國 立 交 通 大 學

電 信 工 程 學 系

碩 士 論 文

合作式自動重傳請求使用投機式束波成形

Cooperative ARQ via Opportunistic Beamforming

研 究 生：李晉豪

指導教授：伍紹勳　博士

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合 作 式 自 動 重 傳 請 求 使 用 投 機 式 束 波 成 形

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研 究 生：李晉豪　　Student：Jing-Haw Li

指導教授：伍紹勳　　Advisor：Dr. Sau-Hsuan Wu

國 立 交 通 大 學

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合作式自動重傳請求使用投機式束波成形研究

生:李晉豪　　　　　　　　指導教授:伍紹勳

國立交通大學電信工程學係 碩士班

摘要

針對於合作式通訊上在解碼傳送與放大傳送的兩個協定上使用合作式自動重傳投機式束波成形來探討錯誤率與有效的傳送資料量，藉由錯誤率的分析可知，在解碼傳送的協定上，使用兩個最佳的接力端幫助傳輸可到達 的錯誤率與使用所有解碼成功的接力端來幫助傳輸是幾乎一樣的。然而，針對於放大傳送的協定而言，假設全部可用的接力端爲M個，使用一個最佳的接力端與全部使用所有的接力端的錯誤率有M!倍的差距。因此，我們探討使用i個最佳的接力端來傳輸的效能與使用所有接力端的效能，此外， 我們利用錯誤率分析的結果，來分析合作式重傳機制在時間上與空間上的 自由度，最後可知，對於解碼傳送協定，使用type-IV的機制可以得到最好 的傳送資料量，而針對放大傳送協定，是要使用更多的接力端才可以增加傳送資料量。

Cooperative ARQ via Opportunistic Beamforming

Student:Jing-Haw Li Advisor:Dr.Sau-Hsuan Wu

Department of Communication Engineering
National Chiao Tung University

Abstract

The outage probability and effective throughput of cooperative automatic request for retransmission (ARQ) are investigated for opportunistic cooperative beamforming using i relays (OC-BF-i) for decode-and-forward (DF) and amplify-and-forward (AF) protocols. According to the outage analysis, the outage performance of the proposed opportunistic cooperation schemes of using the two relay nodes is almost indistinguishable from that of using all decoding relays. However, the cooperative scheme with total relays number M for AF protocol has M! SNR offset gap between opportunistic relaying and using all relays transmission. Thus, we also discuss the outage performance for AF OC-BF-i. Motivated by outage result, cooperative ARQ schemes are developed for opportunistic cooperative beamforming for these two protocols to exploit the spatial and temporal cooperative diversities simultaneously. Analysis shows that the effective way to improve throughput is using type-IV ARQ scheme for DF OC-BF scheme. For AF OC-BF, the throughput can be more effectively improved by using more best relays.

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Chapter 1

Introduction

In wireless communications, the signal qualities often suffer from severe channel fades. The fading effects of wireless channels can be effectively compensated through the diversity. The signal is transmitted over the independent channel realization with suitable receiver combining to average the channel fading. The spatial diversity can be exploited by using multiple-input and multiple-output (MIMO) antennas [1]. However, due to the space limitation of mobile devices, it is sometimes impractical to use multiple antennas to obtain spatial diversities. In [2, 3], the spatial diversity of virtual antenna arrays is introduced by using cooperative relaying. This work obtains the repetition based cooperative diversity algorithm and space-time-code (STC) diversity algorithm to achieve full diversity for decode-and-forward (DF) protocol. It is further shown in [4, 5] that the full diversity of cooperative transmission can be achieved even using the node that with the best channel link quality between relays and destination for relaying often referred to as the opportunistic relaying.

Extended from the concept of opportunistic relaying, cooperative beamforming (Co-BF) schemes are also introduced in [6, 7]. Motivated by the simplicity and effectiveness of opportunistic relaying, some more recent efforts have been made to improve the performance of opportunistic relaying by choosing more best relays. Approximated analysis for the outage probabilities of some opportunistic cooperative beamforming (OC-BF) schemes can be found in [8] and it provides that the performance using best two relays can achieve the performance with all potential relays for beamforming. However, the work in [7] provides that the performance gap between opportunistic relaying and cooperative beamforming (Co-BF) with M relays is $M!$ times for amplify-and-forward (AF) protocol.

In view of the advantage of opportunistic relaying, we investigate the outage performance of (DF) and (AF) beamforming by choosing the best few nodes for relaying. For brevity, this beamforming scheme is referred to as the opportunistic cooperative beamforming (OC-BF) in the sequel. Our results show that, in contrast to the DF OC-BF scheme, the performance of the AF OC-BF scheme doesn't quickly converge to that of the AF Co-BF as the number of best nodes increases. The gap between the outage probabilities of the OC-BF and the Co-BF is characterized by using the diversity analysis in high signal to noise ratio (SNR) regime. In addition, the AF OC-BF performs better than the DF OC-BF when the number of beamforming nodes is greater than one and the performance of the AF opportunistic relaying is very closed to that of the DF opportunistic relaying. Thus, for DF OC-BF, choosing the best two relays is sufficient to achieve the performance of DF Co-BF. While for AF protocols, it requires more best relays to achieve the outage performance of AF Co-BF.

In contrast to the rich research results in the outage analysis for cooperative transmission, the performance of cooperative automatic request for retransmission (ARQ) is rather less investigated. The average throughput of a cooperative hybrid-ARQ (HARQ) scheme is reported in [9] using the distributed space-time-code (DSTC) scheme. Motivated by the effectiveness of opportunistic cooperation and the extra degrees of freedom brought upon by ARQ. In this research, we attempt to simultaneously exploit the spatial and temporal diversities via cooperative ARQ. We proposes several ARQ transmission schemes to analyze the corresponding outage and the effective throughput of cooperative

ARQ for the DF and AF opportunistic cooperative beamforming schemes.

Our results show that the outage performance can be improved a lot by adding more one best relay for AF OC-BF and the outage performance by using best two relays can achieve the performance of Co-BF for DF protocol. Based on these results, we further investigate the outage performance and characterize the average throughput for cooperative ARQs of OC-BF, respectively. Analysis shows that the effective throughput can be increased by using ARQs scheme for DF and AF scheme in low SNR regime. Beside, the throughput can be improved a lot by adding more one best relay than using more one time ARQ scheme for AF protocol.

Chapter 2

System Model

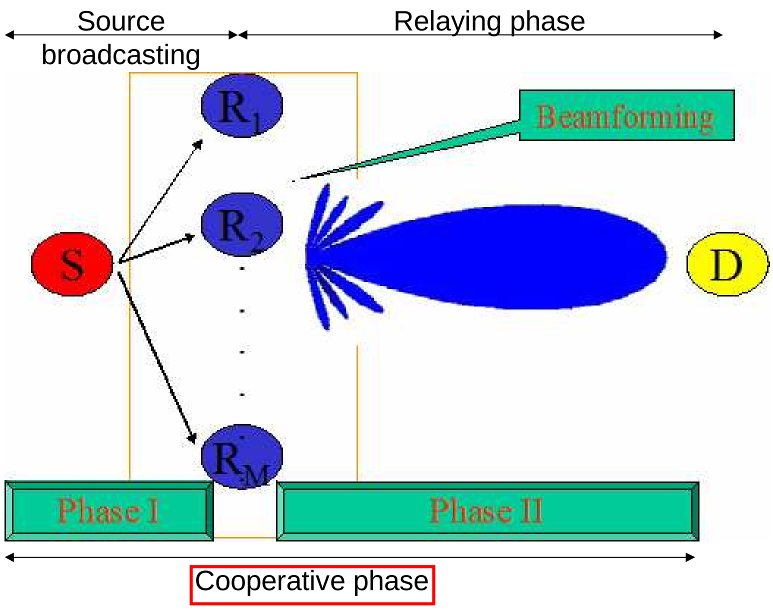

Figure 2.1: Cooperative system.

We consider a cooperative relay network with two phase transmission. There are M potential relays in the network to help re-transmit signals by using AF and DF protocols as shown in Fig. 2.1. In phase I, the source transmit signal to all relay nodes and destination. For DF protocol, the decoded correctly relay nodes collaboratively retransmit signal to destination. However, the relay node in AF protocol amplify the received signal to destination. Following, we individual describes the system model as follow:

2.1 System Model for DF Protocol

The cooperative relay network with two phase transmission for DF protocol. The source broadcasts the signal to all potential relays and destination in the phase one transmission. The received signals at the relay node i and destination are

$$r_i = \sqrt{P_s} h_{s,i} x + n_{s,i}, i = 1, 2 \cdots M. \quad (2.1)$$

$$r_{d,1} = \sqrt{P_s} h_{s,d} x + n_{s,d} \quad (2.2)$$

where the $\sqrt{P_s}$ is the transmitted power at the source node. The $h_{s,i}$ and $h_{s,d}$ are the source to relay and source to destination complex Gaussian channel with zeros mean and variance equal to one. The $n_{s,i}$ and $n_{s,d}$ are the additive white Gaussian noise (AWGN) with zero mean and variance N_0. In the phase II that we called the relaying phase, the successfully decoded relays transmit signal to destination collaboratively. For brevity in the sequel, we called the set that obtains the successfully decoded relays as decoding set,i.e.,$\mathcal{D}(\mathcal{S})$. The received signal to destination in relaying phase is

$$r_{d,2} = \sum_{i \epsilon \mathcal{D}(\mathcal{S})} \sqrt{P_r} h_{i,d} w_{i,DF} x + n_d \quad (2.3)$$

where $\sqrt{P_r}$ is the total relay power and $h_{i,d}$ is between relay and destination complex Gaussian channel with zero mean and variance one, and the n_d is the AWGN noise at the destination. The $w_{i,DF}$ is the beamforming weight to combat the channel fading and $\sum_{i=1}^{|\mathcal{D}(\mathcal{S})|} |w_{i,DF}|^2 = 1$. We assume that the all relay nodes know the channel state information (CSI) between the relay nodes and destination with perfect feedback from destination. The total received signal to noise ratio (SNR) can be shown as:

$$SNR_d = \frac{P_s|h_{s,d}|^2}{N_0} + \frac{P_r|\sum_{i\epsilon\mathcal{D}(\mathcal{S})} h_{i,d}w_{i,DF}|^2}{N_0} \tag{2.4}$$

The optimum weight can be obtained by maximizing received SNR after two phase combing. By cauchy inequality with equality hold, the $w_{i,DF} = kh_{i,d}\dagger$ under constrain $\sum_{i=1}^{|\mathcal{D}(\mathcal{S})|} |w_{i,DF}|^2 = 1$ is shown as

$$\begin{aligned} k^2 \sum_{i=1}^{|\mathcal{D}(\mathcal{S})|} |h_{i,d}|^2 = 1 \\ k = \frac{1}{\sqrt{\sum_{i=1}^{|\mathcal{D}(\mathcal{S})|} |h_{i,d}|^2}} \end{aligned} \tag{2.5}$$

The optimum weight is given by:

$$w_{i,DF} = \frac{h_{i,d}^{\dagger}}{\sqrt{\sum_{i\epsilon\mathcal{D}(\mathcal{S})} |h_{i,d}|^2}} \tag{2.6}$$

Substituting the optimum weight into equation (2.4), we can obtain the maximum received SNR as

$$SNR_{max,DF} = \frac{P_s|h_{s,d}|^2}{N_0} + \frac{\sum_{i\epsilon\mathcal{D}(\mathcal{S})} P_r|h_{r_i,d}|^2}{N_0} \tag{2.7}$$

$$= (1-\alpha)\gamma\alpha_0 + \alpha\{\sum_{i\epsilon\mathcal{D}(\mathcal{S})} \gamma\beta_i\} \tag{2.8}$$

where $\gamma = \frac{P}{N_0}$, $P_s = (1-\alpha)P$, $P_r = \alpha P$ and $\beta_i = |h_{i,d}|^2$ and $\alpha_0 = |h_{s,d}|^2$ are $\sim Exp(1)$ The α is the factor to adjust power between source and relay nodes ie. $0 < \alpha < 1$. $w = (1-\alpha)\gamma\alpha_0$ is exponential distribution with parameter $\lambda_0 = \frac{1}{(1-\alpha)\gamma}$ and $\alpha\gamma\beta_i$ is also exponential random variable with parameter $\frac{1}{\gamma\alpha}$

2.2 System Model for AF Protocol

In the beginning of the transmission, the source node broadcasts signal to all relays and the destination. The received signals at the relay node i and the destination are

$$\begin{aligned} r_i &= \sqrt{P_s} h_{s,i} x + n_{s,i}, i = 1, 2 \cdots M. &(2.9)\\ r_{d,1} &= \sqrt{P_s} h_{s,d} x + n_{s,d} &(2.10) \end{aligned}$$

where the $\sqrt{P_s}$ is the transmitted power at the source node. The $h_{s,i}$ and $h_{s,d}$ are complex Gaussian channel of the source to relay and source to destination channel. The $n_{s,i}$ and $n_{s,d}$ are the additive white Gaussian noise (AWGN) with zero mean and variance N_0. The relays received signal from the phase I and amplify the signal to destination in the relaying phase. The signal to be transmitted from the relay is

$$x_i = \frac{r_i}{\sqrt{E\{|r_i|^2\}}} = \frac{\sqrt{P_s} h_{s,i} x + n_{s,i}}{\sqrt{P_s |h_{s,i}|^2 + N_o}} \tag{2.11}$$

All relay nodes collaboratively transmit signal to the destination, with the $i - th$ relay weighted by $w_{i,AF}$. Thus, the received signal at the destination is

$$\begin{aligned} r_{d,2} &= \sum_{i=1}^{M} \sqrt{P_r} w_{i,AF} h_{i,d} x_i + n_d \\ &= \sum_{i=1}^{M} \frac{\sqrt{P_s P_r} w_{i,AF} h_{s,i} h_{i,d} x}{\sqrt{P_s |h_{s,i}|^2 + N_0}} + \sum_{i=1}^{M} \frac{\sqrt{P_r} w_{i,AF} h_{i,d} n_{s,i}}{\sqrt{P_s |h_{s,i}|^2 + N_0}} + n_d \\ &= \sum_{i=1}^{M} w_{i,AF} \tilde{h}_i x + \tilde{n}_d &(2.12) \end{aligned}$$

where the P_r is the total relay power, $h_{i,d}$ is the complex Gaussian channel between relay and destination , and the n_d is the AWGN noise at the destination. To keep the total relay power is P_r, thus the beamforming weight satisfy $\sum_{i=1}^{M} |w_{i,AF}|^2 = 1$. In equation(2.12), we define an equivalent channel and noise term to simply the expression.

The equivalent channel through relay i is

$$\tilde{h}_i = \frac{\sqrt{P_s P_r} h_{s,i} h_{i,d}}{\sqrt{P_s |h_{s,i}|^2 + N_0}} \tag{2.13}$$

and equivalent noise $\tilde{n_d}$ is circularly symmetric Gaussian distributed as

$$\tilde{n}_d \sim CN(0, (1 + \sum_{i=1}^{M} |w_{i,AF}|^2 |\mathbf{H}_{i,i}|^2) N_0) \tag{2.14}$$

where $\mathbf{H}$ is the diagonal matrix whose $i-th$ element $(\mathbf{H}_{i,i})$, $\mathbf{H}_{i,i} = \frac{\sqrt{P_r} h_{i,d}}{\sqrt{P_s |h_{s,i}|^2 + N_0}}$.

Maximum ratio combing (MRC) of the signal over two transmission phases. Thus, the total received signal to noise ratio (SNR) can be shown as

$$SNR_d = \frac{P_s |h_{s,d}|^2}{N_0} + \frac{|\sum_{i=1}^{M} w_i \tilde{h}_i|^2}{N_0 (1 + \sum_{i=1}^{M} |w_i \mathbf{H}_{i,i}|^2)} \tag{2.15}$$

we can rewrite the second term in equation(2.15) as

$$\frac{\mathbf{w}^\dagger \mathbf{h} \mathbf{h}^\dagger \mathbf{w}}{N_0 \mathbf{w}^\dagger (\mathbf{I} + \mathbf{H}\mathbf{H}^\dagger) \mathbf{w}}. \tag{2.16}$$

where $\mathbf{w} \triangleq [w_1 w_2 \cdots w_M]^T$, $\mathbf{h} \triangleq [h_1 h_2 \cdots h_M]^T$

From [7], the optimum beamforming weight is given by

$$\mathbf{w}^\dagger = \frac{(\mathbf{I} + \mathbf{H}\mathbf{H}^\dagger)^{-1} \mathbf{h}}{||(\mathbf{I} + \mathbf{H}\mathbf{H}^\dagger)^{-1} \mathbf{h}||_2} \tag{2.17}$$

Substituting the optimum beamforming weight into the total received SNR formula, thus it can find the maximum received SNR as follow:

$$\begin{aligned} SNR_{opt} &= \frac{P_s |h_{s,d}|^2}{N_o} + \sum_{i=1}^{M} \frac{P_s P_r |h_{s,i}|^2 |h_{i,d}|^2}{P_s |h_{s,i}|^2 + P_r |h_{i,d}|^2 + N_0} \\ &= (1-\alpha)(\gamma \alpha_0) + \sum_{i=1}^{M} \frac{\alpha (1-\alpha) \gamma^2 \alpha_i \beta_i}{1 + (1-\alpha) \gamma \alpha_i + \alpha \gamma \beta_i} \end{aligned} \tag{2.18}$$

where the $\gamma = \frac{P}{N_0}$, and $\alpha_i = |h_{s,i}|^2$, $\beta_i = |h_{i,d}|^2$ are $\sim Exp(1)$. Especially, the $w = (1-\alpha)(\gamma\alpha_0)$ is exponential distribution with parameter $\lambda_0 = \frac{1}{(1-\alpha)\gamma}$. $(1-\alpha)\gamma\alpha_i$ and $\alpha\gamma\beta_i$ are also exponential distribution with parameter with parameter $\lambda_1 = \frac{1}{(1-\alpha)\gamma}$, $\lambda_2 = \frac{1}{\alpha\gamma}$

Chapter 3

Outage Analysis for Opportunistic Cooperative Beamforming

The outage analysis is widely studied of the cooperative scheme for decode-and-forward (DF) and amplify-and-forward (AF) protocols. The outage probability of the cooperative beamforming using (AF) scheme wa studied in [7] and compared with opportunistic relaying that choose the best relay between the relay and destination link. It investigates the performance gap between cooperative beamforming and opportunistic relaying and shows the SNR gap is $M!$ times. In addition, the outage probability of cooperative beamforming using the decode-and-forward (DF) scheme was also investigated in [8]. In particular, the approximation formula of the outage probability was provided in [10], where it using the best k out of M relays collaboratively transmit signal to destination. It provides that the outage performance of opportunistic cooperative beamforming using best two relays (OC-BF-2) which chooses from relay to destination link is almost indistinguishable from cooperative beamforming. Motivated by this research result, we major investigate outage probability that using best k out of M relays to transmit signal to the destination for (AF) protocol in this section. Our result shows that the performance gap can be reduced by using more best relays and the gap can reduced over half by adding more one relay node for AF protocol.

3.1 AF Protocol

Assume that there are total M relays in the network. In phase I, the source node broadcasts the signal to the relays and the destination. By choosing the best relay which channel quality is the best between relay to destination link. In the end of phase II, the receiver uses the channel information to combine the two phases signal by (MRC). The entire exact SNR is shown as

$$SNR_{O-R} = (1-\alpha)(\gamma\alpha_0) + \max_{i\epsilon(1,2\cdots M)} \frac{\alpha(1-\alpha)\gamma^2\alpha_i\beta_i}{1+(1-\alpha)\gamma\alpha_i+\alpha\gamma\beta_i} \tag{3.1}$$

To simplify the analysis in the high SNR regime, the second term in equation (3.1) can be expressed the harmonic mean random variable as

$$\frac{\alpha(1-\alpha)\gamma^2\alpha_i\beta_i}{1+(1-\alpha)\gamma\alpha_i+\alpha\gamma\beta_i} \sim \frac{\alpha(1-\alpha)\gamma^2\alpha_i\beta_i}{(1-\alpha)\gamma\alpha_i+\alpha\gamma\beta_i}$$

The outage probability is

$$\begin{aligned} P_{out} &= P\{SNR < 2^{(2R)}-1\} = P\{SNR < \delta\} \\ &\approx P\{\gamma\alpha_0 + \max_{i\epsilon(1,2\cdots M)} \frac{\alpha(1-\alpha)\gamma^2\alpha_i\beta_i}{(1-\alpha)\gamma\alpha_i+\alpha\gamma\beta_i} < \delta\} \\ &= \int_0^{\delta} P\{\max_{i\epsilon(1,2\cdots M)} Z_i < \delta - w\} f_W(w)dw \\ &= \int_0^{\delta} \{P\{Z_i < \delta - w\}\}^M f_W(w)dw \end{aligned} \tag{3.2}$$

where $\delta \triangleq 2^{(2R)}-1, Z_i \triangleq \frac{\alpha(1-\alpha)\gamma^2\alpha_i\beta_i}{(1-\alpha)\gamma\alpha_i+\alpha\gamma\beta_i}$ and $w \triangleq (1-\alpha)\gamma\alpha_i \sim Exp(\lambda_0)$ with pdf $f_W(w) = \lambda_0 e^{(-\lambda_0 w)}$, $(1-\alpha)\gamma\alpha_i \sim Exp(\lambda_1)$,$\alpha\gamma\beta_i \sim Exp(\lambda_2)$

From equation(3.2), we need obtain the probability density function(PDF) or cumulative distribution(CDF) of the harmonic random variables to calculate outage probability. The probability density function(PDF) or cumulative distribution(CDF) of the harmonic random variables is obtained by [11] and it are summarized bellow in theorem 1 and theorem 2.

Theorem 1. {*The CDF of Harmonic Mean of Two Exponential Random Variables [11]*}

Let the X_1 and X_2 be two independent random variables with the parameter λ_1 and λ_2 respectively, i.e. $X_i \sim Exp(\lambda_i), i = 1, 2$. Then the cumulative distribution function(CDF) of $X = \frac{X_1 X_2}{X_1 + X_2}$ is given by

$$F(x) = 1 - 2x\sqrt{\lambda_1\lambda_2} exp(-x(\lambda_1 + \lambda_2))K_1(2x\sqrt{(\lambda_1\lambda_2)}) \tag{3.3}$$

Theorem 2. {*The PDF of Harmonic Mean of Two Exponential Random Variables [11]*} *Let the X_1 and X_2 be two independent random variables with the parameter λ_1 and λ_2 respectively, i.e. $X_i \sim Exp(\lambda_i), i = 1, 2$. Then the probability density function(PDF) of $X = \frac{X_1 X_2}{X_1 + X_2}$ is given by*

$$f(x) = \lambda_1\lambda_2 x \exp(-x(\lambda_1 + \lambda_2))\{\frac{\lambda_1 + \lambda_2}{\sqrt{\lambda_1 \lambda 2}} K_1(2x\sqrt{\lambda_1\lambda_2}) + 2K_0(2x\sqrt{\lambda_1\lambda_2})\} \tag{3.4}$$

where the $K_0(\cdot)$ and $K_1(\cdot)$ are the zeroth-order modified Bessel function of the second kind and the first-order modified Bessel function of the second kind.

From [12], the function of $K_1(\cdot)$ can be approximated as $K_1(x) \approx \frac{1}{x}$ for the small x. Thus, we can approximate the CDF of the harmonic random variable as $F(x) \approx 1 - exp\{-x(\lambda_1 + \lambda_2)\}$ when x is small. Obviously, the harmonic mean random variable can be approximated as exponential random variable with parameter $(\lambda_1 + \lambda_2)$.

Substituting theorem 1 into the (3.2), the outage probability can be expressed as

$$
\begin{aligned}
P_{out,O-R} &= \int_0^{\delta} \{P\{Z_i < \delta - w\}\}^M f_W(w)dw \\
&= \int_0^{\delta} \{1 - 2(\delta - w)\sqrt{(\lambda_1\lambda_2)}e^{(-(\delta-w)(2(\lambda_1+\lambda_2)))}K_1(2(\delta - w)\sqrt{(\lambda_1\lambda_2)})\}^M f_W(w)dw
\end{aligned}
$$

Because $2(\delta - w)\sqrt{(\lambda_1\lambda_2)} = \frac{2(\delta-w)\alpha(1-\alpha)}{\gamma}$ is small in the high SNR regime when R is fixed, $K_1(x)$ can be approximated as $\frac{1}{x}$ when x is small. As a result, the harmonic mean random variables can be approximated as exponential random variables with parameter $(\lambda_1 + \lambda_2)$. Thus, the outage can be expressed as

$$
P_{out,O-R} \approx \int_0^{\delta} \{1 - e^{(-(\delta-w)(2(\lambda_1+\lambda_2)))}\}^M f_W(w)dw
$$

According to Binomial theorem,

$$
(x + y)^n = \sum_{k=0}^{n} C_k^n x^k y^{n-k} \tag{3.5}
$$

we can obtain the outage probability for opportunistic relaying is given by

$$
P_{out,O-R} = \sum_{i=0}^{M} C_i^M (-1)^i \lambda_0 \frac{e^{(-(\lambda_1+\lambda_2)\delta i)}}{(\lambda_0 - (\lambda_1 + \lambda_2)i)} \{e^{-\lambda_0-(\lambda_1+\lambda_2)\delta}\}. \tag{3.6}
$$

where Z_i is exponentially distributed with parameter $\lambda_1 + \lambda_2$. For simplify the following derivation, we define an order statistic set $\{Z_1, Z_2, \cdots, Z_M\}$ with order $Z'_M > Z'_{M-1} > \cdots > Z'_1$

3.1.1 Cooperative Beamforming (Co-BF)

Different from the opportunistic relaying that chooses the best relay from the relay to destination channel link, the cooperative beamforming uses all potential relays to help transmission in the phase II. The outage formula at the high SNR regime can be approximated as follow:

$$\begin{aligned} P_{out,Co-BF} &\approx P\{(1-\alpha)\gamma\alpha_0 + \sum_{i=1}^{M} Z_i < \delta\} \\ &= \int_0^{\delta} P\{V < \delta - w\} f_W(w)dw \\ &= \frac{(2\lambda)^M}{(M-1)!}\int_0^{\delta}\int_0^{\delta-w} v^{M-1}e^{-(\lambda_1+\lambda_2)v}dv f_W(w)dw \end{aligned} \tag{3.7}$$

where V is sum of M exponential random variables, and $V = \sum_{i=1}^{M} Z_i$ is the gamma distribution $\sim \Gamma(M, 1/(\lambda_1+\lambda_2))$ with pdf $f_v(v) = \frac{(\lambda_1+\lambda_2)^M v^{(M-1)} e^{-(\lambda_1+\lambda_2)v}}{(M-1)!}$. From [13], the integral formula is shown bellow:

$$\int_0^u x^n e^{-ax}dx = \frac{n!}{a^{n+1}} - e^{-ua}\sum_{k=0}^{n}\frac{n!}{k!}\frac{u^k}{a^{n-k+!}} \tag{3.8}$$

$$\int_0^a x^{b-1}(a-x)^{c-1}e^{\beta x}dx = B(c,b)a^{c+b-1}{}_1F_1(b; c+b; \beta a) \tag{3.9}$$

where $B(\alpha,\beta) = \frac{(\alpha-1)!(\beta-1)!}{(\alpha+\beta-1)!}$ is the Beta function and ${}_1F_1(\alpha;\beta;z) = \frac{z^{(1-\beta)}\int_0^z e^t t^{\alpha-1}(z-t)^{\beta-\alpha-1}dt}{B(\alpha,\beta-\alpha)}$ is the Confluent hypergeometric function.

Thus, we can obtain the outage probability of cooperative beamforming for AF protocol in high SNR approximation is give by

$$P_{out,Co-BF} = \{(1-e^{(-\lambda_0\delta)})\} - \lambda_0 e^{(-(\lambda_1+\lambda_2)\delta)}\sum_{k=0}^{M-1}\frac{(\lambda_1+\lambda_2)^k}{k!}Y_k \tag{3.10}$$

where $Y_k = \int_0^{\delta} e^{(\lambda w)}(\delta-w)^k dw$ and we can use equation (3.9) to express as

$$\begin{aligned} Y_k &= \int_0^{\delta} e^{(\lambda_0-(\lambda_1+\lambda_2)w)}(\delta-w)^k dw \\ &= B(k+1,1)\delta^{(k+1)}{}_1F_1(1; k+2; (\lambda_0-(\lambda_1+\lambda_2))\delta). \end{aligned} \tag{3.11}$$

Similar results for the above two cases at high SNR regime are also provided in [7] and it also emphasizes the performance gap between these two case are $M!$ times.

3.1.2 Opportunistic Cooperative Beamforming Using Two Relays (OC-BF-2)

Motivated by the research result that provided in [10], it shows that the outage probability uses best two relays which chooses from the relay to destination link can obtain the similar performance as cooperative beamforming in (DF) protocol. Here, we investigate the outage performance that chooses best two relays (OC-BF-2) for (AF) protocol and compare it with opportunistic relaying and cooperative beamforming case.

$$
\begin{aligned}
P_{out,Oc-BF-2} = &= P\{(1-\alpha)\gamma\alpha_0 + \max_{i,j}(Z_i + Z_j) < \delta\} \\
&= \int_0^{\delta} P\{(Z'_M + Z'_{M-1}) < \delta - w\} f_W(w)dw \qquad (3.12)
\end{aligned}
$$

From (3.12), we need to obtain the joint probability density function of Z'_M and Z'_{M-1} in the order statistics set $\{Z'_1, Z'_2....Z'_M\}$ where $Z_{(i)} < Z_{(j)}, i < j$. From [14], it provides that the joint probability density function for any order of i, and j.

Theorem 3. *Let $X_{(1)}, X_{(2)}.....X_{(n)}$ be the order statics of independent and identically distributed continuous random variables X_1, X_2,....X_n, with the common probability density function and probability distribution function f, F, respectively. Then for $x < y$, $f_{i,j}(x, y)$, the joint probability density function of $X_{(i)}$ and $X_{(j)}$ $(i < j)$, is given by*

$$
\begin{aligned}
f_{i,j}(x,y) &= \frac{n!}{(i-1)!(j-i-1)!(n-j)!} f(x)f(y) \\
&\times [F(x)]^{i-1}[F(y)-F(x)]^{j-i-1}[1-F(y)]^{n-j} \qquad (3.13)
\end{aligned}
$$

Substituting the theorem 3 into (3.12), the outage probability of OC-BF-2 is shown as follow

$$\begin{aligned} P_{out,Oc-BF-2} &\approx \int_0^{\delta}\int_0^{\frac{\delta-w}{2}}\int_{Z'_{M-1}}^{\delta-w-Z'_{M-1}} f_{Z'_{M-1},Z'_M}(Z'_{M-1},Z'_M)dZ'_{M-1}dZ'_M dw \\ &= \frac{M!}{(M-2)!}(\lambda_1+\lambda_2)\lambda_0 e^{-\lambda_0\delta}\{C-D\}. \end{aligned} \tag{3.14}$$

where C and D are given by

$$C = \sum_{p=0}^{M-2} C_p^{M-2}\frac{(-1)^p}{(\lambda_1+\lambda_2)(2+p)}\{\frac{e^{(\lambda_0\delta)}-1}{\lambda_0} - \frac{1-e^{(-\lambda_1+\lambda_2(1+p/2)-\lambda_0\delta)}}{\lambda_1+\lambda_2(1+p/2)-\lambda_0}\}$$

$$D = \begin{pmatrix} \frac{1}{(\lambda_1+\lambda_2-\lambda_0)^2} - e^{(-(\lambda_1+\lambda_2-\lambda_0)\delta)}\{\frac{\delta}{(\lambda_1+\lambda_2-\lambda_0)} + \frac{1}{(\lambda_1+\lambda_2-\lambda_0)^2}\} & ,q=0 \\ \sum_q^{M-2} C_q^{M-2}\frac{(-1)^q}{(\lambda_1+\lambda_2)q}\{\frac{1-e^{(-(\lambda_1+\lambda_2-\lambda_0)\delta)}}{(\lambda_1+\lambda_2-\lambda_0)} - \frac{1-e^{-(\delta t)}}{t}\} & ,q\neq 0 \end{pmatrix}$$

$$t = \{\tfrac{3}{2}(\lambda_1+\lambda_2)-\lambda_0\}q$$

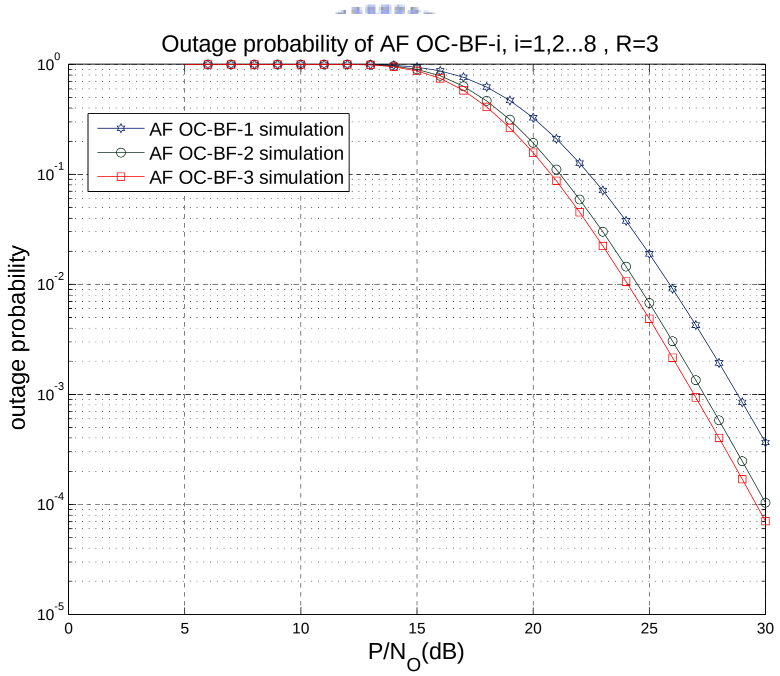

Figure 3.1: The comparison the outage performance between O-R,OC-BF-2 and Co-BF for AF protocol (M=3, R=3, $\alpha = 0.5$).

The outage probability with 2.5 dB gap between AF Co-BF and AF OC-BF-2 with $M = 8$ at $R = 3$. From Fig.3.1 and Fig.3.2, we can observe that the SNR gap between

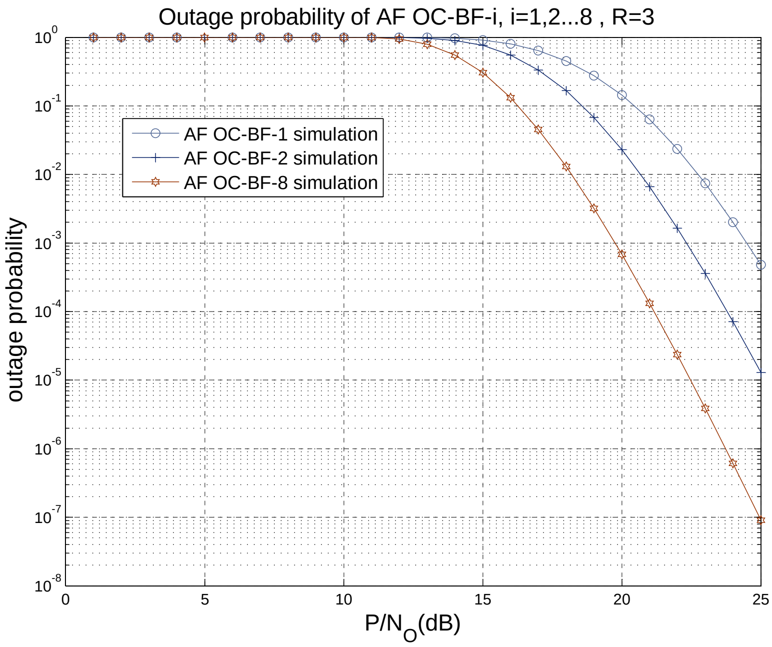

Figure 3.2: The comparison the outage performance between O-R, OC-BF-2 and Co-BF for AF protocol, (M=8, R=3,$\alpha = 0.5$).

CO-BF and OC-FB-2 will be larger when the total relay number becomes large.

3.1.3 Opportunistic Cooperative Beamforming Using i Relays (OC-BF-i)

From Fig. 3.1, it is obviously that uses more best relays can improve the outage performance. Although, the OC-BF-2 can improve the performance a lot in small M. However, When the M becomes large, the SNR gap between the OC-BF-2 and Co-BF becomes larger. According to the above observation, we investigates the outage performance for opportunistic cooperative beamforming using i relays to reduce the performance gap.

$$\begin{aligned} P_{out,Oc-BF-i} &= P\{(1-\alpha)\gamma\alpha_0 + \sum_{i=M-i-1}^{M} Z_i^{'} < \delta\} \\ &= \int_0^{\delta} P\{\sum_{i=M-i-1}^{M} Z_i^{'} < (\delta - w)\} f_W(w)dw \end{aligned} \tag{3.15}$$

From equation (3.15), we should obtain the CDF of sum of top i largest exponential random variables.

From [15], the CDF of sum of the top order statistics for independent exponential random variables is given in theorem 4.

Theorem 4. *Let the* $\{X_{(1)} < X_{(2)} \cdots < X_{(n)}\}$ *be the order statistics from n independent distributed exponential random variables with parameter* μ *and the partial top sum is defined as* $T_i = \sum_{j=i+1}^{n} X_{(j)}, 0 \le i \le n-1$. *The complementary cumulative density function (CCDF) of the* T_i *is given as follow:*

$$P\{T_i > t\} = \sum_{j=1}^{i} W_j e^{\{\frac{c_j}{c_{i+1}}\mu t\}} \frac{1}{(n-i-1)!} \int_0^{t\mu} e^{(d_j y)} y^{(n-i-1)} dy + \sum_{k=0}^{n-i-1} e^{(-\mu t)} \frac{(\mu t)^k}{k!}.$$

where $c_j = n - j + 1$, $d_j = \frac{i+1-j}{n-i}$ and $w_j = \frac{1}{n-j+1} \frac{n!}{(n-i)!} \frac{(-1)^{i-j}}{(j-1)!(i-j)!}$

Substituting theorem 4 into (3.15), the opportunistic cooperative beamforming using i relays can be shown as follow:

$$\begin{aligned} P_{out,Oc-BF-i} &= \int_0^{\delta} P\{T_{M-i-1} < (\delta - w)\} f_W(w) dw \\ &= \int_0^{\delta} \{1 - P\{T_{M-i-1} > (\delta - w)\}\} f_W(w) dw \qquad (3.16) \end{aligned}$$

where $w = (1-\alpha)\gamma\alpha_0 \sim Exp(\lambda_0)$ and $Z_i \sim Exp(\lambda_1 + \lambda_2)$.

The numerical simulation is given in Fig. 3.3. and it provides that the performance gap can be reduced by using more best relays. However, the gap is reduced less when using more best relays to help transmission. Thus, we should take suitable relay number to achieve the target of performance.

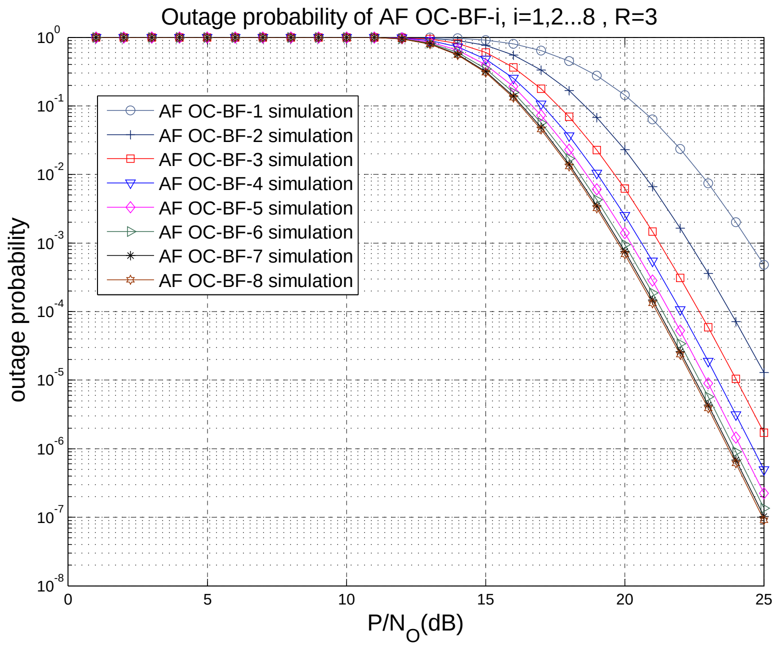

Figure 3.3: Outage performance of AF OC-BF-i, $(i = 1, 2 \cdots 8)$ (M=8, R=3, $\alpha = 0.5$).

3.2 DF Protocol

The outage probability for DF protocol is shown as

$$P_{out} = \sum_{i=0}^{M} P\{\frac{1}{2}log(1 + SNR) < R||\mathcal{D}(\mathcal{S})| = i\}P\{|\mathcal{D}(\mathcal{S})| = i\} \tag{3.17}$$

The $P\{|\mathcal{D}(\mathcal{S})| = i\}$ is the probability of decoding size, and $P\{\frac{1}{2}log(1+SNR) < R||\mathcal{D}(\mathcal{S})| = i\}$ is the conditional outage probability when the size of decoding set is equal to i. The probability of relay decoded correctly is given by

$$P_{relay_i} = P\{\frac{1}{2}log(1 + (1 - \alpha)\gamma\alpha_i) > R\} = e^{-\lambda_0\delta} \tag{3.18}$$

where $\alpha_i = |h_{s,r_i}|^2$ is exponential distribution with parameter λ_0. Thus, the probability of decoding set size can be given by

$$P(|\mathcal{D}(\mathcal{S})|) = C^{M}_{|\mathcal{DS}|}(e^{-\lambda_0\delta})^{|\mathcal{D}(\mathcal{S})|}(1 - e^{-\lambda_0\delta})^{M-|\mathcal{D}(\mathcal{S})|}. \tag{3.19}$$

To simplify the analysis, we only discuss the conditional outage probability in following case. The outage probability for DF protocol by equation(3.17).

3.2.1 Opportunistic Relaying (O-R)

The received SNR of opportunistic relaying case is given by

$$\begin{aligned} SNR_{O-R,DF} &= \frac{P_s|h_{s,d}|^2}{N_0} + \frac{\max_{i\epsilon\mathcal{D}(\mathcal{S})} P_r|h_{i,d}|^2}{N_0} \\ &= (1-\alpha)\gamma\alpha_0 + \alpha \max_{i\epsilon\mathcal{D}(\mathcal{S})} \gamma\beta_i \end{aligned}$$

To simplify the following discussion, we define the order static set $\{X_1, X_2, \cdots X_{|\mathcal{D}(\mathcal{S})|}\}$ with order $\{X'_{|\mathcal{D}(\mathcal{S})|} > X'_{|\mathcal{D}(\mathcal{S})|-1} > X'_2, \cdots > X'_1\}$ The conditional outage probability for opportunistic relaying is

$$\begin{aligned} P_{conditional,O-R} &= P\{SNR < \delta\} = P\{(1-\alpha)\gamma\alpha_0 + \alpha \max_{i\epsilon\mathcal{D}(\mathcal{S})} \gamma\beta_i\} \\ &= \int_0^\delta P\{X'_{\mathcal{D}(\mathcal{S})} < \delta - w\} f_W(w)dw \\ &= \sum_{i=0}^{|\mathcal{D}|} (-1)^i C_i^{|\mathcal{DS}|} \lambda_0 e^{-\lambda_2 \delta i} \{\frac{(1-e^{((\lambda_0-\lambda_2)k)\delta})}{(\lambda_0-\lambda_2)k}\} \end{aligned} \tag{3.20}$$

where $X_i = \alpha\gamma\beta_i, i\epsilon\mathcal{D}(\mathcal{S})$ and $X_i \sim Exp(\lambda_2)$,$\lambda_2 = 1/(\alpha\gamma)$

3.2.2 Cooperative Beamforming (Co-BF)

The received SNR of cooperative beamforming case is given by

$$\begin{aligned} SNR_{Co-BF,DF} &= \frac{P_s|h_{s,d}|^2}{N_0} + \frac{\sum_{i,j\epsilon\mathcal{D}(\mathcal{S})} P_r\{|h_{i,d}|^2\}}{N_0} \\ &= (1-\alpha)\gamma\alpha_0 + \alpha \sum_{i\epsilon\mathcal{D}(\mathcal{S})} \gamma(\beta_i) \end{aligned}$$

The conditional outage probability for cooperative beamforming is

$$
\begin{aligned}
P_{conditional,CO-BF} &= P\{(1-\alpha)\gamma\alpha_0 + \alpha \sum_{i\epsilon\mathcal{D}(\mathcal{S})} \gamma\{\beta_i\}\} \\
&= \int_0^{\delta} P\{\sum_{i\epsilon\mathcal{D}(\mathcal{S})} X_i < \delta - w\} f_W(w)dw \\
&= \int_0^{\delta} P\{Q < \delta - w\} f_W(w)dw \qquad (3.21)
\end{aligned}
$$

where Q is sum of $|\mathcal{D}(\mathcal{S})|$ exponential random variables and Q is the gamma distribution $\Gamma(|\mathcal{D}(\mathcal{S})|, 1/\lambda)$

3.2.3 Opportunistic Cooperative Beamforming Using Two Relays (OC-BF-2)

The received SNR of (OC-BF-2) is given by

$$
\begin{aligned}
SNR_{OC-BF,DF} &= \frac{P_s|h_{s,d}|^2}{N_0} + \frac{\max_{i\epsilon\mathcal{D}(\mathcal{S})} P_r\{|h_{r_i,d}|^2 + |h_{r_j,d}|^2\}}{N_0} \\
&= (1-\alpha)\gamma\alpha_0 + \alpha \max_{i\epsilon\mathcal{D}(\mathcal{S})} \gamma(\beta_i + \beta_j)
\end{aligned}
$$

The conditional outage probability for opportunistic relaying is

$$
\begin{aligned}
P_{conditional,OC-BF-2} &= P\{(1-\alpha)\gamma\alpha_0 + \alpha \max_{i\epsilon\mathcal{D}(\mathcal{S})} \gamma\{\beta_i + \beta_j\}\} \\
&= \int_0^{\delta} P\{X'_{|\mathcal{D}(\mathcal{S})|} + X'_{|\mathcal{D}(\mathcal{S})|-1} < \delta - w\} f_W(w)dw \qquad (3.22)
\end{aligned}
$$

By using theorem 3, we can obtain the conditional outage probability in equation (3.23).

3.2.4 Opportunistic Cooperative Beamforming Using i Relays (OC-BF-i)

The conditional outage of (OC-BF-i) is shown as

$$
\begin{aligned}
P_{conditional,OC-BF-i} &= P\{(1-\alpha)\gamma\alpha_0 + \alpha \sum_{|\mathcal{D}(\mathcal{S})-i-1|}^{|\mathcal{D}(\mathcal{S})|} \gamma\{\beta_i\}\} \\
&= \int_0^{\delta} P\{\sum_{k=|\mathcal{D}(\mathcal{S})-i-1|}^{|\mathcal{D}(\mathcal{S})|} X_k^{'} < \delta - w\} f_W(w)dw
\end{aligned} \tag{3.23}
$$

By using the theorem 4, we can obtain the $P\{\sum_{k=|\mathcal{D}(\mathcal{S})-i-1|}^{|\mathcal{D}(\mathcal{S})|} X_k^{'} > \delta - w\}$. Thus, substituting equation (3.23) into equation (3.17), the outage probability of (OC-BF-i) for DF protocol can be obtained.

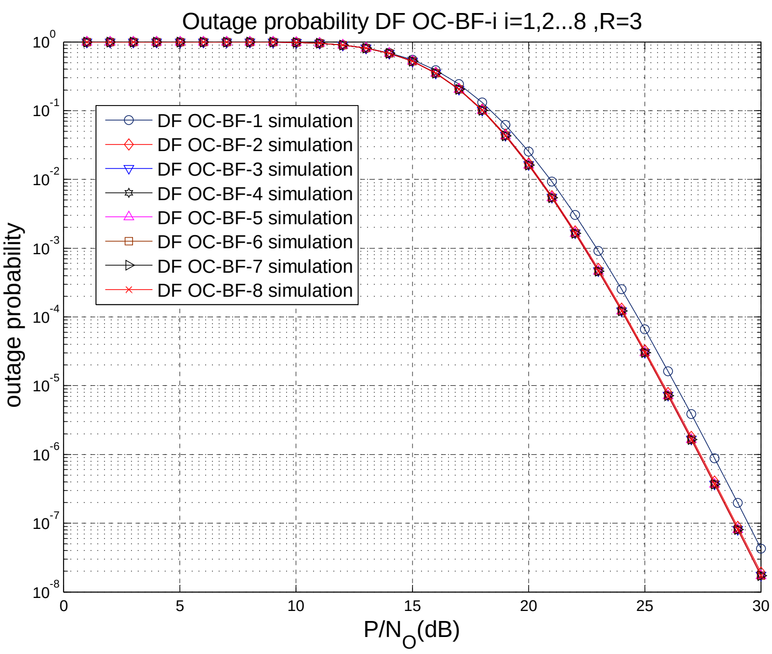

Figure 3.4: Outage performance of DF OC-BF-i, ($i = 1, 2 \cdots 6$) (M=8, R=3, $\alpha = 0.5$).

3.3 Cooperative Diversity and SNR Offset Analysis

From previous discussion of outage probability for DF and AF protocols, we will analyze cooperative diversity and SNR offset in OC-BF-i. The diversity gain can be defined as

$$\lim_{SNR\to\infty} \frac{\log\left(P_{out}\{(SNR,R)\}\right)}{\log\{(SNR)\}} = d \tag{3.24}$$

Thus, the outage probability in high SNR regime can be expressed as

$$P_{out} \approx C \cdot (SNR)^{-d} \tag{3.25}$$

where d is the diversity gain and C is the SNR offset

3.3.1 Analysis Diversity and Offset Gain for Opportunistic Cooperative Beamforming {OC-BF-i}

AF protocol:

From equation (3.16), the outage probability general from in OC-BF-i is sum of largest i out of M exponential random variables. Thus,

$$\begin{aligned} P_{out,Oc-BF-i} &= P\{\gamma\alpha_0 + \sum_{i=M-i-1}^{M} Z_i' < \delta\} \\ &= \int_0^{\delta} P\{T_{M-i-1} < (\delta - w)\} p_W(w) dw \end{aligned} \tag{3.26}$$

where $T_{M-i-1} = Z_M' + Z_{M-1}' + \cdots Z_{M-i-1}'$

From [15], we can obtain the joint probability density function for i out of M order statistics $f_{Z'_M Z'_{M-1} \cdots Z'_{M-i-1}}(Z'_M, Z'_{M-1}, \cdots Z'_{M-i-1})$

$$\begin{aligned}
&f_{Z'_M Z'_{M-1} \cdots Z'_{M-i-1}}(Z'_M, Z'_{M-1}, \cdots Z'_{M-i-1}) \\
&= \frac{M!}{(M-i)!}(\lambda_1+\lambda_2)^i e^{-(\lambda_1+\lambda_2)(Z'_M + Z'_{M-i-1} \cdots Z'_{M-i-1})}(1-e^{-(\lambda_1+\lambda_2)Z'_{M-i-1}})^{M-i}
\end{aligned}$$

where $Z'_M > Z'_{M-1} > \cdots > Z'_{M-i-1}$ and $U = \delta - w$

$$\begin{aligned}
&P\{Z'_M + Z'_{M-1} + \cdots Z'_{M-i-1} < U\} \\
&= \int_0^{\frac{U}{i}} \int_{Z'_{M-i-1}}^{\frac{U-Z'_{M-i-1}}{i-1}} \cdots \int_{Z'_{M-i}}^{U-Z'_{M-i-1}-Z'_{M-i}-\cdots Z'_M} f_{Z'_M Z'_{M-1} \cdots Z'_{M-i-1}}(Z'_M, Z'_{M-1}, \cdots Z'_{M-i-1}) \\
&dZ'_M dZ'_{M-1} \cdots dZ'_{M-i-1} \\
&= \int_0^{\frac{U}{i}} \int_{Z'_{M-i-1}}^{\frac{U-Z'_{M-i-1}}{i-1}} \cdots \int_{Z'_{M-i}}^{U-Z'_{M-i-1}-Z'_{M-i}-\cdots Z'_M} \\
&\times \frac{M!}{(M-i)!}(\lambda_1+\lambda_2)^i e^{-(\lambda_1+\lambda_2)(Z'_M + Z'_{M-i-1} \cdots Z'_{M-i-1})} \\
&\times (1-e^{-(\lambda_1+\lambda_2)Z'_{M-i-1}})^{M-i} dZ'_M dZ'_{M-1} \cdots dZ'_{M-i-1} \qquad (3.27)
\end{aligned}$$

We assume $\lambda = 1/\gamma$ and the the offset gain can be expressed as

$$\begin{aligned}
&\lim_{SNR \longrightarrow \infty} \frac{P_{out}}{SNR^{-(M+1)}} = \lim_{\lambda \longrightarrow 0} \frac{P_{out}}{\lambda^{M+1}} \\
&= \lim_{\lambda \longrightarrow 0} \frac{\int_0^\delta P\{Z'_M + Z'_{M-1} + \cdots Z'_{M-i-1} < (\delta - w)\} \lambda_0 e^{-\lambda_0 w} dw}{\lambda^{M+1}} \\
&= \int_0^\delta \lim_{SNR \longrightarrow \infty} \frac{P\{Z'_M + Z'_{M-1} + \cdots Z'_{M-i-1} < (\delta - w)\} \lambda_0 e^{-\lambda_0 w} dw}{\lambda^{M+1}} \\
&= \int_0^\delta \lim_{SNR \longrightarrow \infty} \int_0^{\frac{U}{i}} \int_{Z'_{M-i-1}}^{\frac{U-Z'_{M-i-1}}{i-1}} \cdots \int_{Z'_{M-i}}^{U-Z'_{M-i-1}-Z'_{M-i}-\cdots Z'_M} \\
&\times \frac{M!}{(M-i)!}(\lambda_1+\lambda_2)^i e^{-(\lambda_1+\lambda_2)(Z'_M + Z'_{M-i-1} \cdots Z'_{M-i-1})} \\
&\times (1-e^{-(\lambda_1+\lambda_2)Z'_{M-i-1}})^{M-i} dZ'_M dZ'_{M-1} \cdots dZ'_{M-i-1} \lambda_0 e^{-\lambda_0 w} dw / \lambda^{M+1}
\end{aligned}$$

Submitting joint probability into equation(3.28), the above equation can be reduced by L'Hospital's rule. Thus, the offset gain formula can be expressed as

$$\int_0^{\delta}\int_0^{\frac{U}{i}}\int_{Z'_{M-i-1}}^{\frac{U-Z'_{M-i-1}}{i-1}}\cdots\int_{Z'_{M-i}}^{U-Z'_{M-i-1}-Z'_{M-i}-\cdots Z'_{M}}$$
$$\times\frac{M!}{(M-i)!}(\lambda_1+\lambda_2)^M(Z'_{M-i-1})^{M-i}dZ'_M dZ'_{M-1}\cdots dZ'_{M-i-1}\lambda_0 dw \quad (3.28)$$

In order to change the range of integral in equation (3.28), we set

$$\left\{\begin{array}{c} t_M = Z'_M - Z'_{M-1} \\ t_{M-1} = Z'_{M-1} - Z'_{M-2} \\ \vdots \\ t_{M-i-1} = Z'_{M-i-1} \end{array}\right\} \quad (3.29)$$

$$\int_0^{\delta}\int_0^{\frac{U}{i}}\int_{Z'_{M-i-1}}^{\frac{U-Z'_{M-i-1}}{i-1}}\cdots\int_{Z'_{M-i}}^{U-Z'_{M-i-1}-Z'_{M-i}-\cdots Z'_{M}}$$
$$\times\frac{M!}{(M-i)!}(\lambda_1+\lambda_2)^M(Z'_{M-i-1})^{M-i}dZ'_M dZ'_{M-1}\cdots dZ'_{M-i-1}\lambda_0 dw$$
$$=\int_0^{\delta}\int_0^{\frac{U}{i}}\int_0^{\frac{U-(M-i-1)t_{(M-i-1)}}{i-1}}\cdots\int_0^{U-(M-i-1)t_{(M-i-1)}-(M-i)t_{(M-i)}-\cdots(M)t_{(M)}}$$
$$\times\frac{M!}{(M-i)!}(\lambda_1+\lambda_2)^M(t_{M-i-1})^{M-i}dt_M dt_{M-1}\cdots dt_{M-i-1}\lambda_0 dw$$

By using the formula in Appendix A, we can obtain the offset gain as

$$\lim_{SNR\longrightarrow\infty}\frac{P_{out}}{SNR^{-(M+1)}}=\int_0^{\delta}\frac{(\delta-w)^M}{i!i^{M-i}}(\lambda_1\lambda_2)^M\lambda_0 dw$$
$$=\frac{(\lambda_1+\lambda_2)^M\lambda_0}{i!i^{M-i}}\frac{\delta^{M+1}}{M+1}=\{\frac{1}{(1-\alpha)}\}\{\frac{1}{1-\alpha}+\frac{1}{\alpha}\}^M\frac{(\lambda\delta)^{M+1}}{i!i^{M-i}}\frac{1}{M+1} \quad (3.30)$$

From above expression, we can obtain the diversity $M+1$ and offset gain $\{\frac{1}{(1-\alpha)}\}\{\frac{1}{1-\alpha}+\frac{1}{\alpha}\}^M\frac{(\delta)^{M+1}}{i!i^{M-i}}\frac{\delta^{M+1}}{M+1}$.

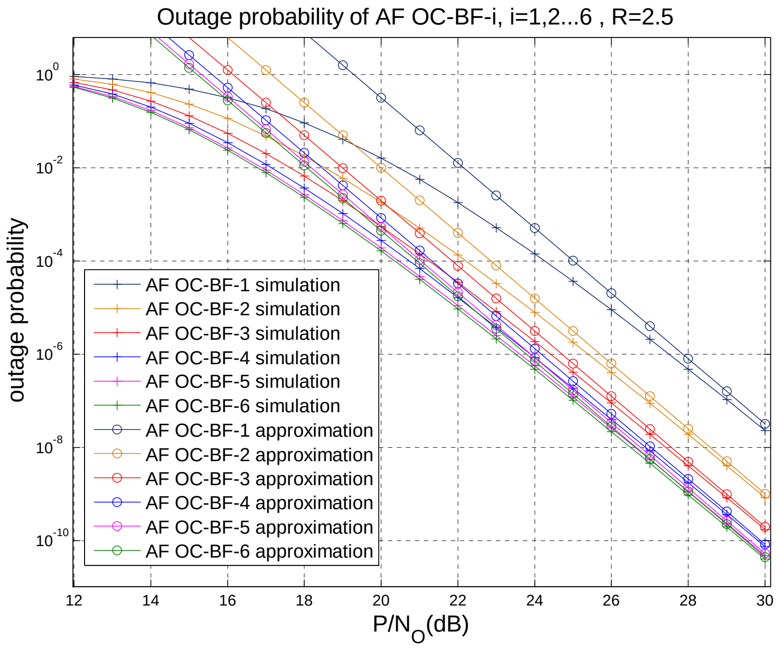

Figure 3.5: Offset and diversity analysis for AF protocol, (M=6, R=2.5, $\alpha = 0.5$)

DF protocol:

Because the conditional outage for DF protocol is similar as outage probability for AF protocol. By using the similar analysis method, we can obtain the SNR offset in high SNR regime can be obtain as

$$\sum_{|\mathcal{D}(\mathcal{S})|} \frac{C^M_{|\mathcal{D}(\mathcal{S})|}}{|\mathcal{D}(\mathcal{S})|+1} \{\frac{1}{1-\alpha}\}\{\frac{1}{\alpha}\}^M (\frac{\delta}{\gamma})^{M+1} \frac{1}{i! i^{|\mathcal{D}\mathcal{S}|-i}} \tag{3.31}$$

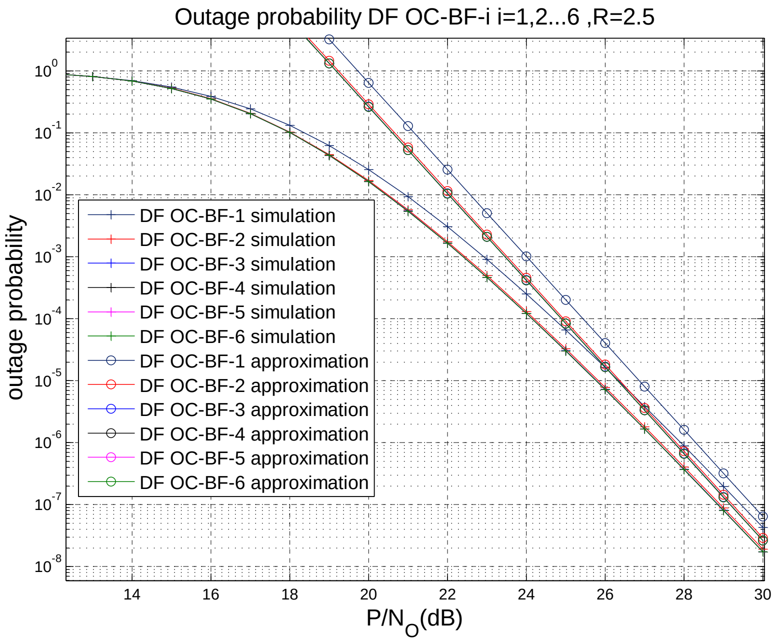

Figure 3.6: Offset and diversity analysis for DF protocol, (M=6, R=2.5, $\alpha = 0.5$)

Chapter 4

Cooperative ARQ Scheme for Decode-and-Forward and Amplify-and-Forward Protocols

We extend the results in the previous section to develop ARQ schemes for DF and AF protocols, and analyze the outage performance for corresponding cooperative ARQ scheme. Here, we refer to the broadcasting phase and the relay phase as the cooperative phase, and the ARQs that follow, as the ARQs phases. Here, we proposed two phase (TP) and one phase (OP) ARQ scheme for DF and AF protocols. The two phase ARQ scheme is means that the cooperative phase with source broadcasting phase and relaying phase. Another one in cooperative phase only has source broadcasting phase without relaying phase. Beside, we also proposed several types for DF and AF protocols and the detail discussion is given in following section. The following type of cooperative ARQ scheme classify as shown in Fig4.1

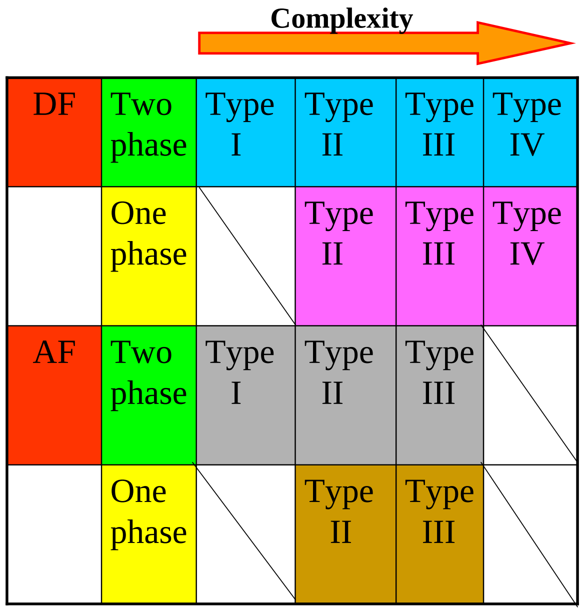

Figure 4.1: The classify of cooperative ARQ scheme

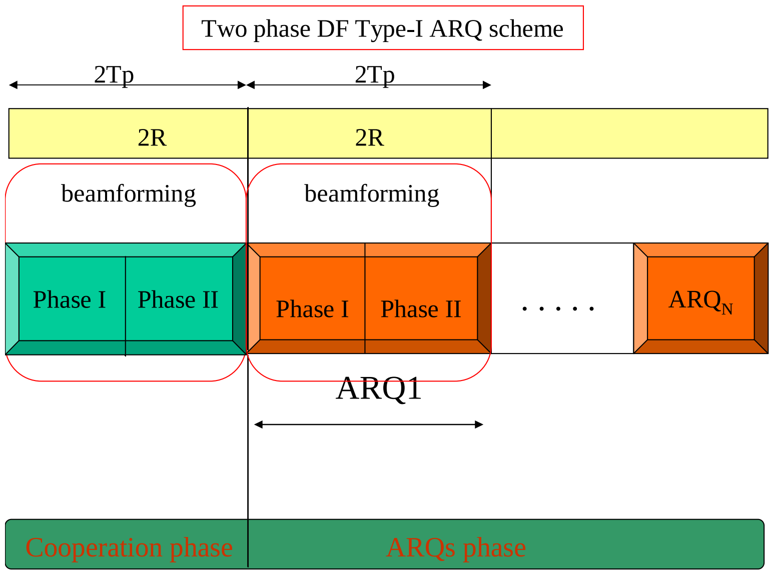

Figure 4.2: TP DF Type-I ARQ scheme

4.1 TP DF ARQ Scheme

The cooperative phase obtains source broadcasting phase and the relaying phase. The cooperative phase takes two times transmission time to transmit signal to destination. At the destination, the receiver uses (MRC) to combine two phase signals.

4.1.1 TP DF Type-I ARQ Scheme

To reduce the complexity of relay, we assume that the relay is memoryless. Each relay does not keep the received signal in previous phase. Thus, each ARQ phase including source broadcasting phase and relaying phase. Especially, the cooperative phase and each ARQ phase take two times transmission. At the destination, the receiver use MRC to combine two phase signal. The TP DF type-I ARQ scheme is proposed as in Fig.4.2

Because the channel between each ARQ phase and cooperative phase are independent. Thus, the outage probability after N times ARQ phases is given by

$$P_{out,N} = \{P_{out}\}^{(N+1)} \tag{4.1}$$

where P_{out} is the outage probability for cooperative phase with two phase combing equation(3.17).

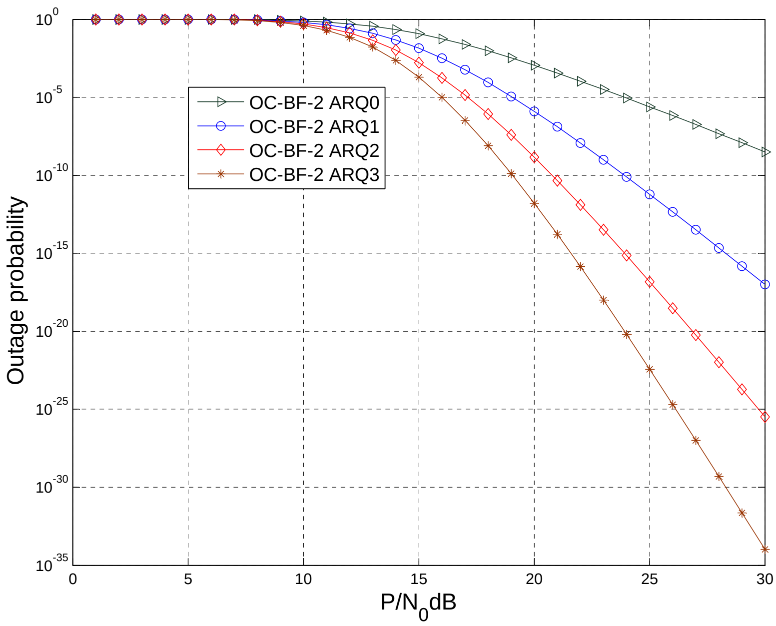

Figure 4.3: Outage probability for TP DF type-I OC-BF-2 in different ARQ times, $[M = 5, R = 2, \alpha = 0.5]$

4.1.2 TP DF Type-II ARQ Scheme

The cooperative phase includes the broadcasting phase and relaying phase so it takes two times transmission time (T_p). In the broadcasting phase, the source broadcasts the signal to all relays and destination. In relaying phase, the decoded correctly relay transmits signal to destination and the receiver uses the MRC to combine two phase received signal at the destination. Different from DF type-I scheme, the relays can keep the information in cooperative phase. Thus , each ARQ phase uses the same relay as the cooperative phase to re-transmission as shown in Fig.4.4.

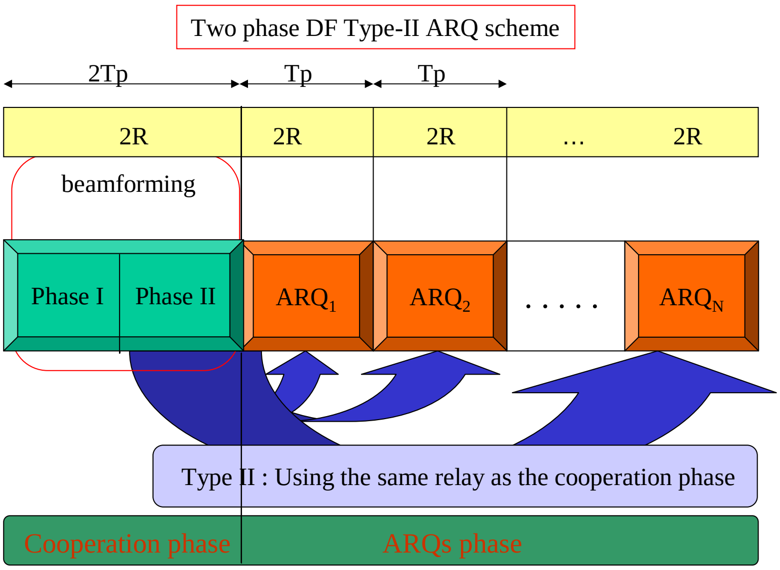

Figure 4.4: TP DF type-II ARQ scheme

Now, we describes the situation of different size of decoding set. If $|\mathcal{D}(\mathcal{S})| > 1$, it is simply equal to relaying phase of OC-BF, but in sequel ARQs phase it uses the same relay as the relaying phase of cooperation phase. If not, the source will re-broadcast the signal in each ARQ phase, until the $|\mathcal{D}| \geq 1$. Then, the following ARQ phase will select the relay nodes and retransmits signal in the similar way as relaying phase. The relay nodes in each ARQ is the same as cooperative phase. Especially, the cooperative phase includes two phases and each ARQ phase includes one phase transmission.