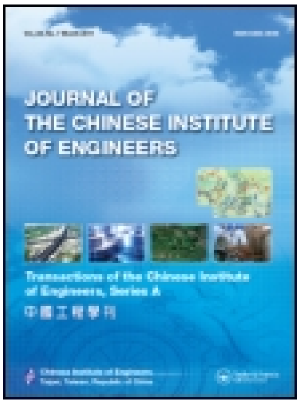

Journal of the Chinese Institute of Engineers
Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/tcie20

A multi-stage IIR time domain equalizer for OFDM systems with ISI

Wen-Rong Wu [a] & Chun-Fang Lee [b]

[a] Department of Communication Engineering , National Chiao Tung University , Hsinchu, 30010, Taiwan, R.O.C.

[b] Department of Communication Engineering , National Chiao Tung University , Hsinchu, 30010, Taiwan, R.O.C. Phone: 886-3-5731647 Fax: 886-3-5731647 E-mail:

Published online: 04 Mar 2011.

A MULTI-STAGE IIR TIME DOMAIN EQUALIZER FOR OFDM SYSTEMS WITH ISI

Wen-Rong Wu and Chun-Fang Lee*

ABSTRACT

In an orthogonal frequency division multiplexing (OFDM) system, it is known that when the delay spread of the channel is larger than the cyclic prefix (CP) size, intersymbol interference will occur. The time-domain equalizer (TEQ), designed to shorten the channel impulse response (CIR), is a common device to solve this problem. Conventionally, the TEQ is treated as a finite-impulse-response (FIR) filter, and many TEQ design methods have been proposed. However, a wireless channel typically has multi-path responses, exhibiting FIR characteristics. Thus, the corresponding TEQ will have an infinite impulse response (IIR), and the FIR modeling of the TEQ is inefficient, i.e., the required order for the TEQ will be high. The conventional approach will then suffer from the high computation complexity problem, both in the derivation of TEQ and in the operation of channel shortening. In this paper, we propose a new scheme to overcome these problems. In the derivation of the TEQ, we propose to use a multistage structure, replacing a high-order TEQ with a cascade of several low-order TEQs. In the shortening operation, we propose to use an IIR TEQ approximating a high-order FIR TEQ. Since the ideal TEQ exhibits low-order IIR characteristics, the order of the IIR TEQ can be much lower than the FIR TEQ. Simulations show that while the proposed method can reduce computational complexity significantly, its performance is almost as good as existing methods.

Key Words: orthogonal frequency division multiplexing, time domain equalizer, infinite impulse response, steiglitz-McBride method.

I. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) is a multicarrier transmission technique popularly used in wireless systems, such as IEEE 802.11g, DAB, DVB, etc. The OFDM divides the available signal band into many subchannels and allows a subcarrier to be used in each subchannel for data transmission. In general, a cyclic prefix (CP) is added in front of an OFDM symbol to avoid intersymbol interference (ISI). The CP length is at least equal to or greater than the length of the channel impulse response (CIR). Since the CP will reduce the transmission efficiency, a large-size CP is not desirable. Thus, the choice of the CP size is often a compromise between the tolerated length of the CIR, and the data throughput. As a result, in some scenarios the length of the CIR will exceed the CP range. When this happens, ISI is induced and the system performance is degraded. A simple remedy for this problem is to apply a time domain equalizer (TEQ) such that the CIR can be shortened into the CP range. Fig. 1 shows a typical OFDM system, with a TEQ added for the purpose of channel shortening.

TEQ development originated from the community of wireline communications (e.g. ADSL). The modulation scheme in wireline applications is called discrete multi-tone (DMT). DMT is essentially the same as OFDM. The main difference lies in the fact that the DMT performs bit-loading while the OFDM does not. With bit-loading, the transmission can approach the maximum channel capacity. Many algorithms have been proposed for the design of a TEQ in a DMT system (Chow and Cioffi, 1992; Melsa *et al.*, 1996; Al-Dhahir and Cioffi, 1996; Arslan *et al.*, 2001; Van Acker *et al.*, 2001; Henkel, *et al.*, 2002;

*Corresponding author. (Tel: 886-3-5731647; Fax: 886-3-5710116; Email: wrwu@faculty.nctu.edu.tw; jefflee.cm89g@nctu.edu.tw)

The authors are with the Department of Communication Engineering, National Chiao Tung University, Hsinchu 30010, Taiwan, R.O.C.

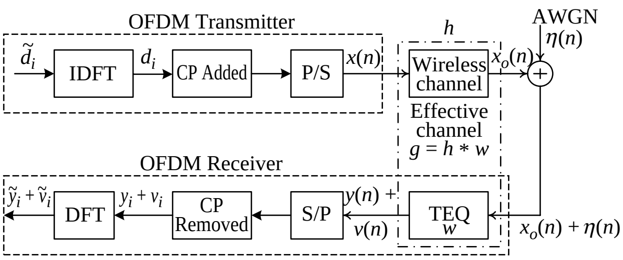

Fig. 1 An OFDM system with TEQ

Vanbleu *et al*., 2004; Martin *et al*., 2005; Kim and Powers, 2005; Wu and Lee, 2005). Recently, some methods have been extended to OFDM systems (Zhang and Ser, 2002; Leus and Moonen, 2003; Yang and Kang, 2006; Lee and Wu, 2007; Wu and Lee, 2007; Rawal, *et al*., 2007; Rawal and Vijaykumar, 2008). For a DMT system, a TEQ design minimizing the mean-squared-error (MMSE) was first developed by Chow and Cioffi (1992). The MMSE method allows an adaptive structure and its computational complexity is low. Treating the TEQ design as a pure channel shortening problem, Melsa *et al*. (1996) proposed a criterion to maximize the shortened signal-to-noise ratio (SSNR), defined as the ratio of the energy of the TEQ shortened response inside and outside the CP range. This method is then referred to as the maximized SSNR (MSSNR) method. The work proposed by Al-Dhahir and Cioffi (1996) first considers capacity maximization in TEQ design. With a geometric SNR (GSNR) maximization, a constrained nonlinear optimization problem was obtained. Taking both residual ISI and channel noise into account, a method referred to as maximum bit rate (MBR) (Arslan *et al*., 2001; Vanbleu *et al*., 2004) was then proposed. To improve the performance, the inter-carrier interference (ICI) is taken into consideration by Henkel *et al*. (2002). It was found that the aforementioned methods shared a common mathematical framework based on the maximization of a product of generalized Rayleigh quotients (Martin, *et al*., 2005).

The methods mentioned above all conduct the TEQ design in the time-domain. Another approach, treating the problem in the frequency-domain, was first proposed by Acker *et al*. (2001) for DMT systems, and later by Leus and Moonen (2003) for MIMO OFDM systems. This method, referred as per-tone equalization (PTEQ), allows an equalizer designed for the signal in each tone. Using the computational advantage of the fast Fourier transform technique, TEQ filtering operations can be effectively implemented in the discrete-Fourier transform (DFT) domain. It has been shown that the PTEQ scheme can outperform conventional TEQ schemes. However, PTEQ requires a large-size memory for storage and is potentially higher in computational complexity (Martin, *et al*., 2005). Another method called sub-symbol equalization (SSE) (Kim and Powers, 2005) also designs the TEQ in the frequency domain. It uses the conventional zero-forcing frequency domain equalizer to obtain the equalized time-domain signal. The drawback of the SSE is that it is only applicable to a certain type of channels.

As mentioned, for OFDM systems, no bit-loading is conducted and the purpose of the TEQ is just to reduce the ISI. The bit-error-rate (BER) is then the criterion to evaluate the performance of an OFDM system. In general, the larger the SSNR, the smaller the ISI and the smaller the BER we can have. Thus, the MSSNR criterion, which cannot achieve the maximum capacity in DMT systems, is adequate in OFDM systems. The MSSNR TEQ for OFDM systems has been studied by Yang and Kang (2006). With the original MSSNR method, the TEQ length must be constrained to be smaller than or equal to the CP length. In a work by Yang and Kang (2006), a modified MSSNR TEQ method was proposed to solve the problem. Using this method, the limitation of the TEQ tap length can be removed. Furthermore, an adaptive TEQ method based on the least mean-square (LMS) algorithm is also proposed to track the channel variation. Since the convergence of the LMS algorithm is slow, the QR-recursive least square (QR-RLS) algorithm is further proposed by Rawal and Vijaykumar (2008) for TEQ adaptation.

For all methods discussed above, the TEQ is treated as a finite-impulse-response (FIR) filter. However, a wireless channel typically has multi-path responses, exhibiting FIR characteristics. It can be shown that the ideal TEQ has infinite impulse response (IIR) characteristics, and its order is low. As a result, the FIR modeling of the TEQ is inefficient. To achieve a high SSNR, the TEQ order must be high. Conventional approaches then suffer from the high computational complexity problem, either in the derivation of TEQ or in the operation of channel shortening.

In this paper, we propose a low-complexity scheme to overcome the problem mentioned above. The basic idea is to use an IIR TEQ instead of an FIR TEQ for the channel shortening. However, the direct derivation of an IIR TEQ from the channel response is a difficult job. In this paper, we propose to use a two-step approach. In the first step, we derive a high-order FIR TEQ. In the second step, we convert the FIR TEQ into a low-order IIR TEQ. In the derivation of the FIR TEQ, we propose to use a multi-stage (MS) structure. Instead of a single-stage (SS) high-order TEQ, we propose to use a cascade of several low-order TEQs. For conventional TEQ design methods such as proposed by Melsa *et al*. (1996) or Arslan *et al*., (2001), matrix operations are frequently required, and the computational complexity is $O(N^3)$ (Yang and Kang, 2006) where N is the TEQ order. Thus, if N is large, the required computational complexity is high. With our MS structure, the computational complexity for the FIR TEQ derivation can be dramatically reduced. Since the ideal TEQ exhibits low-order IIR characteristics, the order required for an IIR TEQ will be much lower than that of an FIR TEQ. To convert an FIR filter into an equivalent IIR form, we apply the Steiglitz McBride method (SMM) (Steiglitz and McBride, 1965) to do the job. Simulations show that while the proposed method can reduce the computational complexity significantly, performance remains excellent. In this paper, we will mainly use the MSSNR method (Melsa *et al*., 1996) as our TEQ design method. It can produce good BER performance for OFDM systems (Yang and Kang, 2006). Note that the idea of the IIR TEQ was first proposed in the works of Wu and Lee (2007) and Lee and Wu (2007). In Rawal *et al*. (2007), an IIR TEQ based on the QR-RLS adaptive algorithm was also proposed. However, it is well-known that the stability of an adaptive IIR filter cannot be guaranteed. This is different from the SMM we use, where convergence is guaranteed (Stoica and Söderström, 1981; Cheng and Stonick, 1994; Netto *et al*., 1995; Regalia *et al*., 1997).

This paper is organized as follows. In Section II, we give the general signal model of an OFDM system. In Section III, we briefly review the IIR characteristics of the TEQ, derive the MS FIR TEQ, detail the proposed IIR TEQ scheme, and analyze its complexity. Section IV shows the simulation results. Finally, Section V draws conclusions.

II. SIGNAL MODEL

Let M be the DFT size, L the CP length, $K = M + L$ the OFDM symbol length, I the channel length, and N the TEQ length. In addition, let n be the signal index, i the OFDM symbol index, both in the time domain, and k the subchannel index in the DFT domain, where $0 \le k \le M - 1$. Let $*$ be the linear convolution operation, and denote $[\cdot]^T$, and $[\cdot]^H$ as the transpose, and the Hermitian operation for a vector or matrix, respectively. Denote $\boldsymbol{0}_p$ as the $p \times 1$ zero column vector, $\boldsymbol{1}_p$ the $p \times 1$ unity column vector, $\boldsymbol{0}_{pxq}$ the $p \times q$ zero matrix, and $\boldsymbol{I}_p$ the $p \times p$ identity matrix.

A common model of an OFDM system with TEQ design is shown in Fig. 1. On the OFDM transmitter side, denote the i-th transmitted data symbol as $\tilde{\boldsymbol{d}}_i = [\tilde{d}_i(0), \cdots, \tilde{d}_i(M-1)]^T$, where $\tilde{d}_i(k)$ is the $(k+1)$-th element of $\tilde{\boldsymbol{d}}_i$. Taking the M-point inverse DFT (M-IDFT) to $\tilde{\boldsymbol{d}}_i$, we can then obtain the corresponding time domain signal, denoted as $\boldsymbol{d}_i$. That is, $\boldsymbol{d}_i = [d_i(0), \cdots, d_i(M-1)]^T = \frac{1}{M}\boldsymbol{F}^H\tilde{\boldsymbol{d}}_i$, where $\boldsymbol{F} = [\boldsymbol{f}(0), \boldsymbol{f}(1), \cdots, \boldsymbol{f}(M-1)]$ is an $M \times M$ DFT matrix, $\boldsymbol{f}(k) = [1, e^{-j2\pi k/M}, \cdots, e^{-j2\pi(M-1)k/M}]^T$. Subsequently, appending the CP and conducting parallel-to-serial conversion, we obtain the transmitted signal $x(n)$. Here, $n = iK + l$, and

$$x(iK+l) = \begin{cases} d_i(l+M-L)\,, & \text{for } 0 \le l \le L-1\,, \\ d_i(l-L)\,, & \text{for } L \le l \le K-1\,, \end{cases} \quad (1)$$

where $d_i(l)$ is the $(l+1)$-th element of $\boldsymbol{d}_i$. The signal $x(n)$ is then transmitted over a wireless channel with FIR and corrupted by the additive white Gaussian noise (AWGN).

Let the wireless CIR be represented as $\boldsymbol{h} = [h(0), \cdots, h(I-1)]^T$, and AWGN as $\eta(n)$. $x(n)$ is assumed independent of the noise $\eta(n)$. Denote the noise-free channel output signal as $x_o(n)$, where $x_o(n) = x(n) * h(n)$. At the receiver side, both $x_o(n)$ and $\eta(n)$ are first filtered by an N-tap TEQ. Let the TEQ coefficients be denoted as $\boldsymbol{w} = [w(0), \cdots, w(N-1)]^T$. Also let the corresponding TEQ-filtered output of $x_o(n)$ and that of the channel noise be $y(n)$ and $v(n)$, where $y(n) = x_o(n) * w(n)$ and $v(n) = \eta(n) * w(n)$, respectively. Moreover, without loss of generality, let the synchronization delay be zero in the following paragraphs. Performing the serial-to-parallel conversion and removing the CP, we can obtain the i-th received signal-only OFDM symbol as $\boldsymbol{y}_i = [y(iK+L), \cdots, y((i+1)K-1)]^T$. Let the corresponding i-th noise symbol vector at the TEQ input and output be $\eta_i = [\eta(iK+L), \cdots, \eta((i+1)K-1)]^T$ and $\boldsymbol{v}_i = [v(iK+L), \cdots, v((i+1)K-1)]^T$, respectively.

From Fig. 1, we can see that the transmitted signal $x(n)$ passes the wireless channel, $h(n)$, and the TEQ, $w(n)$. Let $g(n) = h(n) * w(n)$ be the equivalent channel response (ECR), and $\boldsymbol{g} = [g(0), \cdots, g(J-1)]^T$, where $J = I + N - 1$. Assume that $J < M$, and we can decompose $\boldsymbol{g}$ into $\boldsymbol{g} = \boldsymbol{g}_S + \boldsymbol{g}_I$, where $\boldsymbol{g}_S = [g(0), \cdots, g(L-1), \boldsymbol{0}_{J-L}^T]^T$ is the desired shortened channel response, and $\boldsymbol{g}_I = [\boldsymbol{0}_L^T, g(L), \cdots, g(J-1)]^T$ the residual ISI response.

We can then express $\boldsymbol{g}_S$ and $\boldsymbol{g}_I$ in terms of the channel matrix $\boldsymbol{H}$ and the TEQ vector $\boldsymbol{w}$ as $\boldsymbol{g}_S = \boldsymbol{D}_S\boldsymbol{H}\boldsymbol{w}$ and $\boldsymbol{g}_I = \boldsymbol{D}_I\boldsymbol{H}\boldsymbol{w}$, respectively, where $\boldsymbol{D}_S = \text{diag}[\boldsymbol{1}_L^T, \boldsymbol{0}_{J-L}^T]$, $\boldsymbol{D}_I = \boldsymbol{I}_J - \boldsymbol{D}_S = \text{diag}[\boldsymbol{0}_L^T, \boldsymbol{1}_{J-L}^T]$, and

$$\boldsymbol{H} = \begin{bmatrix} h(0) & 0 & 0 & \cdots & 0 \\ h(1) & h(0) & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ h(I-1) & h(I-2) & h(I-3) & \cdots & h(I-N) \\ 0 & h(I-1) & h(I-2) & \cdots & h(I-N+1) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & h(I-1) \end{bmatrix}_{J \times N} . \quad (2)$$

Here, diag[·] denotes a diagonal matrix with the vector inside the bracket as its diagonal elements. We can reexpress $\boldsymbol{g}_S$ and $\boldsymbol{g}_I$ as $\boldsymbol{g}_S = [g_S(0), \cdots, g_S(J-1)]^T$, and $\boldsymbol{g}_I = [g_I(0), \cdots, g_I(J-1)]^T$, respectively, where $g_S(l)$ is the $(l+1)$-th element of $\boldsymbol{g}_S$, and $g_I(l)$ that of $\boldsymbol{g}_I$. Let $y_S(n)$, $y_I(n)$ be the desired part and the residual ISI part of $y(n)$. Thus we have $y(n) = y_S(n) * y_I(n)$, where $y_S(n) = x(n) * g_S(n)$, and $y_I(n) = x(n) * g_I(n)$. Consequently, we can also decompose $\boldsymbol{y}_i$ as

$$\boldsymbol{y}_i = \boldsymbol{y}_{S,\,i} + \boldsymbol{y}_{I,\,i}, \tag{3}$$

where $\boldsymbol{y}_{S,\,i} = [y_S(iK+L), \cdots, y_S((i+1)K-1)]^T$, and $\boldsymbol{y}_{I,\,i} = [y_I(iK+L), \cdots, y_I((i+1)K-1)]^T$.

III. PROPOSED IIR TEQ METHOD

In this section, we first describe the IIR characteristics of the TEQ in Section III.1. Then we derive the MS FIR TEQ in Section III.2. Based on the result, we then derive the proposed IIR TEQ scheme in Section III.3. Finally, we analyze the computational complexity of the proposed scheme in Section III.4.

1. IIR Characteristic of the TEQ

A typical wireless channel generally has multipath responses, which can be modeled as an FIR system. In this paragraph, we show that the TEQ for an FIR channel will exhibit an IIR characteristic. Recall that a wireless CIR $\boldsymbol{h} = [h(0), \cdots, h(I-1)]^T$ has an FIR where the channel length exceeds the CP size, that is, $I > L$. Without loss of generality, we let $h(0) = 1$. Denote the transfer function of the channel as $H(z)$. Then,

$$\begin{aligned} H(z) &= 1 + h(1)z^{-1} + \cdots + h(I-1)z^{-I+1} \\ &= (1 - z_1 z^{-1})(1 - z_2 z^{-1}) \cdots (1 - z_{I-1} z^{-1}), \end{aligned} \tag{4}$$

where $z_1, \cdots, z_{I-1}$ are $I-1$ zeros of $H(z)$ and $|z_1| \le |z_2| \ldots \le |z_{I-1}|$. We can further express $H(z)$ as a cascade of three FIR channels, i.e., $H(z) = H_0(z)H_1(z)H_2(z)$ where $H_0(z)$ has m_0 zeros all located inside the unit circle, $H_1(z)$ has m_1 zeros all located on the unit circle, and $H_2(z)$ has m_2 zeros all located outside the unit circle. Note that $m_0 + m_1 + m_2 = I - 1$. Now, suppose we want to shorten the wireless channel into the CP range. In other words, the TEQ must shorten at least $I - L$ channel taps. We have three cases to discuss, i.e., (i) $I - L \le m_0$, (ii) $m_0 < I - L \le m_0 + m_2$, (iii) $m_0 + m_2 < I - L$. For case (i), the TEQ can be an IIR filter having $I - L$ poles of $\{z_1, \cdots, z_{I-L}\}$. Denoting the transfer function of the TEQ as $W(z)$, we can have

$$W(z) = \frac{1}{(1 - z_1 z^{-1})(1 - z_2 z^{-1}) \cdots (1 - z_{I-L} z^{-1})}. \tag{5}$$

In this case, $I - L$ zeros of $H(z)$ is canceled by $I - L$ poles of $W(z)$, and the channel response can be perfectly shortened. For case (ii), we can let m_0 zeros of $H(z)$ be canceled by m_0 poles of $W(z)$ obtained from $H_0(z)$. However, there are $I - L - m_0$ zeros cannot be canceled. Note that if we substitute for z with z^{-1} in $H_2(z)$, the resultant transfer function will have its zeros located inside the unit circle. This indicates that the zeros of $H_2(z^{-1})$ can also be canceled by an IIR filter if the time index goes from 0 to $-\infty$. Although the IIR filer is not realizable, it can be approximated by an non-causal FIR filter. Thus, we have the TEQ as

$$W(z) = \frac{W_0(z)}{(1 - z_1 z^{-1})(1 - z_2 z^{-1}) \cdots (1 - z_{m_0} z^{-1})}, \tag{6}$$

where $W_0(z)$ is the FIR filter designed to cancel the response of the $I - L - m_0$ zeros. In this case, the channel can be shortened, but not perfectly. The performance depends on the dimension of $W_0(z)$. As is known, zeros on the unit circle cannot be canceled. Thus, for case (iii), the channel response cannot be shortened into the CP range. Since the number of the taps to be shortened is generally much smaller than the channel length itself, case (i) will be observed in most environments.

From the above discussion, we conclude that the TEQ possesses an IIR characteristic in wireless channels. Note that this property is quite different from the wireline applications where the CIR can be modeled as a low-order IIR system (Crespo and Honig, 1991). Thus, a low-order FIR TEQ can effectively shorten the channel. This is also the main difference between the application of the TEQ in DMT and OFDM systems.

2. Derivation of MS FIR TEQ

As shown in Fig. 1, the objective of the TEQ is to shorten the CIR length I to the CP size, L. As discussed, for wireless channels, the required FIR TEQ order for the desired shortening may be long. As we will see, the derivation of the MSSNR TEQ relies on matrix operations having the computational complexity of $O(N^3)$. If N is large, the computational complexity will be high. Here, we propose an MS structure to alleviate this problem. We approach the original SS TEQ with a cascade of multiple TEQs. It is simple to see that the TEQ order in each stage can be made much smaller than that of the original one. Let the number of stages be V, the TEQ vector in the l-th stage be $\boldsymbol{w}_l$, and its order be N_l, that is, $\boldsymbol{w}_l = [w_l(0), \cdots, w_l(N_l - 1)]^T$, where $1 \le l \le V$. In each stage, we can derive the TEQ using the conventional MSSNR method.

For each individual stage of the MS structure, let the ECR at the l-th stage be denoted as $\boldsymbol{g}_l$. Then, $\boldsymbol{g}_l = \boldsymbol{g}_{l-1} * \boldsymbol{w}_l$, where $\boldsymbol{g}_0 = \boldsymbol{h}$ and $1 \le l \le V$. Here, the convolution operator '$*$' is applied for vectors. In the l-th stage, the TEQ shortens the CIR for designated P_l taps. In other words, after the l-th TEQ, the length of target-impulse-response becomes $I - \Sigma_{i=1}^{l} P_i$. Hence, the total target-shortening-length is $\Sigma_{l=1}^{V} P_l = I - L$ and the overall equivalent TEQ length is $\Sigma_{l=1}^{V} N_l - V + 1$. Furthermore, the overall TEQ response $\boldsymbol{w}$ is equal to the cascade of the individual TEQs, that is, $\boldsymbol{w} = \boldsymbol{w}_1 * \boldsymbol{w}_2 * \cdots \boldsymbol{w}_V$.

As mentioned in Section II, assume that the synchronization delay is zero, and let $\boldsymbol{g}_l = [g_l(0), \cdots, g_l(J_l - 1)]^T$, where J_l is the ECR length at the l-th stage, and $J_l = J_{l-1} + N_l - 1$, $1 \le l \le V$. Note that $J_0 = I$ is the original CIR length. We can then decompose $\boldsymbol{g}_l$ into two parts, the desired shortened channel response $\boldsymbol{g}_{S,l} = [g_l(0), \cdots, g_l(L_l - 1), \boldsymbol{0}_{J_l - L_l}^T]^T$ and the residual ISI $\boldsymbol{g}_{I,l} = [\boldsymbol{0}_{L_l}^T, g_l(L_l), \cdots, g_l(J_l - 1)]^T$, where $L_l = I - \Sigma_{j=1}^{l} P_l$. That is, $\boldsymbol{g}_l = \boldsymbol{g}_{S,l} + \boldsymbol{g}_{I,l}$. Then, we can rewrite $\boldsymbol{g}_{S,l}$ and $\boldsymbol{g}_{I,l}$ as

$$\boldsymbol{g}_{S,l} = \boldsymbol{D}_{S,l}\boldsymbol{H}_l\boldsymbol{w}_l,$$

$$\boldsymbol{g}_{I,l} = \boldsymbol{D}_{I,l}\boldsymbol{H}_l\boldsymbol{w}_l, \tag{7}$$

where $\boldsymbol{D}_{S,l} = \mathrm{diag}[\boldsymbol{1}_{L_l}^T, \boldsymbol{0}_{J_l - L_l}^T]$, $\boldsymbol{D}_{I,l} = \mathrm{diag}[\boldsymbol{0}_{L_l}^T, \boldsymbol{1}_{J_l - L_l}^T]$, and $\boldsymbol{H}_l$ a $J_l \times N_l$ matrix consisting of a shift version of the ECR $\boldsymbol{g}_{l-1}$,

$$\boldsymbol{H}_l = \begin{bmatrix} g_{l-1}(0) & 0 & \cdots & 0 \\ g_{l-1}(1) & g_{l-1}(0) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ g_{l-1}(J_{l-1}-1) & g_{l-1}(J_{l-1}-2) & \cdots & g_{l-1}(J_{l-1}-N_l) \\ 0 & g_{l-1}(J_{l-1}-1) & \cdots & g_{l-1}(J_{l-1}-N_l+1) \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & g_{l-1}(J_{l-1}-1) \end{bmatrix}_{J_l \times N_l}. \tag{8}$$

The SSNR at the TEQ output of the l-th stage for the OFDM receiver is then defined as

$$SSNR_l = \frac{\boldsymbol{g}_{S,l}^H \boldsymbol{g}_{S,l}}{\boldsymbol{g}_{I,l}^H \boldsymbol{g}_{I,l}} = \frac{\boldsymbol{w}_l^H \boldsymbol{H}_l^H \boldsymbol{D}_{S,l}^H \boldsymbol{D}_{S,l} \boldsymbol{H}_l \boldsymbol{w}_l}{\boldsymbol{w}_l^H \boldsymbol{H}_l^H \boldsymbol{D}_{I,l}^H \boldsymbol{D}_{I,l} \boldsymbol{H}_l \boldsymbol{w}_l} = \frac{\boldsymbol{w}_l^H \boldsymbol{A}_l \boldsymbol{w}_l}{\boldsymbol{w}_l^H \boldsymbol{B}_l \boldsymbol{w}_l}, \tag{9}$$

where $g_{S,l}^H \boldsymbol{g}_{S,l}$ is the desired signal power, $\boldsymbol{g}_{I,l}^H \boldsymbol{g}_{I,l}$ the residual ISI power, $\boldsymbol{A}_l = \boldsymbol{H}_l^H \boldsymbol{D}_{S,l}^H \boldsymbol{D}_{S,l} \boldsymbol{H}_l$, and $\boldsymbol{B}_l = \boldsymbol{H}_l^H \boldsymbol{D}_{I,l}^H \boldsymbol{D}_{I,l} \boldsymbol{H}_l$.

The optimal TEQ for the MSSNR method can be obtained through the maximization of the SSNR. The rows of $\boldsymbol{D}_{I,l}\boldsymbol{H}_l$ are formed by the shifted version of the CIR and the rank of $\boldsymbol{D}_{I,l}\boldsymbol{H}_l$ is $J_l \times N_l$. Consequently, the matrix $\boldsymbol{B}_l$ is of full rank $N_l \times N_l$ and also positive definite. Hence, $\boldsymbol{B}_l$ can be decomposed by using the Cholesky decomposition, that is, $\boldsymbol{B}_l = \mathbb{B}_l \mathbb{B}_l^H$. We can define a vector $\boldsymbol{y}_l = \mathbb{B}_l^H \boldsymbol{w}_l$, and then $\boldsymbol{w}_l = (\mathbb{B}_l^H)^{-1} \boldsymbol{y}_l$. Thus, $\boldsymbol{w}_l^H \boldsymbol{B}_l \boldsymbol{w}_l = \boldsymbol{y}_l^H \boldsymbol{y}_l$, and $\boldsymbol{w}_l^H \boldsymbol{A}_l \boldsymbol{w}_l = \boldsymbol{y}_l^H (\mathbb{B}_l)^{-1} \boldsymbol{A}_l (\mathbb{B}_l^H)^{-1} \boldsymbol{y}_l = \boldsymbol{y}_l^H \mathbb{A}_l \boldsymbol{y}_l$, where $\mathbb{A}_l = (\mathbb{B}_l)^{-1} \boldsymbol{A}_l (\mathbb{B}_l^H)^{-1}$. As a result, $\mathrm{SSNR}_l = \boldsymbol{y}_l^H \mathbb{A}_l \boldsymbol{y}_l / \boldsymbol{y}_l^H \boldsymbol{y}_l$ has a form of Raleigh quotient. It is well known that optimal $\boldsymbol{y}_{l,O}$ maximizing the quotient SSNR_l can be obtained by choosing the eigenvector corresponding to the maximum eigenvalue of $\mathbb{A}_l$ (Chong and Zak, 2001). Thus, the optimal TEQ vector $\boldsymbol{w}_{l,O}$ is

$$\boldsymbol{w}_{l,O} = (\mathbb{B}_l^H)^{-1} \boldsymbol{y}_{l,O} \tag{10}$$

and the corresponding optimal SSNR_l is

$$SSNR_{l,O} = \frac{\boldsymbol{w}_{l,O}^H \boldsymbol{A}_l \boldsymbol{w}_{l,O}}{\boldsymbol{w}_{l,O}^H \boldsymbol{B}_l \boldsymbol{w}_{l,O}} = \lambda_l, \tag{11}$$

where λ_l is the maximum eigenvalue of $\mathbb{A}_l$. Different from DMT systems, the MSSNR TEQ has been shown to have good performance in OFDM systems (Yang and Kang, 2006).

After deriving TEQ vectors $\{\boldsymbol{w}_{1,O}, \boldsymbol{w}_{2,O}, \cdots, \boldsymbol{w}_{V,O}\}$ for all V stages, we can have the equivalent optimal TEQ vector $\boldsymbol{w}_O$ as

$$\boldsymbol{w}_O = \boldsymbol{w}_{1,O} * \boldsymbol{w}_{2,O} * \cdots \boldsymbol{w}_{V,O} \tag{12}$$

This result is also shown in Fig. 2.

3. Derivation of IIR TEQ

As shown in Section III.1, the TEQ for the wireless channel possesses a low-order IIR property. Thus, a conventional FIR TEQ achieving satisfactory performance requires a high order. This will require heavy computations in the shortening operation. To solve the problem, we then propose to convert the

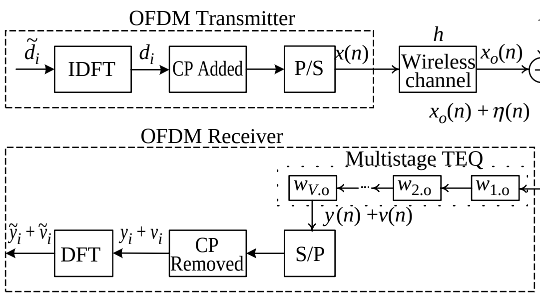

Fig. 2 An OFDM system with multistage TEQ

FIR TEQ obtained in Eq.(12) to an equivalent IIR TEQ. By doing so, we can effectively reduce the required computational complexity for the shortening operation. Here, we use the Steiglitz-McBride method (SMM) to do the job.

The SMM is an iterative method for IIR system identification (Steiglitz and McBride, 1965). Its structure is shown in Fig. 3, in which $c(n)$, $x(n)$, and $r(n)$ denote the impulse response, the input signal, and the output signal of the plant, respectively. Here, the plant is an IIR system and its transfer function can be represented as a rational function as

$$C(z) = \frac{A(z)}{B(z)} . \tag{13}$$

Also let

$$C_m(z) = \frac{A_m(z)}{B_m(z)} \tag{14}$$

be the estimated transfer function of the plant in the m-th iteration, where $A_m(z) = \Sigma_{j=0}^{Q} \alpha_j(m) z^{-j}$, $B_m(z) = 1 - \Sigma_{j=1}^{P} \beta_j(m) z^{-j}$. Note that Q and P are the orders of $A(z)$ and $B(z)$, respectively. Assume that in the (m-1)-th iteration, optimal $B_{m-1}(z)$ and $A_{m-1}(z)$ have been obtained. To conduct the m-th iteration, the SMM first filters the plant output, $r(n)$, and its input, $x(n)$, with $1/B_{m-1}(z)$. The resultant outputs, $u(n)$ and $v(n)$, are then fed to $B_m(z)$ and $A_m(z)$, respectively. Optimal $B_m(z)$ and $A_m(z)$ can then be obtained by minimizing the average-squared-error (ASE) power of the two outputs. It is simple to see that if the algorithm converges, i.e., $B_{m-1}(z) = B_m(z)$, then the plant is identified as $A_m(z)/B_m(z)$.

Put the unknown parameters $\beta_j(m)$ and $\alpha_j(m)$ together to form a vector $\boldsymbol{\Theta}(m)$ as

$$\boldsymbol{\Theta}(m) = [\beta_1(m), \cdots, \beta_P(m), \alpha_0(m), \cdots, \alpha_Q(m)]^T, \tag{15}$$

and also define a vector $\boldsymbol{\Phi}(n)$ as

$$\boldsymbol{\Phi}(n) = [v(n-1), \cdots, v(n-P), u(n), \cdots, u(n-Q)]^T. \tag{16}$$

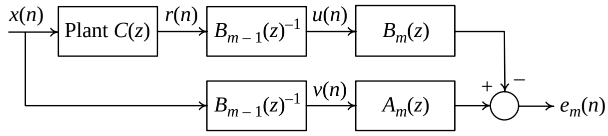

Fig. 3 System model for Steiglitz McBride method

Define the error signal between the filtered outputs of $u(n)$ and $v(n)$ as $e_m(n)$. Then, we have

$$\begin{aligned} e_m(n) &= \sum_{j=0}^{Q} \alpha_j(m) v(n-j) - u(n) + \sum_{j=1}^{P} \beta_j(m) u(n-j) \\ &= \boldsymbol{\Phi}^T(n)\boldsymbol{\Theta}(m) - u(n) . \end{aligned} \tag{17}$$

If we collect the observations of $u(n)$ and $v(n)$ in a time window with size N', we can then have N' samples of the error signal which can be expressed as

$$\boldsymbol{e}_m(n) = \boldsymbol{\Psi}(n)\boldsymbol{\Theta}(m) - \boldsymbol{u}(n), \tag{18}$$

where $\boldsymbol{e}_m(n) = [e_m(n), e_m(n-1), \cdots, e_m(n-N'+1)]^T$, $\boldsymbol{u}(n) = [u(n), u(n-1), \cdots, u(n-N'+1)]^T$, and $\boldsymbol{\Psi}(n) = [\boldsymbol{\Phi}(n), \boldsymbol{\Phi}(n-1), \cdots, \boldsymbol{\Phi}(n-N'+1)]^T$. Thus, we can use the least-squares (LS) method to obtain the optimal estimate of $\boldsymbol{\Theta}(m)$. The criterion for the LS method is to minimize the ASE power, denoted as $\xi[\boldsymbol{\Theta}(m)]$, given by Steiglitz and McBride (1965),

$$\xi[\boldsymbol{\Theta}(m)] = \|\boldsymbol{e}_m(n)\|^2 = \|\boldsymbol{\Psi}(n)\boldsymbol{\Theta}(m) - \boldsymbol{u}(n)\|^2, \tag{19}$$

The solution to the LS Eq. (19) can be written as

$$\boldsymbol{\Theta}(m) = (\boldsymbol{\Psi}^T(n)\boldsymbol{\Psi}(n))^{-1}\boldsymbol{\Psi}^T(n)\boldsymbol{u}(n). \tag{20}$$

Then, $1/B_m(z)$ is used to filter $r(n)$ and $x(n)$, and $u(n)$ and $v(n)$ is obtained for the LS solution in the next iteration. Since the SMM is an iterative algorithm, it requires an initial estimate of $B_0(z)$. A simple method for this problem is just to let $B_0(z) = 1$. In this case, $v(n)$ is the input of the plant which is $x(n)$, and $u(n)$ is the corresponding output, i.e., $u(n) = r(n)$. For IIR filter design, stability is always an issue. The stability and the convergence of the SMM have been investigated. Interested readers may refer to the works by Stoica and Söderström (1981), Cheng and Stonick (1994), Netto *et al*. (1995), and Regalia *et al*. (1997).

We summarize the procedure of the proposed TEQ design method as follows. Firstly, we apply the MS structure and use the conventional MSSNR method to obtain an FIR TEQ $\boldsymbol{w}_{l,O}$ for each stage, where $1 \le l \le V$. By cascading the multiple stages of TEQs $\boldsymbol{w}_{l,O}$, we can obtain the equivalent optimal TEQ

$\mathbf{w}_O$ in Eq. (12). Treating $\mathbf{w}_O$ as the impulse response of an IIR plant, we can then apply the SMM to convert the FIR TEQ into an equivalent IIR TEQ, efficiently.

4. Complexity Analysis

In this section, we discuss the issue of computational complexity of the proposed algorithm. We first compare the design complexity of the conventional SS and the proposed MS FIR TEQ. For fair comparison, we let the order of the conventional SS TEQ be equal to the equivalent order of the MS TEQ. The computational complexity of the SS MSSNR TEQ method is shown to be $38N^3/3 + IN^2$ (Yang and Kang, 2006), where N is the SS TEQ length. Thus, that of the proposed MS method is $38\Sigma_{l=1}^{V} N_l^3/3 + I\Sigma_{l=1}^{V} N_l^2$, where N_l is the l-th stage length of the proposed TEQ, V the number of multi-stages, and I the length of the CIR. Hence, the MS approach can greatly reduce the required computational complexity. As an example, we let $N = 16$, $V = 3$, and $I = 25$. The computational complexity of the MS TEQ is only 13.8% of that of the SS TEQ. The improvement comes from the fact that the computational complexity of the MSSNR method is $O(N^3)$. As a result, when N is large, the complexity grows fast.

We now consider the computational complexity of the SMM. For simplicity, let the data window size of the SMM, denoted as N', be equal to the FIR TEQ filter order N. It can be shown that the computational complexity of the SMM is $O(m[(P + Q + 1)^3 + (P + Q + 1)^2N + (P + Q + 1)N])$, where m is the iteration number. Although the computational complexity of the SMM has the same order as that of the MSSNR, its actual complexity will be much lower. This is due to two facts. First, as we will see in the next section, the SMM converges very fast, usually within five iterations. Second, in typical applications, $P + Q$ is usually much smaller than N. As a result, the overhead introduced by the SMM is not significant.

We now evaluate the computational complexity during the shortening operation. Note that the shortening operation has to be conducted for every input data sample. It solely depends on the number of taps in the TEQ. Thus, the computational complexity for the conventional FIR TEQ is $O(N)$, while that for the proposed IIR TEQ is $O(P + Q + 1)$. Since $P + Q$ is usually much smaller than N, the computational complexity of the IIR TEQ is much smaller than the FIR TEQ. Using a typical example, the proposed algorithm can save approximately up to 70% of the computations without compromising the BER performance (Lee and Wu, 2007). When M and L are large, as found in many practical OFDM systems, the reduction in computational complexity can be very significant.

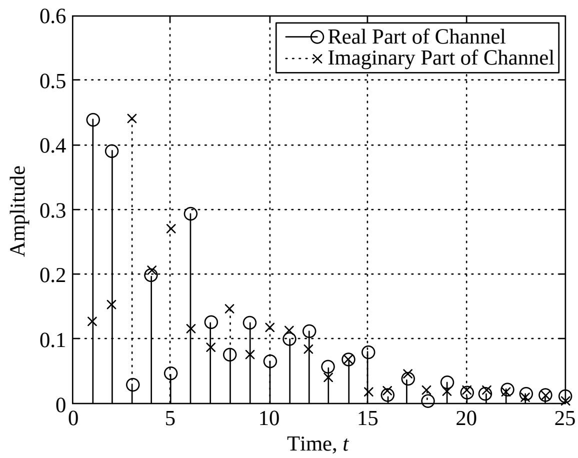

Fig. 4 A typical wireless channel impulse response

IV. SIMULATIONS

The simulation setup is described as follows. The OFDM system we use has symbol size of 64, and CP size of 16. The wireless channel is generated using an exponential-decay power profile. The channel is quasi-static and its response changes for every OFDM packet. In our simulations, we assume that the CIR is known or can be well estimated. The length of the wireless CIR is assumed to be 25, exceeding the CP size. A typical wireless CIR is shown in Fig. 4. Channel noise is modeled as the AWGN, and added at the channel output. All FIR TEQs considered in the simulations have an order of 16. They are designed with the MSSNR method (Melsa *et al*., 1996), which has been shown to be a good compromise between complexity and BER performance (Yang and Kang, 2006). In the figures shown, N and D stand for the number of zeros and poles used in the IIR TEQ, respectively.

In the first set of simulations, we evaluated the impact of the number of poles and zeros used in the IIR TEQ, and the convergence rate of the SMM. Fig. 5 shows the relationship between the ASE power and the iteration numbers, under the variation of the pole/zero order of the IIR TEQ. We can see that as the number of poles (or zeros) increases, the error power decreases. This is not surprising since more degrees of freedom can be used to reduce the ASE power. Fig. 6 shows the relationship between the residual ISI power and the iteration numbers, under the same setting as that in Fig. 5. Since the residual ISI power is not the criterion to be minimized, an IIR TEQ with higher order does not necessarily yield a smaller residual ISI power. Note that the residual ISI power relates to the BER, directly. Also shown in Figs. 5 and 6, we can see that the SMM converges to a stable value very quickly. The required number of iterations is typically below 5. We then consider the BER

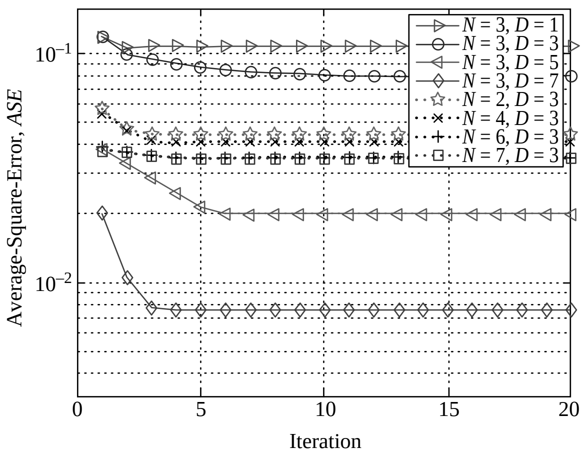

Fig. 5 Average-squared-error of IIR TEQ fitted with SMM (for various pole/zero orders)

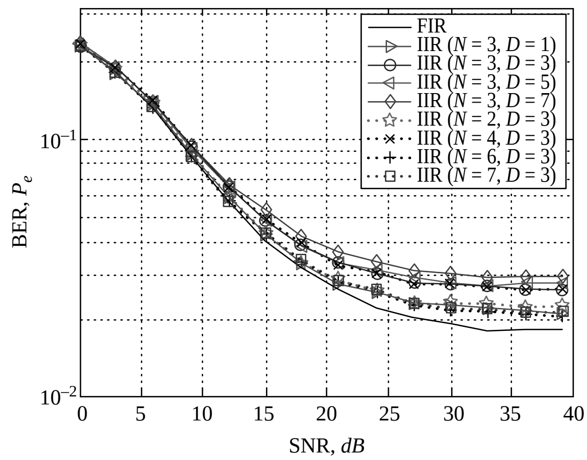

Fig. 7 BER performance of IIR TEQ fitted with SMM (for various pole/zero orders)

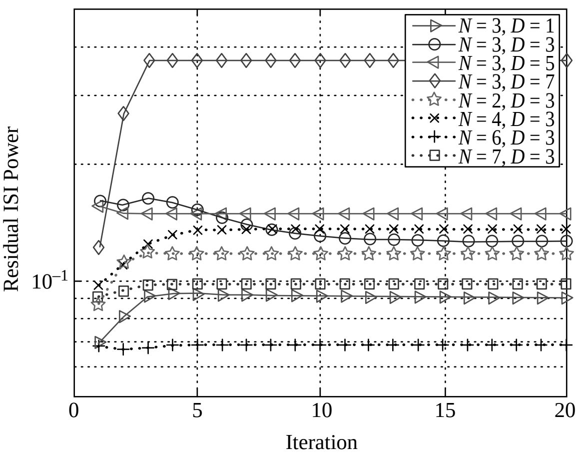

Fig. 6 Residual ISI power of IIR TEQ fitted with SMM (for various pole/zero orders)

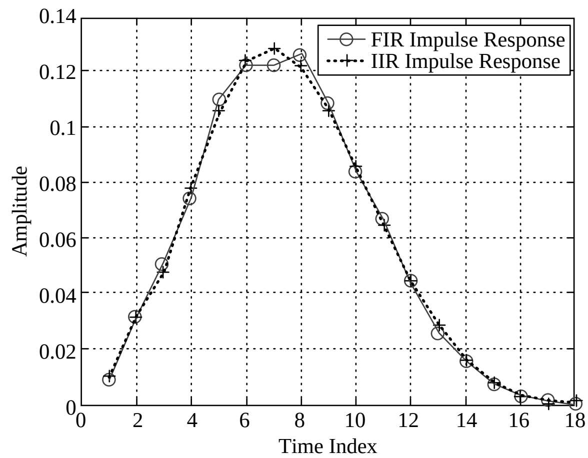

Fig. 8 Impulse response of an FIR TEQ and its fitted IIR TEQ

performance of the IIR TEQs discussed above, as shown in Fig. 7. The behavior of the BER performance in Fig. 7 is similar to that of the residual ISI power in Fig. 6. This is consistent with the assertion we just mentioned. Note that the choice of the order of the IIR TEQ is a compromise between the BER performance and the computational complexity. From simulations, we found that a good choice for the numbers of zeros and poles are 3 and 3, respectively. Fig. 8 shows an example of the impulse responses of the FIR filter and its equivalent IIR filter (fitted with the SMM). Here, the number of poles is 3, that of zeros is also 3, and the iteration number used in the SMM is 5. We can see that the fitted IIR TEQ can approach the original FIR TEQ well.

The performance and the computational complexity of the proposed algorithm depend on the parameters it uses such as the number of stages, the filter order at each stage, and the target channel length to be shortened (TLS) at each stage. Before the actual application of the proposed algorithm, we need to determine those parameters. We then need some design guidelines in order to obtain optimal results. Since theoretical analysis is difficult, we use simulations to do the job here. Table 1, 2, and 3 show the different parameter settings for simulations. The second column in the tables numbers the test TEQs used in the simulations, and the third column gives the number of stages used in the MS structure. The fourth column gives the order of the TEQ used at each stage, in which the notation $\lceil a, b, \cdots \rceil$ indicates that the TEQ order for the first stage is a, that for the second stage is b, and so on. The last column gives the TLS, where the notation $\lfloor c, d, \cdots \rfloor$ indicates that the TLS for the first stage is c, that for the 2nd stage is d, and so on.

The BER performance of the MSSNR (Melsa *et al.*, 1996) and the proposed method are then evaluated. All the simulations are evaluated with

Table 1 Simulation scenario 1 (for various IIR orders)

Fig. #	TEQ #	Multistage order	TEQ order per stage	TLS per stage
Fig. 9	TEQ #1a	2	⌈8, 9⌉	⌊4, 5⌋
	TEQ #1b	3	⌈6, 6, 6⌉	⌊3, 3, 3⌋
	TEQ #1c	4	⌈5, 5, 5, 4⌉	⌊3, 2, 2, 2⌋
	TEQ #1d	5	⌈4, 4, 4, 4, 4⌉	⌊2, 2, 2, 2, 1⌋

Table 2 Simulation scenario 2 (for various pole/zero orders per stage)

Fig. #	TEQ #	Multistage order	TEQ order per stage	TLS per stage
Fig. 10	TEQ #2a	2	⌈16, 16⌉	⌊4, 5⌋
	TEQ #2b	2	⌈13, 16⌉	⌊4, 5⌋
	TEQ #2c	2	⌈11, 16⌉	⌊4, 5⌋
	TEQ #2d	2	⌈8, 16⌉	⌊4, 5⌋
	TEQ #2e	2	⌈6, 16⌉	⌊4, 5⌋
	TEQ #2f	2	⌈4, 16⌉	⌊4, 5⌋
Fig. 11	TEQ #3a	2	⌈16, 16⌉	⌊4, 5⌋
	TEQ #3b	2	⌈16, 13⌉	⌊4, 5⌋
	TEQ #3c	2	⌈16, 11⌉	⌊4, 5⌋
	TEQ #3d	2	⌈16, 9⌉	⌊4, 5⌋
	TEQ #3e	2	⌈16, 6⌉	⌊4, 5⌋
	TEQ #3f	2	⌈16, 4⌉	⌊4, 5⌋

Table 3 Simulation Scenario 3 (for various TLS per stage)

Fig. #	TEQ #	Multistage order	TEQ order per stage	TLS per stage
Fig. 12	TEQ #4a	2	⌈16, 16⌉	⌊7, 2⌋
	TEQ #4b	2	⌈16, 16⌉	⌊6, 3⌋
	TEQ #4c	2	⌈16, 16⌉	⌊5, 4⌋
	TEQ #4d	2	⌈16, 16⌉	⌊4, 5⌋
	TEQ #4e	2	⌈16, 16⌉	⌊3, 6⌋
	TEQ #4f	2	⌈16, 16⌉	⌊2, 7⌋

1000 OFDM packets, where each OFDM packet contains 60 OFDM symbols. We first see the effect of the number of processing stages. Table 1 shows the parameter setting for this purpose. Here, we let the equivalent order of the MS TEQ be the same in all settings. The number of stages we tried are 2, 3, 4, and 5, corresponding to TEQ #1a, #1b, #1c, and #1d, respectively. The equivalent TEQ filter order is 16 for all 4 test TEQs. The TEQ filter orders are ⌈8, 9⌉, ⌈6, 6, 6⌉, ⌈5, 5, 5, 4⌉, and ⌈4, 4, 4, 4, 4⌉, respectively. And the TLSs for the test TEQs are ⌊4, 5⌋, ⌊3, 3, 3⌋, ⌊3, 2, 2, 2⌋, ⌊2, 2, 2, 2, 1⌋, respectively. Fig. 9 shows the BER performance comparison for settings in Table 1. We can see that as the number of stages increases, although the amount of computation can be reduced, the BER performance degrades. It is apparent that the BER performance for the SS TEQ (the plot of MSSNR TEQ) is superior to that of the multistage ones. This is not surprising since the original MSSNR design is a joint optimization approach (for all tap weights), while the MS structure is not. From Fig. 9,

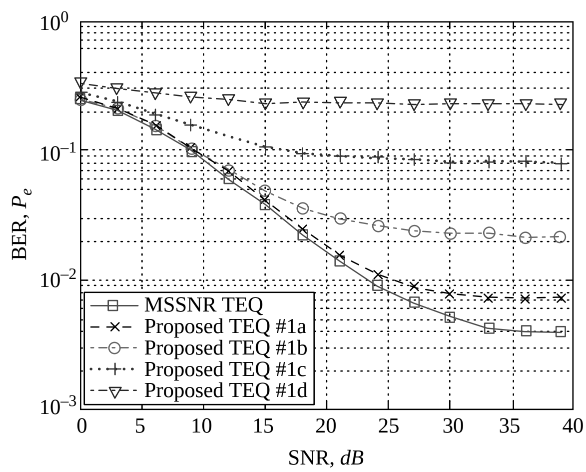

Fig. 9 BER performance of experiment #1 (for various stage numbers)

we can see that it is adequate to let the number of stages be 2 or 3 (that is, TEQ #1a and #1b), a good compromise between complexity and BER performance.

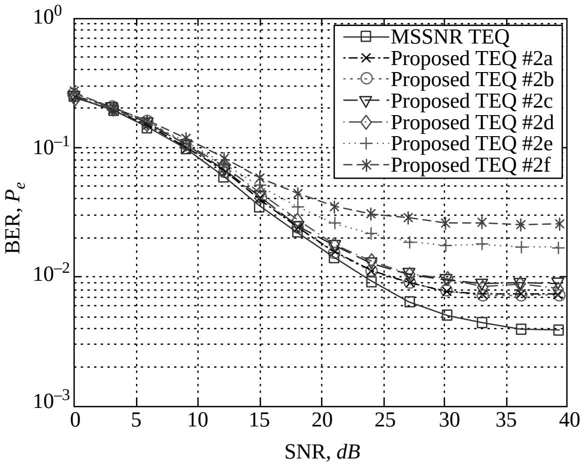

Fig. 10 BER performance of experiment #2 (for various TEQ orders in the first stage)

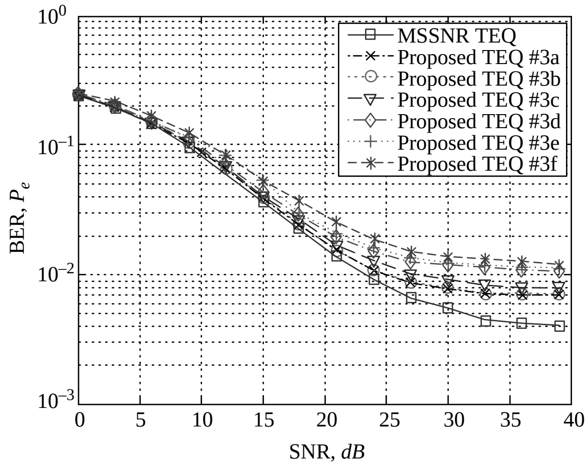

Fig. 11 BER performance of experiment #3 (for various TEQ orders in the second stage)

We then evaluate the effect of the filter order used at each stage. Table 2 gives the setting for simulations. Here, the number of stages is set as 2, the highest order for each stage is set as 16, and the TLSs for the test TEQs are all fixed to $\lfloor 4, 5 \rfloor$. Figs. 10 and 11 show the simulation results. From the figures, we can see that the larger the filter order, the better the BER performance we can have. However, as the filter order of one stage increases, the computational complexity increases accordingly. Thus, there is a compromise between the TEQ order and the performance. Also from Fig. 11, we can see that as the filter order at the second stage decreases (that in the first stage is fixed), the performance degrades, but the degradation is not severe. In contrast, from Fig. 10, we see that as the filter order of the first stage decreases (that in the second stage is fixed), the performance degradation is more severe. This is because the residual ISI of the first stage will propagate to the second stage, and the TEQ in the second stage cannot compensate for that effect completely. Thus, the TEQs in early stages play more important roles than those in following stages. We should give a higher order for the TEQs in the early stages. On the other hand, the shortening work is also relatively easier at early stages, and a higher order for the TEQ may be not required. In summary, we may let the TEQ order be roughly equal for all stages. This is an important property the MS structure has.

Table 3 shows the settings of the TEQ in scenarios with various TLSs. Here, the number of stages is still set to 2, and the TEQ tap length for both stages is set to 16. Fig. 12 shows the simulation results. We see that if the TLS of the first stage is in a smaller order, such as the case of TEQ #4d, #4e and #4f, the BER performance is generally better than that of other cases. The reason is similar to the results in Figs. 10 and 11. As the TLS of the first stage increases, the residual ISI of the first stage will become larger and it propagates to the second stage. The TEQ in the second stage cannot compensate for that effect. However, if the TLS of the first stage becomes too small, the corresponding TLS of the second stage becomes large and the required filter order of the second stage becomes high. Then the computational complexity of the TEQ will be increased. With a larger residual ISI, no matter whether in the first or second stage, the performance of the TEQ will be degraded. Thus, it is better to distribute the required TLS to all stages, evenly. This is another important property the MS structure has.

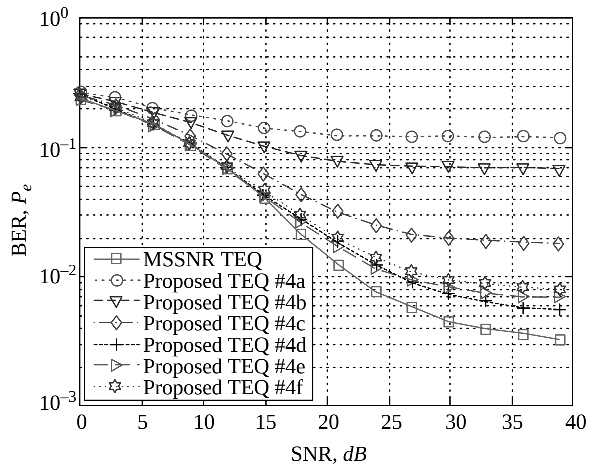

Fig. 12 BER performance of experiment #4 (for various TLS per stage)

Based on the simulation results, we can obtain some design guidelines for the MS design. Firstly, the number of stages used should not be too large, i.e., 2 or 3. Secondly, the filter order for each stage

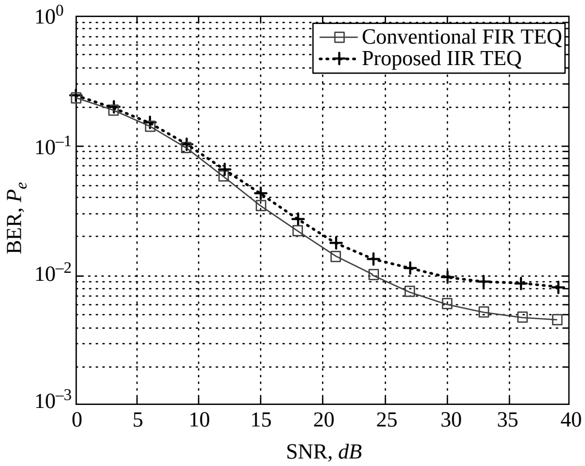

Fig. 13 BER comparison of conventional FIR TEQ and proposed IIR TEQ

can be made roughly equal. The order is selected with a compromise between complexity and performance. For example, an appropriate filter order for a two-stage structure may be $\lceil 8, 9 \rceil$. Thirdly, the total TLS can also be evenly distributed to all TEQs. In other words, the TLS for each stage can also be set roughly equal. Or, that in early stages is somewhat lower. For example, an appropriate TLS value for a two-stage structure can be $\lfloor 4, 5 \rfloor$ or $\lfloor 3, 6 \rfloor$.

According to the above design guidelines, we can determine proper values for the parameters. It turns out that the number of stages is 2, the filter order per stage is $\lceil 8, 9 \rceil$, the TLS is $\lfloor 4, 5 \rfloor$. Fig. 13 shows the simulation results with the settings. As we can see, the BER performance of the proposed IIR TEQ is slightly worse than that of the original FIR TEQ. The complexity ratio of the IIR TEQ compared to that of the FIR TEQ in TEQ derivation, and in shortening, is only 33% and 37%, respectively. We can then conclude that the proposed IIR TEQ is much more efficient than the conventional FIR TEQ.

V. CONCLUSIONS

In this paper, we propose to use the IIR TEQ for channel shortening in OFDM systems. The objective is to reduce the computational complexity of the conventional FIR TEQ. Since the direct derivation of the IIR TEQ is difficult, we use a simpler two-step approach. In the first step, we use a multistage structure to obtain the FIR TEQ. In the second step, we use the SMM to convert the FIR TEQ into an equivalent IIR TEQ. Since the ideal TEQ exhibits low-order IIR characteristics, the order of the IIR TEQ can be much lower than that of the FIR TEQ. Also, the TEQ derivation with the MS structure can be much more efficient than the conventional SS structure. We then obtain a low-complexity TEQ, both in derivation and the shortening phase. Simulations show that while the proposed method can reduce the computational complexity significantly, its performance is almost as good as that of the existing methods.

NOMENCLATURE

$A(z)$, $A_m(z)$	zero part of $C(z)$ and $C_m(z)$, respectively
$\mathbb{B}_l$	Cholesky decomposition of $\boldsymbol{B}_l$
$B(z)$, $B_m(z)$	pole part of $C(z)$ and $C_m(z)$, respectively
$c(n)$	impulse response of an IIR plant.
$C(z)$	transfer function of an IIR system.
$C_m(z)$	estimated transfer function of an IIR system $C(z)$ in the m^{th} iteration
$\boldsymbol{D}_S$	masked window to $\boldsymbol{g}_S$, $\boldsymbol{D}_S = \text{diag}[\boldsymbol{1}_L^T, \boldsymbol{0}_{J-L}^T]$
$\boldsymbol{D}_I$	masked window to $\boldsymbol{g}_I$, $\boldsymbol{D}_I = \boldsymbol{I}_J - \boldsymbol{D}_S = \text{diag}[\boldsymbol{0}_L^T, \boldsymbol{1}_{J-L}^T]$
$\boldsymbol{D}_{S,l}$	masked window to $\boldsymbol{g}_{S,l}$, $\boldsymbol{D}_{S,l} = \text{diag}[\boldsymbol{1}_{L_l}^T, \boldsymbol{0}_{J_l-L_l}^T]$
$\boldsymbol{D}_{I,l}$	masked window to $\boldsymbol{g}_{I,l}$, $\boldsymbol{D}_{I,l} = \boldsymbol{I}_{J_l} - \boldsymbol{D}_{S,l} = \text{diag}[\boldsymbol{0}_{L_l}^T, \boldsymbol{1}_{J_l-L_l}^T]$
$\boldsymbol{d}_i$	i^{th} transmitted data symbol by taking M-IDFT to $\tilde{\boldsymbol{d}}_i$ on the transmitter side
$d_i(k)$, $\tilde{d}_i(k)$	$(k+1)^{\text{th}}$, element of $\boldsymbol{d}_i$ and $\tilde{\boldsymbol{d}}_i$, respectively
$\tilde{\boldsymbol{d}}_i$	i^{th} transmitted OFDM symbol on the transmitter side
$e_m(n)$	error signal between $u(n)$ and $v(n)$
$\boldsymbol{e}_m(n)$	column vector formed by $e_m(n)$
$\boldsymbol{F}$	$M \times M$ DFT matrix
$\boldsymbol{f}(k)$	k^{th} column vector of the DFT matrix $\boldsymbol{F}$
$g(n)$	equivalent channel response
$\boldsymbol{g}$	vector form of the equivalent channel response
$\boldsymbol{g}_l$	l^{th} stage of the equivalent channel response
$\boldsymbol{g}_S$	desired shortened channel response
$\boldsymbol{g}_I$	residual ISI response
$\boldsymbol{g}_{S,l}$	desired shortened channel response at the l^{th} stage
$\boldsymbol{g}_{I,l}$	residual ISI response at the l^{th} stage
$g_S(l)$, $g_I(l)$	$(l+1)^{\text{th}}$ element of $\boldsymbol{g}_S$ and $\boldsymbol{g}_I$, respectively
$\boldsymbol{H}$	channel matrix
$\boldsymbol{H}_l$	l^{th} stage channel matrix
$\boldsymbol{h}$	channel impulse response
$h(n)$	$(n+1)^{\text{th}}$ element of $\boldsymbol{h}$
$H(z)$	transfer function of channel $\boldsymbol{h}$
$H_0(z)$, $H_1(z)$, $H_2(z)$	transfer function contained m_0, m_1, and m_2 zeros located inside the unit circle, respectively
I	channel length
i	index of the OFDM symbol
$\boldsymbol{I}_p$	a $p \times p$ identity matrix
J	length of the equivalent channel response

J_l	ECR length of the l^{th} stage
K	length of an OFDM symbol with CP
k	subchannel index in the frequency domain
L	length of CP
M	length of an OFDM symbol, also the DFT size
N	TEQ length
N_l	TEQ length at the l^{th} stage
n	time domain index of the signal
P, Q	order of $B(z)$ and $A(z)$, respectively
P_l	number for the TEQ to shorten the CIR
$r(n)$	output signal of an IIR system
SSNR_l	shortened SNR at the l^{th} stage
$\text{SSNR}_{l,O}$	optimal shortened SNR at the l^{th} stage
$u(n)$	output signal that $r(n)$ passes through $B_{m-1}^{-1}(z)$
$\boldsymbol{v}_i$	i^{th} noise symbol vector at the TEQ output
$v(n)$	TEQ-filtered output of the channel noise $\eta(n)$
V	number of the multi-stage
$\boldsymbol{w}$	TEQ vector
$\boldsymbol{w}_l$	TEQ vector in the l^{th} stage
$\boldsymbol{w}_O$	optimal TEQ vector
$\boldsymbol{w}_{l,O}$	optimal TEQ vector in the l^{th} stage
$W(z)$	transfer function of a TEQ vector $\boldsymbol{w}$
$W_0(z)$	transfer function of an FIR filter
$w(n)$	element of TEQ vector
$x(n)$	transmitted signal
$x_0(n)$	noise-free channel output signal
$y(n)$	TEQ-filtered output of $x_0(n)$
$y_S(n)$, $y_I(n)$	desired part and residual ISI part of $y(n)$, respectively
$\boldsymbol{y}_i$	i^{th} received signal-only OFDM symbol
$\boldsymbol{y}_{S,i}$, $\boldsymbol{y}_{I,i}$	desired part and residual ISI part of $\boldsymbol{y}_i$, respectively
$\alpha_j(m)$, $\beta_j(m)$	parameter of $A_m(z)$ and $B_m(z)$, respectively
η_i	i^{th} noise symbol vector at the TEQ input
$\eta(n)$	AWGN signal
λ_l	maximum eigenvalue of $\mathbb{A}_l$
$\boldsymbol{\Theta}$	column vector formed by the filter parameters $\alpha_j(m)$ and $\beta_j(m)$
$\boldsymbol{\Phi}$	column vector formed by the signal $u(n)$ and $v(n)$

REFERENCES

Al-Dhahir, N., and Cioffi, J. M., 1996, "Optimum Finite-length Equalization for Multicarrier Transceivers," *IEEE Transactions on Communications*, Vol. 44, No. 1, pp. 56-64.

Arslan, G., Evans, B. L., and Kiaei, S., 2001, "Equalization for Discrete Multitone Transceivers to Maximize Bit Rate," *IEEE Transactions on Signal Processing*, Vol. 49, No. 12, pp. 3123-3135.

Cheng, M. H., and Stonick, V. L., 1994, "Convergence, Convergence Point and Convergence Rate for Steiglitz-McBride Method; a Unified Approach," *IEEE International Conference on Acoustics, Speech, and Signal Processing-94*, Adelaide, South Australia, Vol. 3, pp. 477-480.

Chong, K. P., and Zak, S. H., 2001, *An Introduction to Optimization*, 3rd ed., John Wiley & Sons, Hoboken, New Jersey, USA.

Chow, J., and Cioffi, J. M., 1992, "A Cost Effective Maximum Likelihood Receiver for Multicarrier Systems," *International Conference Communication*, Chicago, USA, Vol. 2, pp. 948-952.

Crespo, P., and Honig, M., 1991, "Pole-Zero Decision Feedback Equalization with a Rapidly Converging Adaptive IIR Algorithm," *IEEE Journal on Selected Areas Communications*, Vol. 9, No. 6, pp. 817-829.

Henkel, W., Taubock, G., Odling, P., Borjesson, P. O., and Petersson, N., 2002, "The Cyclic Prefix of OFDM/DMT - An Analysis," *International Zurich Seminar on Broadband Communications, Access, Transmission, Networking*, Zurich, Switzerland, Vol. 2, pp. 22.1-22.3.

Kim, J., and Powers, E. J., 2005, "Subsymbol Equalization for Discrete Multitone Systems," *IEEE Transactions on Communications*, Vol. 53, No. 9, pp. 1551-1560.

Lee, C. F., and Wu, W. R., 2007, "A Multistage IIR Time Domain Equalizer for OFDM Systems," *Proceedings of IEEE Region 10 Annual Conference, Speech and Image Technologies for Computing and Telecommunications., TENCON '07*, Taipei, Taiwan, pp. 1-5.

Leus, G., and Moonen, M., 2003, "Per-Tone Equalization for MIMO OFDM Systems," *IEEE Transactions on Signal Processing*, Vol. 51, No. 11, pp. 2965-2975.

Martin, R. K., Vanbleu, K., Ding, M., Ysebaert, G., Milosevic, M., Evans, B. L., Moonen, M., and Johnson, C. R., 2005, "Unification and Evaluation of Equalization Structures and Design Algorithms for Discrete Multitone Modulation Systems," *IEEE Transactions on Signal Processing*, Vol. 53, No. 10, pp. 3880-3894.

Melsa, P. J., Younce, R. C., and Rohrs, C. E., 1996, "Impulse Response Shortening for Discrete Multitone Transceiver," *IEEE Transactions on Communications*, Vol. 44, No. 12, pp. 1662-1672.

Netto, S. L., Diniz, P. S. R., and Agathoklis, P., 1995, "Adaptive IIR Filtering Algorithms for System Identification: A General Framework," *IEEE Transactions on Education*, Vol. 38, No. 2, pp. 54-66.

Rawal, D., and Vijaykumar, C., 2008, "QR-RLS Based Adaptive Channel TEQ for OFDM Wireless LAN," *Proceedings of International Conference on Signal Processing, Communication and Networking*, Chennai, India, pp. 46-51.

Rawal, D., Vijaykumar, C., and Arya, K. K., 2007, "QR-RLS Based Adaptive Channel Shortening IIR-TEQ for OFDM Wireless LAN," *Proceeding Wireless Communication and Sensor Networks*, Havana, Cuba, pp. 21-26.

Regalia, P. A., Mboup, M. and Ashari, M., 1997, "On the Existence of Stationary Points for the Steiglitz-McBride Algorithm," *IEEE Transactions on Automatic Control*, Vol. 42, No. 11, pp. 1592-1596.

Steiglitz, K., and McBride, L. E., 1965, "A Technique for the Identification of Linear System," *IEEE Transactions on Automatic Control*, Vol. AC-10, No. 7, pp. 461-464.

Stoica, P. and Söderström, T., 1981, "The Steiglitz-McBride Identification Algorithm Revised - Convergence Analysis and Accuracy Aspects," *IEEE Transactions on Automatic Control*, Vol. AC-26, No. 6, pp. 712-717.

Van Acker, K., Leus, G., Moonen, M., van de Wiel, O., and Pollet, T., 2001, "Per Tone Equalization for DMT-based Systems," *IEEE Transactions on Communications*, Vol. 49, No. 1, pp. 109-119.

Vanbleu, K., Ysebaert, G., Cuypers, G., Moonen, M., and Acker, K. Van, 2004, "Bitrate Maximizing Time-domain Equalizer Design for DMT-based Systems," *IEEE Transactions on Communications*, Vol. 52, No. 6, pp. 871-876.

Wu, W. R., and Lee, C. F., 2005, "Time Domain Equalization for DMT Transceivers: A New Result," *2005 International Sumposium on Intelligent Signal Processing and Communication Systems*, Hong-Kong, China, pp. 553-556.

Wu, W. R., and Lee, C. F., 2007, "An IIR Time Domain Equalizer for OFDM Systems," *IEEE VTS Asia Pacific Wireless Communications Symposium 2007*, Hsin-Chu, Taiwan, pp. 1-5.

Yang, L., and Kang, C. G., 2006, "Design of Novel Time-Domain Equalizer for OFDM Systems," *IEICE Transactions on Communications*, Vol. E89-B, No. 10, pp. 2940-2944.

Zhang, J. M., and Ser, W., 2002, "New Criterion for Time-domain Equalizer Design in OFDM Systems," *IEEE International Conference on Acoustics, Speech, and Signal Processing-'02*, Orlando, USA, Vol. 3, pp. 2561-2564.

Manuscript Received: May 21, 2009
Revision Received: Nov. 12, 2009
and Accepted: Dec. 12, 2009