdots & \ddots & \vdots \\
  (x_{n_i} - x_{on}) & (x_{n_i} - x_{on})^2 & \ldots & (x_{n_i} - x_{on})^{n_i}
\end{bmatrix} \begin{bmatrix}
  a_{f1} \\
  a_{f2} \\
  \vdots \\
  a_{fn_i}
\end{bmatrix} \equiv [C] \cdot [A]
\]
So,

\[ [A] = [c]^{-1} \cdot [D] \]  \hspace{1cm} (C-2)

Where the matrix \([A]\) is the constant coefficient matrix.

Theoretically, when \(x\) is equal to \(x_{on}\), \(f(x) = f(x_{on})\) is equal to zero for \(n=1, 2, 3, \) and so on. In physically, the numerical-error results display that only close to zero in some degree. Assuming that

\[ V(x) = \frac{1 + \overline{S}_{11 \text{ inf}}(x)}{1 - \overline{S}_{11 \text{ zero}}(x)} \] \hspace{1cm} (C-3)

and

\[ U(x) = \frac{1 + \overline{S}_{11 \text{ zero}}(x)}{1 - \overline{S}_{11 \text{ inf}}(x)} \equiv \left( \frac{F_r(x) + j \cdot F_i(x)}{G_r(x) + j \cdot G_i(x)} \right) \] \hspace{1cm} (C-4)

Where the \(F_r(x), F_i(x), G_r(x),\) and \(G_i(x)\) all can be represented by Eq. (C-1), the relative coefficients can also be solved in according with the procedures of Eq. (C-2).

When the \(a_{f1}, a_{f2}, a_{f3}, \ldots \), and \(a_{jo}\) have been resolved in Eq. (C-1), the mainly processed procedures display as follows:
The first, to process the characterization variation frequency points except for at \( x = x_{o1}, x_{o2}, x_{o3}, \ldots \) etc. Setting the numerical error \( f(x_{on}) \) to zero in Eq. (C-1), to get the modified new values of \( f(x) \), applying to the \( F_r(x), F_l(x), G_r(x), G_l(x) \) near at \( x_{o1}, x_{o2}, x_{o3}, \ldots \) etc., respectively, to get the modified new values of \( U(x) \) and the relative values of \( V(x) \), to solve the modified new characteristic impedance \( Z_c(x) \).

The second, to process the error dominating frequency points at \( x = x_{o1}, x_{o2}, x_{o3}, \ldots \) etc. Here, \( U(x) \) occur \( \begin{pmatrix} 0 \\ 0 \end{pmatrix} \) at \( x = x_{o1}, x_{o2}, x_{o3}, \ldots \) etc., to solve the limitation values of \( U(x) \) as \( x \) approach to the \( x_{o1}, x_{o2}, x_{o3}, \ldots \) etc., respectively, in according with L’Hopital’s rule, to get the modified new values of \( U(x) \) and the relative values of \( V(x) \), to solve the modified new \( Z_c(x) \) at \( x = x_{o1}, x_{o2}, x_{o3}, \ldots \), respectively.

Finally, using the modified new \( Z_c \) to get the other guiding properties (\( SWF \) and \( Loss \ (dB/\lambda_g) \)) of transmission line in according with the \( \gamma \) - extracting procedures from Eqs. (A-8) - (A-13) in Appendix A.

For verified purpose, we have used above numerical method and extracting procedures, to process simulated and/or measured S-parameter of a finite length.
traditional MS line, to solve its characterization \((Z_c, \beta/k_o, \alpha/k_o, \text{Loss (dB/mm), and Loss (dB/λg)} \ldots \text{etc.})\). We have also applied the presented numerical method to process the original values of characteristic impedance \(Z_c\), to reduce the influence of numerical error near at frequencies where the line length \(\ell = \frac{n}{2} \lambda_g\), \(n = 1, 2, 3, \ldots\) etc., to get the accurate broadband characterization of transmission line \((Z_c, \text{SWF, and Loss(dB/λg)})\). These examples will be presented and discussed as Appendix D.
Appendix D. Verified Examples of the $\gamma - Z_c$ Extracting Procedure and Numerical Technique for the finite-length TL

**Case 1:**

The measured results across the 5 GHz to 40 GHz band reported by Goldfard and Platzker [66, Fig. 8] are applied to validate the above-mentioned theoretical extracting procedure for obtaining $Z_c$ and $\gamma$.

First, the measured data by Goldfard and Platzker are validated by the mode-matching method (MMM) incorporating the metal mode [67] as shown Fig. D-1(a). Then the full-wave method-of-moment (MOM) results are compared to the MMM data in Fig. D-1(b). Figure D-1(a) shows that the attenuation constants (in dB/mm) obtained by the MMM are in the middle of error bound when operating frequency is below 10 GHz. By contrast the attenuation constants approach the lower limit of the error bound between 10 GHz and 35 GHz. The attenuation constants obtained by the full-wave MOM are slightly below the measurement limits of error below 25 GHz and approach upper limit near 35 GHz. Since the MMM data agree very well with the measurement across the band, they are used as a reference to access the accuracy of the full-wave MOM extracting procedure. Figure D-1(b) indicates that the normalized attenuation constant ($\alpha/k_o$) obtained by the full-wave MOM is
approximately −23.77% at 5 GHz

(a) Loss (dB/mm) vs. f

(b) Dispersion characteristics: \( (\beta / k_o) \), \( (\alpha / k_o) \) vs. f

Fig. D-1. The comparison of losses in GaAs MS for 70-μm width on 100-μm GaAs, using the MMM and 3D Full-Wave MOM Integral Equations. (a) The
losses, using MMM and 3D Full-Wave simulations, compare to previous work [66], (b) The dispersion characteristics $\beta/k$ (solid lines) and $\alpha/k_o$ (dash lines) versus frequency.

GHz and 17.54 % at 35 GHz less than those obtained by the MMM. On the other hand, the maximum deviation of the normalized phase constant ($\beta/k_o$) between two method occurs at the lowest frequency end at 5GHz, where only $-0.57 \%$ deviation from the MMM data is observed. Therefore the validity of the full-wave MOM parameters extracting procedure is validated, provided approximately $\pm 20 \%$ error bound is acceptable for the normalized attenuation constant across the band.
Case 2:

Here, we present an example of a traditional MS line structure that Roger RO4003 substrate of thickness of 0.508 mm, relative dielectric constant $\varepsilon_r$ of 3.38, tan-loss $\tan\delta$ of 0.0022, and cladding conductor thickness of 17.5 µm is employed for the 110-Ω MS line design. The MS line structural parameters are linewidth $W$ of 0.278 mm, ground-plane width of 1.524 mm, and line length $L$ of 48.379 mm.

The comparing guiding property parameters: characteristic impedance ($Z_{co}$ and $Z_c$), slow-wave factor ($SWF$) and loss per guided wavelength ($Loss \ (dB/\lambda_g)$) versus frequency that got from the original data characteristic impedance $Z_{co}$ and the numerical method processed characteristic impedance $Z_c$ for the conventional MS line, show as Fig. D-2(a)-(b).

We find out the original data $Z_{co}$ have larger variation near about at 7.8 GHz ($L \sim 0.5 \ \lambda_g$), which the curve of the real parts Re($Z_{co}$) has a tip ($\sim 131 \ \Omega$) about at 7.7 GHz and a dip ($\sim 82 \ \Omega$) about at 7.8 GHz, simultaneously, the curve of the imaginary parts Im($Z_{co}$) has a dip ($\sim -17.5 \ \Omega$) about at 7.8 GHz, and the derived $SWF$ and $Loss \ (dB/\lambda_g)$ have a little variation about at 7.8 GHz that got from the $Z_{co}$. But the other hand, the processed characteristic impedance $Z_c$ by numerical
Fig. D-2. The Compare of guiding property parameters: (a) $Z_{co}$ and $Z_c$ versus frequency, (b) SWF and Loss($dB/\lambda_g$) versus frequency, that got from the original data - characteristic impedance $Z_{co}$ and the processed characteristic impedance $Z_c$ by numerical technique, for conventional MS line.
technique are smoothly for real part and imaginary part and the derived SWF and Loss (dB/\(\lambda_g\)) are more smooth that got from the \(Z_c\).

Beside, we derived the characteristic impedance \((Z_c)\) that got from the input (or output) voltage standing-wave-ratio (VSWR) at the interested frequencies \((L = \frac{n}{4} \cdot \lambda_g, n = 1, 2, 3\ldots)\) [8] as following expression

\[
Z_c = Z_o \cdot \sqrt{\frac{\text{VSWR}_{\max}}{\text{VSWR}_{\min}}}
\]  

(D-1)

Where the \(Z_o\) is equal to 50 \(\Omega\); the maximum VSWR, \(\text{VSWR}_{\max}\), is equal to 4.764 occur at 3.9 GHz (\(\sim 0.25 \lambda_g\)) and the minimum VSWR, \(\text{VSWR}_{\min}\), is equal to 1.019 occur at 7.8 GHz (\(\sim 0.5 \lambda_g\)), the characteristic impedance \(Z_c\) is approximately equal to 110.165 \(\Omega\) for the case.

The error-percentage of the characteristic impedance \(Z_c\) that got from Eq. (D-1) comparing to our numerical method is smaller than 3.64 \%. So, the validity of the presented numerical method for \(Z_c\)-parameter extraction is verified again.
Summary:

By comparing to the data that get from Mode-Match method (MMM) and HP ADS Line-CAL software analyses, the results are identically well and satisfied to our demand. So, the validity of presented numerical method and extraction procedures are verified.

In the following, we will apply the presented numerical method to process the original characteristic impedance $Z_{co}$, to reduce the serious influence of the numerical error near at $l = \frac{n}{2} \cdot \lambda_g$ and $n = 1, 2, 3$ etc., to get the broadband guided properties ($Z_c, SWF$ and $Loss (dB/\lambda_g)$) of the TLs in this dissertation.